



## Multiproduct firms and scope adjustment in globalization

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### ABSTRACT

A model of heterogeneous firms with variety-specific fixed costs is developed and analyzed to study how multiproduct firms respond to globalization. In contrast with most existing models, the analysis demonstrates that more-productive firms may expand their product scope, which in turn may push up their average costs. A necessary and sufficient condition for scope expansion is that the fixed cost of introducing more varieties increases rapidly with the product scope. With increasing globalization, the percentage of scope-expanding firms diminishes and eventually becomes zero.

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### 1. Introduction

Multiproduct firms dominate production and export in modern economies. In the U.S., 41 percent of manufacturing firms produce more than a single product, but they account for 91 percent of U.S. manufacturing output and 94 percent of U.S. exports. Twelve percent of U.S. firms export more than five products to more than five destinations, and they account for more than 90 percent of export value (Bernard et al., 2010, 2012). Similar observations apply to other countries. Several recent studies have extended the Melitz (2003) firm heterogeneity model to multiproduct firms and pointed out that productivity may differ not only across firms, but also within each firm across various products. Despite diverging focuses and different modeling techniques, almost all those models concluded that multiproduct firms invariably reduce their product scope in response to trade liberalization (Bernard et al., 2011; Eckel and Neary, 2010; Mayer et al., 2011).<sup>1</sup> The logic is simple: Just like the least productive firms (in any single-product firm model) are forced to exit in the face of trade liberalization, the least productive products within each multiproduct firm should also be dropped.

Although there is a consensus among existing theoretical papers about the effects of trade liberalization on product scope, the empirical evidence is much more nuanced. In many cases, scope adjustment was found to depend on firm productivity. After the eurozone was established in 1999, the most productive French firms expanded their export scope while less-productive firms reduced it (Berthou and Fontagne, 2013). With the introduction of the North American Free Trade Agreement, many Mexican firms developed new products for export (Iacovone and Javorcik, 2010). Tariff reduction in Canada induced small or non-exporting Canadian firms to reduce their scope, but had no effect on large or exporting firms (Baldwin and Gu, 2009).

Looking back at existing theoretical models more closely, one realizes that the analogy between within-firm rationalization and cross-firm rationalization hinges on the premise that a multiproduct firm's least productive products are also the least productive in the whole industry. This premise, in turn, is derived from a seemingly innocuous assumption that the cost of introducing new products stays constant as a firm expands. Ample evidence, however, indicates within-firm diminishing returns to product development. Empirical studies have shown that the number of patents or innovations per dollar of R&D investment declines as a firm grows larger or a firm's R&D expenditure increases (Scherer, 1980; Bound et al., 1982; Acs and Audretsch, 1991), and that small firms account for a disproportionately large number of innovations relative to their size (Scherer, 1965; Pavitt et al., 1987; Acs and Audretsch, 1988). Cohen and Klepper (1992, 1996) summarized these findings into "the stylized fact that the average

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<sup>1</sup> There are two exceptions. Nocke and Yeaple (2008) found that smaller firms expand the number of product lines while larger firms reduce it. Feenstra and Ma (2008) showed numerically that, on average, firms expand product scope.

productivity of R&D, measured in terms of the number of patents or innovations per dollar of R&D spending, tends to be lower for larger firms even though R&D tends to increase proportionately with firm size.<sup>2</sup>

As it turns out, the efficiency of product development plays an important role in multiproduct firms' scope choice. We will show in this paper that if new products are increasingly costly to introduce, more-productive firms may expand product scope in response to globalization. This is in sharp contrast with most existing papers which predict unanimous scope contraction. Consider the following scenario. A firm can produce its core product with the productivity that it draws, but incurs a fixed cost of introducing every additional variety, which we call *variety-introduction fee*.<sup>3</sup> Inspired by the above-mentioned empirical findings about R&D efficiency, we assume that the variety-introduction fee rises within each firm as it adds more and more varieties. Furthermore, assume that a variety's marginal cost of production rises as it moves further away from a firm's core competence—the familiar core competence approach (for example, Eckel and Neary, 2010). Given declining production efficiency and rising variety-introduction fee, a firm's optimal scope is then reached when the (gross) profit of its least efficient variety, dubbed the *marginal variety*, is just enough to cover its introduction fee. It then becomes clear that a firm expands its scope in response to globalization if and only if globalization raises its marginal variety's profit.

A high-productivity firm can generate a higher profit for each variety than a low-productivity firm for the corresponding variety. This implies that the former will maintain a larger product scope, and therefore production of its marginal variety must be more efficient in order to cover the greater introduction fee it incurs due to its larger scope. In other words, the efficiency of a firm's marginal variety must increase with the firm's productivity. In the linear demand system considered in the main model, globalization raises a variety's profit if and only if the variety is sufficiently efficient (Melitz and Ottaviano, 2008). Although a firm's marginal variety is its least efficient one, it may nevertheless be quite efficient as compared to the industry's average variety if the firm itself is very productive. In that case, the profit of the marginal variety may increase after globalization, and as a result the firm will expand its product scope. Therefore, scope expansion is related to the speed at which the variety-introduction fee rises. If the variety-introduction fee stays constant or increases only slowly, a firm's marginal variety can never be very efficient and consequently the firm will never expand its scope after trade liberalization. The results and insights are obtained from a model with linear demand, but the analysis will show that they also hold for CES preferences.

Thus, a steeply rising variety-introduction fee is necessary and sufficient for scope expansion (by more-productive firms) in response to globalization. Such a condition is absent in all existing models of

multiproduct firms, and that is why scope expansion has never been predicted. If there is no variety-introduction fee, as Eckel and Neary (2010) and Mayer et al. (2011) have assumed, every firm will extend its product scope down to the point where the marginal variety generates zero profit. This means that all firms' marginal varieties are equally inefficient, and they together constitute the industry's least-efficient varieties. Because trade liberalization reduces the profits of the less-efficient varieties, all firms drop their marginal varieties, i.e., they all reduce their product scopes in response to liberalization. With CES preferences, Bernard et al. (2011) assumed constant entry cost for each variety, while Arkolakis and Muendler (2010) allowed the entry cost to vary with scope. However, the variety-level entry costs in those two studies are market specific, while the variety-introduction fee in our model is assumed to be specific to a variety for all markets. This will lead to different predictions.<sup>4</sup>

In addition to the slope of variety-introduction fee, we have identified a number of other factors which may affect the pattern of scope adjustment. Scope expansion is more likely when cross-firm heterogeneity is skewed towards low productivity, when within-firm heterogeneity is smaller, or when the market size is smaller. The analysis also predicts that globalization always induces scope expansion (by the more-productive firms) initially, but further globalization subsequently reduces the percentage of firms that expand their scope, and eventually all firms reduce scope. When a firm expands its scope in response to globalization, the newly added varieties are farther from its core competence and therefore less efficient than its existing varieties. This may raise the firm's average cost. Thus, by showing the possibility of scope expansion and identifying its necessary and sufficient conditions, this study provides a more complete picture of multiproduct firms' behavior in globalization. It also generates novel predictions that can be tested in future empirical investigations.

Several recent studies have addressed multiproduct firms' scope choices and within-firm rationalization in response to trade liberalization. Bernard et al. (2011) assumed heterogeneous production efficiency both among firms and among each firm's varieties. They found that trade liberalization raises the wage rate, which squeezes the least-productive firms in the industry as well as the least-efficient varieties in each firm, so all firms reduce their scope. Eckel and Neary (2010) also found scope reduction by all firms in response to globalization using a model allowing for cannibalization on the demand side and within-firm diminishing efficiency on the supply side.<sup>5</sup> Mayer et al. (2011) explained how tougher competition induces firms to focus on more successful products, leading to a more skewed product mix as well as a smaller product scope. Their focus is mainly on how competition intensity affects the product mix rather than how trade liberalization affects scope adjustment.<sup>6</sup> As has been explained earlier, all these researchers found scope contraction because they all assumed no variety-introduction fee, which is the distinguishing feature of this study.<sup>7</sup>

<sup>2</sup> Declining R&D efficiency is commonly explained by the inefficiency of large organizations. Such a rationale has also been recognized in other academic fields and comes under different names such as “core rigidity” (Leonard-Barton, 1992), “incumbent's curse” (Chandy and Tellis, 2000) and “organizational inertia” (Tripsas and Gavetti, 2000). Note that R&D efficiency is about the link between R&D input and output. If we look at R&D output, there is also a clear pattern that large firms underperform relative to their size. This can be explained by either lower R&D incentives or lower R&D productivity by large firms. Henderson (1993) disentangled these two effects using a unique field data of the photolithographic alignment equipment industry, and concluded that “while established firms invested substantially more in research than entrants did, they were significantly less effective in their efforts to bring products based on major innovation to commercial success.” Henderson (1993) also cited anecdotal evidence in which established industry leaders such as General Electric, IBM, and DEC failed to introduce the next generation products despite extensive experience and heavy investment in product development. See also Henderson and Clark (1990).

<sup>3</sup> The most straightforward interpretation of the fixed cost is an R&D expenditure or customization cost, but it can also be understood as a management cost that rises due to limited internal resources for managing multiple products. Such internal resource is called the span of control by Lucas (1978), knowledge capital by Klette and Kortum (2004), organizational ability by Maksimovic and Phillips (2001), and organizational capital by Santalo (2001) in the industrial organization literature and by Nocke and Yeaple (2008) in the trade literature.

<sup>4</sup> A variety-introduction fee is more like a firm's investment in product R&D, while market-specific entry cost is more like advertising expense for each variety in each market (Bernard et al., 2011). It is this difference which leads to scope expansion (by more-productive firms) in our model but scope reduction in all existing models. The reason will become clear in Section 5.

<sup>5</sup> Their main model assumes no entries, homogenous firms and Cournot competition, but they show that the result continues to hold with free entry and heterogeneous firms.

<sup>6</sup> Unlike Bernard et al. (2011) and Eckel and Neary (2010) who found scope reduction unconditional, Mayer et al. (2011) were more cautious: “We do not emphasize these results for the extensive margin, because they are quite sensitive to the specifications of fixed production and export costs.”

<sup>7</sup> As in this study, Nocke and Yeaple (2008) also found that some firms may expand scope in response to trade liberalization. They assumed that a firm produces all its varieties at a common unit cost, which increases with the firm's scope, but the speed of the increase depends on the firm's organizational capabilities. Such an assumption generates a negative correlation between intensive and extensive margins, a result which contradicts empirical findings and all other theoretical models. The negative correlation led Nocke and Yeaple (2008) to find that smaller firms (i.e., those with lower output and fewer product lines) expand scope in response to trade liberalization. This study, by contrast, predicts a positive correlation between intensive and extensive margins, so the expanding firms are those with larger sales and greater scope.

Analyses of trade liberalization's impact on scope need to model how the optimum scope is determined. Scope choices can be constrained either on the demand side through cannibalization or on the supply side through diseconomies of scope. On the demand side, Baldwin and Gu (2009), Dhingra (2011), Eckel and Neary (2010) and Feenstra and Ma (2008) all emphasized cannibalization, where new varieties reduce the demand for a firm's existing varieties. Although it is important in oligopolistic industries, cannibalization tends to produce a negative relationship between extensive and intensive margins, which seems at odds with the empirical findings about most industries (e.g., Arkolakis and Muendler, 2010; Bernard et al., 2011; Iacovone and Javorcik, 2010). Furthermore, scope reduction is found whether or not cannibalization is considered, and therefore does not seem to be driven by cannibalization. Partly for this reason, this study focuses on the supply linkage by assuming away cannibalization.

On the supply side, diseconomies of scope postulate that a firm stops extending its scope as the profits of extra varieties diminish and/or the cost of adding new varieties rises. Bernard et al. (2010, 2011), Eckel and Neary (2010), and Mayer et al. (2011) have all assumed declining profit for additional varieties as a firm departs from its core competence (due to declining consumer tastes or declining production efficiency) or variety-specific random shocks. Bernard et al. (2011) and Arkolakis and Muendler (2010) assumed a market-specific entry cost which was constant or varied with scope. Dhingra's (2011) investment in product R&D is a variety-specific introduction fee. However, scope never expands in her model because the R&D investment does not vary with the product scope (cannibalization may also play a role).

Most empirical studies of multiproduct firms have found significant impacts of trade liberalization on firms' scope choices.<sup>8</sup> Although many claimed to find scope-reduction in response to globalization, the empirical results are subject to different interpretations and are by no means inconsistent with the idea that more-productive firms may expand scope. There is also some direct evidence consistent with the findings of this study which cannot be explained by existing theories. Berthou and Fontagne (2013) found that after the eurozone was established in 1999 the most productive French firms increased the number of products exported to eurozone destinations while less-productive French firms concentrated their exports on a smaller range of product lines. Iacovone and Javorcik (2010) have documented how a substantial number of Mexican firms developed new products for export as a response to improved access to foreign markets. Baldwin and Gu (2009) found that tariff cuts in Canada between 1973 to 1997 induced scope contraction by small or non-exporting Canadian firms, but had no effect on large or exporting firms. Bernard et al. (2011) have demonstrated that U.S. firms exposed to more tariff reductions under the Canada–U.S. Free Trade Agreement reduced the number of products they produce relative to firms exposed to fewer tariff reductions. This finding represents the average response by all firms in an industry and does not preclude the possibility that some firms may have actually expanded their scope.<sup>9</sup> Goldberg et al. (2010) have shown that during the 1989–2003 period when profound trade and other reforms took place in India, Indian firms added more products than they dropped, and

<sup>8</sup> Using French data, Mayer et al.'s (2011) empirical investigation focused mainly on the correlation between export destination characteristics and within-firm skewness in export sales. This lends support to the idea of endogenous and variable markup but says little about scope adjustment in response to trade liberalization. Dhingra (2011) showed that in Thailand during 2003–2006, less export-oriented domestic firms increased their product lines in response to a unilateral tariff cut while more export-oriented domestic firms reduced their product lines. But that trade liberalization was unilateral, which can have impacts very different from those of bilateral trade liberalization.

<sup>9</sup> In a previous empirical study, Bernard et al. (2010) demonstrated that high efficiency U.S. manufacturing firms increased scope between 1987 and 1997, while low efficiency firms reduced it. They attributed that finding to idiosyncratic shocks to individual products' productivity, but the pattern is certainly consistent with the result of this study if trade liberalization played an important role in affecting firms' scope choices during that period.

the dropping was unrelated to tariff reduction.<sup>10</sup> Clearly, the previous empirical findings have been far from complete or conclusive. The results of this study point out the possibility of and the conditions for scope expansion, and thus enrich the set of predictions for further empirical investigations.

The organization of the paper is as follows. We set up the main model in Section 2 with linear demand system, and analyze the impact of globalization in Section 3, which establishes the key results of the paper. Section 4 contains some comparative statics results using specific cost distributions. Finally, in Section 5 we discuss CES preferences to demonstrate the robustness of the findings. All proofs are collected in the Appendix A.

## 2. Model

We first describe consumer preferences and production technology, and how they determine firms' optimal decisions, including about product scope. The industry equilibrium can then be derived, which prepares for analysis in the next section of the impact of globalization on multiproduct firms' choices, especially with respect to product scope.

### 2.1. Preference and demand

Assume fully integrated world economy inhabited by  $L$  identical consumers, each having a linear-quadratic preference (à la Melitz and Ottaviano, 2008) for a continuum of differentiated goods indexed by  $j \in \Omega$ , and a homogeneous good chosen as numeraire:

$$U = \check{q}_0 + \alpha \int_{j \in \Omega} \check{q}_j dj - \frac{1}{2} \beta \left( \int_{j \in \Omega} \check{q}_j dj \right)^2 - \frac{1}{2} \gamma \int_{j \in \Omega} \check{q}_j^2 dj,$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are positive constants,  $\Omega$  is the set of all differentiated products supplied to the market, and  $\check{q}_0$  and  $\check{q}_j$  are individual consumptions of the numeraire and product  $j$ , respectively. The parameter  $\gamma$  captures the degree of product differentiation among the differentiated goods. In particular,  $\gamma = 0$  corresponds to perfect substitution, and a larger  $\gamma$  indicates greater differentiation.

Given a market price  $p_j$  for product  $j$ , individual consumption choice gives rise to the market demand for product  $j$ ,  $q_j$  ( $q_j = L\check{q}_j$ ), as

$$p_j = \alpha - \frac{\beta}{L} \int_{j \in \Omega} q_j dj - \frac{\gamma}{L} q_j.$$

Let the aggregate price of all products be  $P \equiv \int_{j \in \Omega} p_j dj$ . Then  $P = \left( \alpha - \frac{\beta}{L} \int_{j \in \Omega} q_j dj \right) M - \frac{\gamma}{L} \int_{j \in \Omega} q_j dj$ , where  $M \equiv \int_{j \in \Omega} dj$  is the total number of products. As a result,  $\frac{\beta}{L} \int_{j \in \Omega} q_j dj = \frac{\alpha M - P}{\gamma + M\beta}$ , and the demand for product  $j$  becomes

$$p_j = A - \frac{\gamma}{L} q_j, \quad \text{where } A \equiv \frac{\alpha\gamma + \beta P}{\gamma + \beta M}. \quad (1)$$

### 2.2. Production and technologies

Assume that labor is the only factor of production and that it is provided by the  $L$  consumers. The numeraire good is produced under constant returns to scale at unit cost and is supplied competitively, which implies a unit wage. There is a continuum of potential firms attempting to enter the differentiated goods industry. They enter by paying an entry fee,  $f > 0$ . After sinking the entry fee, each firm randomly draws its productivity from a cumulative distribution  $G(x)$  on  $[0, X]$ , with smaller  $x$  indicating greater productivity. Since

<sup>10</sup> They attributed the discrepancy between their findings and the predictions of prevailing theories to regulations in India which prevented optimal allocation of resources.

there may be multiple firms that draw the same  $x$ , “firm  $x$ ” will indicate any firm that has productivity level  $x$ , as all firms with the same productivity will behave identically.

After drawing its productivity, a firm can produce a product, dubbed its core competence or core variety/product, at constant unit cost, which is assumed to be  $x$  for firm  $x$ . In this paper, the terms “product” and “variety” will be used interchangeably. In addition to its core variety, a firm can introduce new varieties at some extra fixed cost, which can be understood as the cost for research and development, customization or management (Dhingra, 2011; Klette and Kortum, 2004). Denote the fixed variety-introduction fee as  $h(v) \geq 0$  for the  $v$ th variety and assume  $h'(v) \geq 0$ . If  $h'(v) \equiv 0$ , the fee for introducing a new variety is constant no matter how many varieties a firm already has. If  $h'(v) > 0$  (for  $v > 0$ ), varieties are increasingly costly to introduce.<sup>11</sup> Note that  $h(v)$  does not depend on  $x$ , so all firms face the same fee schedule.<sup>12</sup>

Producing extra varieties may not be as efficient as producing the core product (Eckel and Neary, 2010; Mayer et al., 2011). Suppose that a firm's varieties are indexed with decreasing production efficiency such that the unit production cost of firm  $x$ 's  $v$ th variety is  $c(x, v)$ , with  $c_x \equiv \frac{\partial c(x,v)}{\partial x} > 0$ ,  $c_v \equiv \frac{\partial c(x,v)}{\partial v} \geq 0$ , and  $c(x, 0) = x$  (so variety  $v = 0$  is the firm's core variety). If  $c_v \equiv 0$ , firm produces all its varieties with equal efficiency.<sup>13</sup>

### 2.3. Firm behavior

After drawing its productivity, a firm decides whether to exit or stay in the industry. The staying firms engage in monopolistic competition facing demand as in Eq. (1). When making output choices, individual firms regard the demand intercept,  $A$ , as given. If firm  $x$  stays, it chooses its number of varieties,  $z$ , and each variety's output,  $q(v)$  for all  $v \in [0, z]$ , to maximize its total profit:

$$\max_{z \geq 0, q(v) \geq 0} \Pi(x, z) \equiv \int_0^z \left\{ \left( A - \frac{\gamma}{L} q(v) - c(x, v) \right) q(v) \right\} dv - \int_0^z h(v) dv.$$

For a given variety  $v \in [0, z]$ , the optimal output is  $q(x, v) = \frac{1}{2\gamma}(A - c(x, v))$ , the corresponding price is  $p(x, v) = \frac{1}{2}(A + c(x, v))$ , the absolute markup is  $p(x, v) - c(x, v) = \frac{1}{2}(A - c(x, v))$ , and the variety's profit (gross of its variety-introduction fee) is

$$\pi(x, v) = \frac{L}{4\gamma} (A - c(x, v))^2. \tag{2}$$

Given the output choice for each variety, firm  $x$ 's profit becomes  $\Pi(x, z) = \int_0^z \pi(x, v) dv - \int_0^z h(v) dv$ . Thus, the optimal number of varieties, called the *product scope* and denoted as  $v^*(x)$ , satisfies  $\frac{\partial \Pi(x, z)}{\partial z} = \pi(x, v^*(x)) - h(v^*(x)) = 0$ , or

$$\frac{L}{4\gamma} (A - c(x, v^*(x)))^2 = h(v^*(x)). \tag{3}$$

A firm chooses its optimal scope by comparing the marginal benefit of scope (the left-hand side of Eq. (3)) with the marginal cost (the right-hand side). If the marginal benefit and marginal cost are

both independent of scope, condition (3) will hold for at most one firm. Therefore, to obtain a non-degenerate equilibrium, at least one of the marginal terms must vary with scope. For the marginal benefit to vary,  $c_v \equiv 0$  must be ruled out; and for the marginal cost to vary,  $h' \equiv 0$  must be ruled out. Most existing models of multiproduct firms assume diminishing marginal benefits of scope so that each additional variety generates smaller and smaller profits, which corresponds to  $c_v > 0$  in this model.<sup>14</sup> For simplicity, these studies have also assumed  $h' \equiv 0$ . That is, the marginal cost of scope is constant (and sometimes zero) and, more importantly, identical across all firms. This formulation, by contrast, allows for increasing marginal cost of scope,<sup>15</sup> i.e.,  $h'(v) > 0$ . We refer to the case of  $h'(\cdot) \equiv 0$  as *constant fees*, and the case of  $h'(\cdot) > 0$  as *increasing fees*.

Eq. (3) says that a firm's scope is optimal if the marginal cost of scope,  $h(v^*(x))$ , equals the marginal benefit. We prove in the Appendix A that each active (i.e., staying) firm has a unique choice of  $v^*(x) \geq 0$ . Note that  $\pi(x, v^*(x))$  is the profit of firm  $x$ 's least efficient variety produced, referred to as its *marginal variety*. If all varieties are equally efficient within a firm ( $c_v \equiv 0$ ), any variety can be regarded as the marginal variety. We can then establish the following properties:

**Proposition 1.** (i)  $\frac{dv^*(x)}{dx} < 0$ ; (ii)  $\frac{dc(x, v^*(x))}{dx} > 0$  if  $h' > 0$ , and  $\frac{dc(x, v^*(x))}{dx} = 0$  if  $h' \equiv 0$ .

Part (i) of the proposition says that less-productive firms (with larger  $x$ ) maintain smaller scopes (smaller  $v^*(x)$ ). Refer to Fig. 1 for the case of  $c_v > 0$ . The variety-level profit curve moves down as a firm becomes less productive, leading to a smaller scope regardless of the slope of  $h(v)$  (i.e., increasing fees in panel (a) or constant fees in panel (b)). The result will be the same if  $c_v \equiv 0$ , which will generate flat profit lines. Intuitively, compared to a more-productive firm, a less-productive firm has a higher unit cost of producing a given  $v$ th variety (because  $c_x > 0$ ) and therefore a smaller profit from that variety. Since both firms face the same fee in introducing the  $v$ th variety, condition (3) will hold for the less-productive firm at a smaller  $v^*$ .

Part (ii) of the proposition says that a firm's marginal variety's efficiency increases with the firm's productivity if the variety-introduction fee increases, while all firms' marginal varieties are equally (in)efficient if the fee is constant. Two forces are at work. A less-productive firm has a higher unit cost than a more-productive firm for any given  $v$ . At the same time, a less-productive firm maintains a smaller scope and hence a marginal variety that is closer to its core competence than that of its more-productive counterpart, which implies a lower unit cost. The net effect depends crucially on the slope of the variety-introduction fee. For increasing fees (panel (a)), a more-productive firm's marginal fee is greater because its scope is larger, which implies that its marginal variety must be more profitable, and hence more efficient, than a less-productive firm's marginal variety. By

<sup>14</sup> There may be several reasons why additional varieties bring declining profits. Selling an additional variety may depress the demand for a firm's existing varieties. Such cannibalization can be modeled either as firms' optimization (Baldwin and Gu, 2009; Eckel and Neary, 2010; Feenstra and Ma, 2008) or using a product substitution parameter in the demand function (Dhingra, 2011). A second mechanism, the so-called core competence approach, suggests that the unit cost of producing each extra variety may increase (i.e.,  $c_v > 0$ ). The source of the within-firm heterogeneity in production efficiency can be explained by random draws of unit costs, either at the beginning of the game (Eckel and Neary, 2010; Mayer et al., 2011) or repeatedly (Bernard et al., 2011). A third mechanism could be identical unit cost (within each firm) that is assumed to increase with a firm's scope (Nocke and Yeaple, 2008). The difference between the second and third approaches is that rising unit cost applies only to the marginal variety in core-competence, but to all existing varieties in Nocke and Yeaple (2008). Note that  $c_v = 0$  in Baldwin and Gu (2009), Feenstra and Ma (2008), and Nocke and Yeaple (2008), and  $c_v > 0$  in all other models.

<sup>15</sup> Arkolakis and Muendler (2010) allowed a product's entry cost to vary with scope. Their entry cost is market-specific, while the introduction fee here is related to a specific variety but not any specific market. The importance of this distinction will be clear in Section 5. Arkolakis and Muendler (2010) did not study scope adjustment in response to globalization, but if they had, they would still have concluded that all firms reduce scope. See Section 5.

<sup>11</sup> We have argued in Section 1 that this feature is indeed supported by ample evidence. Alternatively, we can think of the sum of the variety-introduction fees as product (or plant) management costs. The corresponding feature is that the marginal cost of managing additional product is increasing, which is supported by many studies in the management literature. See Footnote 3.

<sup>12</sup> The firms are assumed to be heterogeneous along a single dimension in terms of the productivity they draw. It would be interesting to study what would happen if the firms also differed in their ability to introduce multiple varieties, but this is left for future research.

<sup>13</sup> In this paper, a variety's *efficiency* refers specifically to its unit production cost. This variety-level characteristic is to be distinguished from firm-level characteristic, namely a firm's unit cost of producing its core variety, which is termed a firm's *productivity*. The usage here of the two terms does not carry any normative meaning.

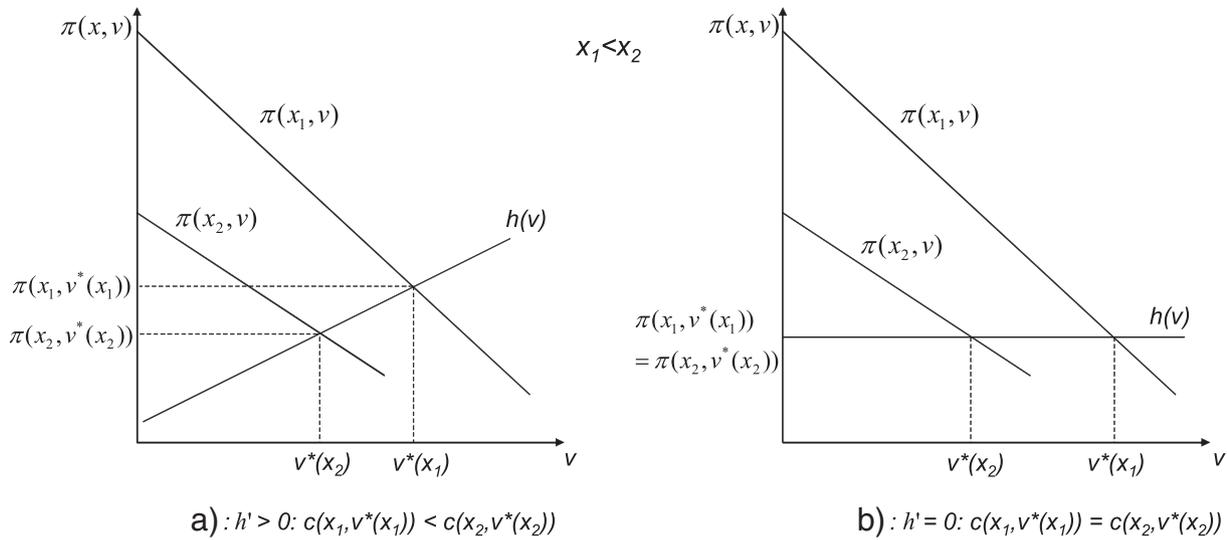


Fig. 1. The efficiency of marginal varieties.

contrast, in the case of constant fees (panel (b)), all firms pay the same marginal fee even though they maintain different scopes. This means that marginal varieties of different firms must be equally efficient.

It is now helpful to clarify the relationship between a variety's profit and its efficiency. A multiproduct firm can be viewed as a collection of varieties with different unit costs of production. As firms differ in the efficiency with which they produce their core varieties, they also differ in their efficiency of producing any given variety with the same index. Nevertheless, a variety's profit,  $\pi(x, v) = \frac{1}{4\gamma}(A - c(x, v))^2$ , is solely determined by its unit cost regardless who produces it and how far away it is from that firm's core competence. That is, if two varieties produced by two different firms have the same unit cost, their profits will be the same.

At the firm level, output is  $Q(x) \equiv \int_0^{v^*(x)} q(x, v) dv$ . Then,  $\frac{dQ(x)}{dx} = \int_0^{v^*(x)} \frac{\partial q(x, v)}{\partial x} dv + q(x, v^*(x)) \frac{dv^*(x)}{dx} < 0$ . So a firm's scope, variety- and firm-level sales all increase with its productivity. As a result, intensive margins (firm-level output) and extensive margins (the number of varieties a firm maintains) are positively correlated, a result commonly found in prior research and supported by empirical studies.

2.4. Entry, exit, and industry equilibrium

Since a firm's profit from each of its varieties,  $\pi(x, v)$ , weakly decreases with  $v$ , a firm produces its core variety if it produces at all. Define

$$x_n \equiv A - \sqrt{\frac{4\gamma h(0)}{L}} \tag{4}$$

such that  $x_n$  satisfies  $\pi(x_n, 0) = \frac{1}{4\gamma}(A - c(x_n, 0))^2 = h(0)$ . That is, a firm with  $x = x_n$  earns zero profit from producing its core variety, so it is indifferent between staying and exiting. Refer to this firm as the industry's *marginal firm* and assume that it stays in the industry. It is then clear that all firms with  $x > x_n$  will exit and all firms with  $x \leq x_n$  will remain in the industry to produce something. The marginal firm is the least productive among all the active firms. Its marginal variety is also its core variety, so the unit cost of its marginal variety is  $c_n = c(x_n, v^*(x_n)) = c(x_n, 0) = x_n$ .

Given the above discussion, Proposition 1(ii) immediately implies the following corollary:

Corollary 1.

- (i) If  $h' > 0$ , then  $c(x, v) < c_n$  for  $x < x_n$ .
- (ii) If  $h' \equiv 0$ , then for all  $x$ ,  $c(x, v) < c_n$  for  $v < v^*(x)$ , and  $c(x, v^*(x)) = c_n$ .

The corollary says that  $c_n$  is the largest unit cost in the industry. For this reason, let us call firm  $x_n$ 's marginal variety the industry's marginal variety—the least efficient among all the varieties produced in this industry. Whether other varieties exist which are equally inefficient depends on  $h'$ . If the variety-introduction fee increases ( $h' > 0$ ), all other varieties are strictly more efficient than the industry's marginal variety. If the variety-introduction fee is constant ( $h' \equiv 0$ ), every firm's marginal variety is as inefficient as the industry's marginal variety, so all firms' marginal varieties constitute the set of the industry's least efficient varieties.

Given the variety-level profit  $\pi(x, v)$  and the optimal scope  $v^*(x)$ , an active firm's profit (net of the variety-introduction fee but gross of the firm-level entry fee) is  $\Pi(x, v^*(x))$ , denoted as  $\Pi(x)$ :

$$\Pi(x) = \int_0^{v^*(x)} \pi(x, v) dv - \int_0^{v^*(x)} h(v) dv.$$

In addition to the firm-level productivity  $x$ , this profit also depends on  $A$  and  $L$ . Although  $A$  is an endogenous variable, each single firm treats  $A$  as given. The Envelope Theorem now suggests some useful properties: Since  $\Pi(x)$  is the value function with  $v^*(x)$  being the choice variable, the impact of a parameter ( $x$ ,  $A$  or  $L$ ) on the value function (when  $v^*(x)$  is optimally chosen) equals the impact when  $v^*(x)$  is fixed. For example,  $\frac{d\Pi(x)}{dx} = \int_0^{v^*(x)} \frac{\partial \pi(x, v)}{\partial x} dv < 0$ , i.e., a firm's total profit increases with its productivity. Also,  $\frac{d\Pi(x)}{dA} = \int_0^{v^*(x)} \frac{\partial \pi(x, v)}{\partial A} dv > 0$ , meaning that a firm's profit increases with demand.

Anticipating  $A$ , a firm's expected profit before sinking the entry fee and knowing its productivity is

$$\Pi^e(A) \equiv \int_0^{x_n} \Pi(x) dG(x),$$

where  $A$  depends in turn on firms' exit and scope/output choices. Firms keep entering the differentiated-product industry until the expected profit equals the entry fee:

$$\Pi^e(A) = f. \tag{5}$$

This free-entry equilibrium condition contains only one unknown endogenous variable,  $A$ . Since  $\Pi(x_n) = 0$  and  $\frac{d\Pi(x)}{dA} > 0$ , then  $\frac{d\Pi^e(A)}{dA} = \Pi(x_n) \frac{dx_n}{dA} + \int_0^{x_n} \frac{d\Pi(x)}{dA} dG(x) = \int_0^{x_n} \frac{d\Pi(x)}{dA} dG(x) > 0$ . Since  $\Pi^e(A)$  strictly increases with  $A$  and  $\Pi^e(A) = 0$  when  $A = \sqrt{4\gamma h(0)}/L$ , Eq. (5) has a unique solution in  $A$ . Once  $A$  has been determined, the number of total varieties,  $M$ , can be solved from the equation  $A = \frac{\alpha\gamma + \beta P(A)}{\gamma + \beta M}$ , where  $P(A) = \frac{1}{c(x_n)} \int_0^{x_n} \int_0^{v^*(x)} p(x, v) dv dG(x)$ .

### 3. The impact of globalization

This discussion has assumed fully integrated world economy. As there is no trade cost (whether variable or fixed), the number of consumers,  $L$ , can be viewed as  $L$  symmetric countries each having a population normalized to unity. The entry of more countries to this free trade zone, referred to hereafter as (further) globalization or trade liberalization, can then be represented by an increase in  $L$ .<sup>16</sup> The following proposition describes how globalization affects the industry equilibrium and variety-level variables.<sup>17</sup>

#### Proposition 2.

- (i)  $\frac{dA}{dL} < 0$  and  $\frac{dx_c}{dL} < 0$ .
- (ii) There exists  $\tilde{c}_\pi \in (0, c_n)$  such that  $\frac{d\pi(x,v)}{dL} > 0$  if and only if  $c(x, v) < \tilde{c}_\pi$ .
- (iii) There exists  $\tilde{c}_q \in (\tilde{c}_\pi, c_n]$  such that  $\frac{dq(x,v)}{dL} > 0$  if and only if  $c(x, v) < \tilde{c}_q$ .

Part (i) of the proposition indicates that globalization intensifies competition and induces the least productive firms to exit. Globalization brings more consumers to the common market. The increased demand raises the production of any given variety and hence each firm's profit. The expected profit increases prompt entry by new firms until the expected profit returns to the original level. A greater number of active firms intensify competition, which reduces the markup of all surviving varieties ( $\frac{1}{2}(A - c(x, v))$ ). In the variety-level demand function depicted in Eq. (1), greater market size is reflected as a flatter slope (smaller  $\frac{1}{2}$  due to greater  $L$ ), while intensified competition is reflected as a lower intercept (smaller  $A$ ).<sup>18</sup>

Since each firm is represented by a number of varieties, globalization affects firm-level choices through its impact on variety-level profits. Note first that it is impossible for every surviving variety's profit to go up or for every surviving variety's profit to go down. In either case, the expected profit,  $\Pi^e$ , will change, which violates the free entry condition (5). Then the question is, which variety's profit will go up and which will go down? Part (ii) of the proposition says that the profits of the industry's more-efficient varieties will increase after globalization, while the profits of the less-efficient varieties will decrease. As discussed above, globalization enlarges the market's size (larger  $L$ ), which tends to raise profits, but it also intensifies competition (smaller  $A$ ), which tends to reduce them. The changes in  $A$  and  $L$  are at the industry level and therefore the same for all varieties, but their impacts on a variety's profit depend on the variety's production efficiency. For a very inefficient variety the enlarged market size is not very useful due to the small markup (a larger  $L$  will not raise the profit (2) by much), but the intensified competition is very damaging (a lower  $A$  drives the already small margin,  $\frac{1}{2}(A - c(x, v))$ , to almost zero). The net effect is a smaller profit. The opposite happens with a more-efficient variety.

So globalization raises the profits of more-efficient varieties and reduces the profits of less-efficient ones.<sup>19</sup> For convenience, refer to the variety whose profit remains unchanged after globalization (the variety with the cutoff cost  $\tilde{c}_\pi$ ) as the industry's *average variety*. A similar force underlies Proposition 2(iii), which says that production of more-efficient varieties expands while that of less-efficient varieties decreases. Note that the cutoff cost for output expansion,  $\tilde{c}_q$ , is greater than the cutoff cost for profit increase,  $\tilde{c}_\pi$ . This implies that the production of some varieties is expanded without increasing their profits. This is because a variety's profit is its production scale multiplied by its markup. The markup always decreases after globalization, so some varieties'

profits will drop even though their production expands. Therefore, if a variety's profit increases, its production scale must increase; conversely, if a variety's scale decreases, its profit must drop. In other words, scale expansion is more pervasive than profit increase at the variety level.

So how do firms adjust their product scopes in response to globalization? The next proposition answers this question.

#### Proposition 3.

- (i)  $\frac{dv^*(x)}{dL} > 0$  if and only if  $c(x, v^*(x)) < \tilde{c}_\pi$ .
- (ii) If  $c(0, v^*(0)) \geq \tilde{c}_\pi$ , then  $\frac{dv^*(x)}{dL} < 0$  for all  $x$ . If  $c(0, v^*(0)) < \tilde{c}_\pi$ , then there exists an  $\tilde{x} > 0$ , which is determined by  $c(\tilde{x}, v^*(\tilde{x})) = \tilde{c}_\pi$ , such that  $\frac{dv^*(x)}{dL} > 0$  if and only if  $x < \tilde{x}$ .

Part (i) of the proposition says that a firm expands its scope after globalization if and only if its marginal variety is sufficiently efficient. To understand this, refer to Fig. 2, which illustrates a firm's scope choices before globalization (subscript 1) and after (subscript 2), for the case of increasing marginal cost of scope ( $h' > 0$ ) and decreasing marginal benefit of scope ( $c_v > 0$ ). Recall that at the optimal scope the marginal cost of scope equals the marginal benefit, which is the marginal variety's profit. Since globalization does not change the marginal cost curve, the optimal scope moves along that curve. If the scope increases ( $v_2^*(x) > v_1^*(x)$ ), the upward-sloping marginal cost implies that the marginal variety's profit must increase:  $\pi_2(x, v_2^*(x)) \geq \pi_1(x, v_1^*(x))$  (with equality if and only if  $h' \equiv 0$ ). On the other hand, the downward-sloping marginal benefit implies that the profit of the pre-globalization marginal variety must be greater than that of the post-globalization marginal variety:  $\pi_2(x, v_1^*(x)) \geq \pi_1(x, v_2^*(x))$  (with equality if and only if  $c_v \equiv 0$ ). Since  $h' \equiv 0$  and  $c_v \equiv 0$  cannot coexist, it must be true that  $\pi_2(x, v_1^*(x)) > \pi_1(x, v_1^*(x))$ , i.e., the profit of the original marginal variety must increase strictly. This immediately implies that the marginal variety must be sufficiently efficient.

While part (i) of the proposition establishes the necessary and sufficient condition for a given firm to expand its scope after globalization, part (ii) characterizes the pattern of scope adjustment by all firms. It says that if the most productive (i.e.,  $x = 0$ ) firm's marginal variety is very inefficient, all firms reduce scope after globalization. Otherwise, the industry's more-productive firms expand their scope while the less-productive firms reduce it. This is best understood by focusing on the case where  $h' > 0$  and  $c_v > 0$ . Recall that the efficiency of marginal varieties increases with a firm's productivity when  $h' > 0$  (Proposition 1(ii)), and that globalization raises a variety's profit if and only if it is sufficiently efficient (Proposition 2(ii)). If the efficiency of the most productive firm's marginal variety is below the cutoff level, the firm will reduce its scope after globalization. Because all other firms' marginal varieties are even less efficient, they all reduce their scopes. Conversely, if the most productive firm's marginal variety is sufficiently efficient, it will expand its scope. Since the marginal firm's marginal variety is the least efficient variety in the industry and must therefore receive smaller profits, by continuity, those firms with sufficiently high productivity will expand their scope while the remaining firms will reduce it.

So less-productive firms always reduce scope. It is possible for all firms to reduce scope, but it is impossible for all firms to expand it simultaneously. And the scope-expanding firms are the industry's more-productive ones.<sup>20</sup> Proposition 3 says that the sufficient and necessary condition for scope expansion is that the most productive firm's marginal variety be sufficiently efficient. Since the efficiency of a firm's marginal variety is endogenous, whether or not that obtains depends on the parameters. There are, however, two situations in which the result is unambiguous. Recall from Proposition 2(ii) that

<sup>20</sup> Nocke and Yeaple (2008) reached the opposite conclusion: It is the less capable and smaller firms that expand scope after globalization. This was due to their special model which assumed that the less efficient varieties were being produced by more capable firms that maintain larger scopes.

<sup>16</sup> Eckel and Neary (2010) took the same approach in modeling trade liberalization.

<sup>17</sup> Statements such as " $z > 0$  if and only if  $c < C$ " are a shorthand expression meaning that  $z > 0$  if  $c < C$ ;  $z = 0$  if  $c = C$ ; and  $z < 0$  if  $c > C$ .

<sup>18</sup> These effects of trade liberalization are now well understood for linear demand systems, as has been discussed by Melitz and Ottaviano (2008) for single-product firms and by Eckel and Neary (2010) for multiproduct firms.

<sup>19</sup> This immediately implies that the (original) marginal firm's profit must drop from zero to negative if it stays. This firm will exit, leading to a smaller cutoff  $x_\pi$ .

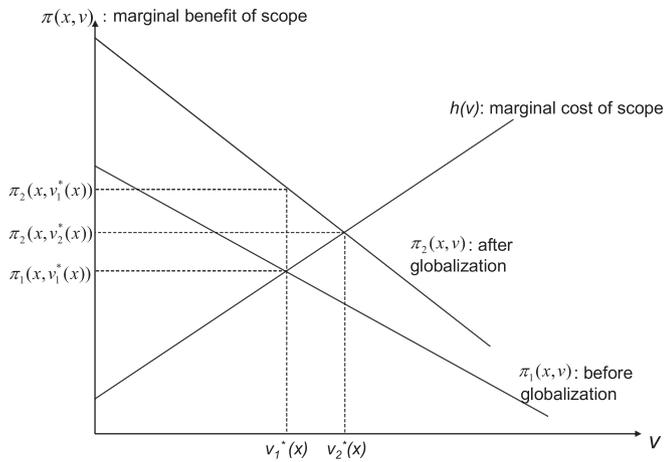


Fig. 2. Scope expansion after globalization.

the profit of the industry's least efficient variety must drop after globalization, while that of its most efficient variety must rise. If the marginal cost of scope is constant ( $h' \equiv 0$ ), every firm's marginal variety is equally inefficient. In that case, the most productive firm's marginal variety is among the industry's least efficient varieties, so all firms reduce scope after globalization. If, on the other hand, all varieties are produced with equal efficiency within each firm ( $c_v \equiv 0$ ), then the most productive firm's marginal variety is as efficient as its core variety, which is the most efficient variety in the industry. In that case, the most productive firm (and other firms that are sufficiently productive) will expand its scope after globalization. So an increasing variety-introduction fee ( $h' > 0$ ) is necessary for scope expansion, but it is not sufficient. These conclusions are summarized in the following proposition.

**Proposition 4.**

- (i) If  $h' \equiv 0$ , then  $\frac{dv^*(x)}{dL} < 0$  for all  $x$ ;
- (ii) if  $c_v \equiv 0$  and  $h' > 0$ , then  $\frac{dv^*(x)}{dL} > 0$  if and only if  $x < \tilde{c}_n$ .

So far this discussion has focused on how multiproduct firms respond to globalization by adjusting their product scope. Adjustments in other decision and performance variables can now easily be established. Proposition 2 has described the pattern at the variety level: In response to globalization, more-efficient varieties' outputs and profits both increase; the scale of moderately efficient varieties increases but profits decrease; less-efficient varieties' scales and profits both decrease; the least efficient varieties are dropped. A similar pattern exists at the firm level: more-productive firms increase both scale (firm-level output) and profits; moderately productive firms expand scale but make less profit; less-productive firms decrease their scale and earn lower profits; the least productive firms exit. Such sorting has been well established for single-product firms; it continues to hold for multiproduct firms. The extra channel of adjustment by multiproduct firms is their scope. If a firm expands its scope, its profit and output at the firm- and variety-level all increase, but the converse is not true.

**4. Parametrization of productivity and costs**

The previous section has demonstrated the possibility of and conditions for scope expansion after globalization. In particular, Proposition 4 shows that  $h' > 0$  is a necessary condition for scope expansion and that  $c_v \equiv 0$  (which requires  $h' > 0$ ) is a sufficient condition. However, this sufficient condition is an extreme case. It would be interesting to understand what happens in the general case in which  $h' > 0$  and  $c_v > 0$ . To this end, turn to the more specific productivity distribution,  $G(x)$ , and the cost functions  $c(x, v)$  and  $h(v)$ . Assume that productivity is Pareto distributed:  $G(x) = \left(\frac{x}{\bar{x}}\right)^k$  where  $k > 0$ . A desirable

property of this distribution is that the cost of active firms also follows Pareto's with the same shape parameter  $k$  on a truncated support  $[0, x_n]$ . If  $k = 1$ , the distribution becomes uniform. A greater  $k$  tilts the draw of  $x$  toward larger values such that a firm is more likely to have low productivity. Also assume that the unit production cost is  $c(x, v) = x + \lambda v$  with  $\lambda \geq 0$ . Then within-firm heterogeneity is captured by  $c_v = \lambda$ . If  $\lambda = 0$ , all varieties within a firm are produced with equal efficiency. A larger  $\lambda$  indicates a faster decline in production efficiency as varieties move away from the core competence. Finally, assume that the variety-introduction fee is  $h(v) = \rho^2 v^2$  with  $\rho \geq 0$ .<sup>21</sup> Then  $\rho = 0$  corresponds to constant fees, and  $\rho > 0$  corresponds to increasing fees.<sup>22</sup>

With the specific functions  $G(x)$ ,  $c(x, v)$  and  $h(v)$  assumed throughout this section, all the equilibrium variables can be determined explicitly. Eq. (3) becomes  $\frac{1}{4\gamma}(A - x - \lambda v^*)^2 = \rho^2 v^{*2}$ . The optimal scope is then

$$v^*(x) = \frac{A - x}{\lambda + 2\rho\sqrt{\frac{\gamma}{L}}}$$

The firm-level profit is  $\Pi(x) = \frac{\lambda + 4\rho\sqrt{\frac{\gamma}{L}}}{12\frac{\gamma}{L}(\lambda + 2\rho\sqrt{\frac{\gamma}{L}})^2}(A - x)^3$ . Because  $h(0) = 0, x_n(A, L) = A$ . After calculating the expected profit, the equilibrium  $A$  can be derived as

$$A = \left(\frac{4\phi\frac{\gamma}{L}(\lambda + 2\rho\sqrt{\frac{\gamma}{L}})^2}{\lambda + 4\rho\sqrt{\frac{\gamma}{L}}}\right)^{\frac{1}{k+3}}$$

where  $\phi = \frac{\Gamma(k+1)(k+2)(k+3)}{2}$  is a technology index that depends on the entry fee  $f$  and the distribution governing the cost draw ( $k$  and  $X$ ). Globalization as represented by an increase in  $L$  will affect  $v^*(x)$  both directly, and indirectly through its impact on  $A$ . Define (for  $\lambda > 0$ )

$$y \equiv \frac{2\rho}{\lambda} \sqrt{\frac{\gamma}{L}}$$

which is a composite parameter depending on market size ( $L$ ) and the cost distributions ( $\lambda$  for the rate of rise of unit production costs and  $\rho$  for the rate of rise of the variety-introduction fee). As will become clear, a larger  $y$  then leads to marginal varieties that are more efficient. The following proposition establishes conditions for scope expansion:

**Proposition 5.**

- (i) Globalization induces scope expansion if and only if  $k > \frac{2 + 3y}{y(1 + 2y)}$ . (6)
- (ii) When (6) is satisfied, the percentage of scope-expanding firms is  $r_v \equiv \frac{1}{k+3} \left(k - \frac{2+3y}{y(1+2y)}\right)$ , with  $\frac{\partial r_v}{\partial k} > 0$  and  $\frac{\partial r_v}{\partial y} > 0$ .
- (iii)  $\frac{dv^*(x)}{dL} > 0$  if and only if  $x < \tilde{x} \equiv \frac{A}{k+3} \left(k - \frac{2+3y}{y(1+2y)}\right)$ , with  $\frac{\partial \tilde{x}}{\partial L} < 0, \frac{\partial \tilde{x}}{\partial \lambda} < 0$ , and  $\frac{\partial \tilde{x}}{\partial \rho} > 0$ .

Part (i) proposes (6) as the necessary and sufficient condition for some firms to expand scope with globalization. The condition is

<sup>21</sup> The use of  $\rho^2$  rather than  $\rho$  is merely for notational convenience. The assumption of  $v^2$  rather than  $v$  is for computational simplicity. If  $h(v) = \rho v$ , the optimal  $v^*(x)$  solved from Eq. (3) contains a square root, which greatly complicates the calculation without yielding any new insight.

<sup>22</sup> Here the case of constant marginal cost of scope is represented by  $\rho = 0$ . All the existing models which assume a constant cost for introducing new varieties adopt the functional form  $h(v) = \theta$  for some  $\theta \geq 0$ . If the demand had been derived from a CES preference,  $\theta$  must be positive (Bernard et al., 2010, 2011). This is because the quantity demanded is positive no matter how high the price. If  $\theta = 0$ , a firm will keep expanding its scope no matter how inefficient the variety is. By contrast, all existing models with linear demand (Eckel and Neary, 2010; Mayer et al., 2011) assume  $\theta = 0$ .

more likely to hold for a larger  $k$  or  $y$ , and the intuition is as follows. It has been established that scope expansion is more likely if marginal varieties are more efficient. In the above equilibrium, a marginal variety's cost is  $c(x, v^*(x)) = x + \frac{1}{4-y}$ . To understand the role of  $y$ , note that  $y$  decreases with the market size ( $L$ ) and the rate of within-firm decline of efficiency ( $\lambda$ ), but increases with the rate of increase of the variety-introduction fee ( $\rho$ ). When the market is smaller (smaller  $L$ , which means a larger  $y$ ), a given variety will generate a smaller profit, leading to smaller scopes. A firm's marginal variety will then be more efficient because it is closer to its core competence. When production efficiency drops more slowly within each firm (a smaller  $\lambda$ , which means a larger  $y$ ), any given variety's efficiency (including that of the marginal variety) will be higher even though the scope is larger. Finally, when the variety-introduction fee rises more quickly (a larger  $\rho$ , which means a larger  $y$ ), a variety's profit can cover its introduction fee only when it is closer to the core, so the marginal variety will again be more efficient.

Now consider the role of  $k$ . As  $k$  increases, more firms will draw lower productivity levels. As a result, the industry's average variety will tend to be less efficient, making it more likely that the efficiency of a firm's marginal variety will exceed that of the industry's average variety.

To summarize, a greater  $y$  implies that marginal varieties are more efficient, while a greater  $k$  implies that the average variety is less efficient. Either will make marginal varieties compare more favorably with the average variety, thus making it more likely that (more-productive) firms will expand their scope in response to globalization.

Part (ii) of the proposition gives the percentage of all active firms that expand their scope. The percentage increases when  $k$  or  $y$  is larger, in the manner just discussed. Thus  $k$  and  $y$  are conducive to scope expansion in two senses: Greater likelihood of scope expansion (part (i)), and a higher percentage of firms expanding their scope (part (ii)).

Part (iii) is more specific, about the scope adjustment of any particular firm. As a general conclusion, it has been established in the previous section that a firm expands scope if and only if it is sufficiently productive. Here part (iii) expresses the cutoff productivity, which enables the identification of which firms will expand their scope and the factors affecting this cutoff level. Consistent with the discussion about condition (6) and in the same way, scope expansion is more likely for any given firm (i.e.,  $\tilde{x}$  is larger) if the market is smaller, its unit costs rise more slowly, or the variety-introduction fee rises more steeply.

Condition (6) always holds when  $L$  is close to 0 ( $y$  will be very large), and always fails when  $L$  is sufficiently large ( $y$  will be very small). Suppose that the initial free trade area is very small, i.e.,  $L$  is very small. Each variety's profit will then be small, and a firm will stop expanding at a marginal variety that is still very efficient. In that case, the most productive firms always expand scope in response to globalization. As globalization further broadens,  $L$  keeps enlarging, and each firm keeps extending its scope to ever lower efficiency levels. When  $L$  is sufficiently large, each variety is so profitable that a firm reaches marginal varieties that are very inefficient (if  $\rho > 0$ ). At that point, further globalization leads to scope contraction by all firms. This dynamic effect of globalization is summarized in the following corollary.

**Corollary 2.** *At the beginning of globalization ( $L$  is very small), more-productive firms always expand their scope. With more countries joining the integrated economy, the percentage of scope-expanding firms drops. Eventually, when  $L$  exceeds some critical level, all firms reduce their scope in response to further globalization.*

The next corollary gives an idea about the maximum fraction of firms expanding their scopes in response to globalization. Notice that  $\lim_{y \rightarrow \infty} \tau_y = \frac{k}{k+3}$ .

**Corollary 3.** *If  $x$  is uniformly distributed ( $k = 1$ ), the percentage of scope-expanding firms never exceeds  $\frac{1}{4}$ .*

Researchers are interested in studying multiproduct firms mainly because within-firm rationalization provides another potential channel for globalization to improve production efficiency, in addition to the cross-firm rationalization highlighted by Melitz (2003) for single-product firms. In this model, cross-firm rationalization generates patterns similar to those found for single-product firms: Output and profits shift from less-productive firms to more-productive ones, resulting in a larger mean and greater variance in the industry's output and profit distributions.<sup>23</sup> A more informative measure of cross-firm rationalization is the industry average cost (weighted by output)<sup>24</sup>:

$$\bar{c} \equiv \frac{\int_0^{x_n} \int_0^{v^*(x)} c(x, v) q(x, v) dv dG(x)}{\int_0^{x_n} \int_0^{v^*(x)} q(x, v) dv dG(x)}$$

The Appendix A shows that  $\frac{d\bar{c}}{dx} < 0$ . Therefore, globalization improves the overall production efficiency of the whole industry.

Within-firm rationalization, however, displays richer patterns depending on whether a firm expands its scope after globalization. On the one hand, each firm's output redistributes between any two surviving varieties in favor of the more-efficient one:  $\frac{q(x, v_1)}{q(x, v_2)}$  increases after globalization if  $v_1 < v_2$ .<sup>25</sup> Such a product-mix effect (Mayer et al., 2011) tends to improve a firm's average efficiency. On the other hand, a firm also adjusts its scope in response to globalization. If a firm reduces its scope, it drops its least efficient varieties, which also tends to improve its average efficiency. If a firm expands its scope, however, it adds new varieties which are less efficient, which reduces its average efficiency. So the product-mix and scope-adjustment effects work in the same direction when a firm shrinks its scope, but in opposite directions when a firm expands its scope.

To determine the effect of globalization on within-firm rationalization, we calculate two variables. The first is the within-firm output concentration represented by a Herfindahl–Hirschman index (HHI)<sup>26</sup>:

$$\eta(x) \equiv \frac{1}{v^*(x)} \int_0^{v^*(x)} \left( \frac{q(x, v)}{Q(x)} \right)^2 dv,$$

<sup>23</sup> The unconditional expected profit will always equal to  $f$  due to the free entry condition, but the conditional expected profit (conditional on a firm being active) increases after globalization due to the drop in  $x_n$ . Because it is less likely to draw an  $x$  that is below  $x_n$ , a firm's expected profit conditional on  $x \leq x_n$  must increase in order for the unconditional expected profit to remain unchanged.

<sup>24</sup> The introduction fee is regarded as a capital cost while the production of varieties involves only labor. For this reason, production efficiency is calculated based on production cost excluding the introduction fee. Even if the introduction fee is included, the qualitative result will not change.

<sup>25</sup>  $\frac{q(x, v_1)}{q(x, v_2)} = \frac{A - c(x, v_1)}{A - c(x, v_2)} = 1 + \frac{c(x, v_2) - c(x, v_1)}{A - c(x, v_2)}$ . If  $v_1 < v_2$ ,  $c(x, v_2) > c(x, v_1)$  and  $\frac{c(x, v_2) - c(x, v_1)}{A - c(x, v_2)}$  increases after globalization because  $A$  drops. Alternatively,  $q(x, v) = \frac{1}{2y}(A - c(x, v))$ , so  $\frac{dq(x, v)}{dv} = -\frac{1}{2y} \frac{dc(x, v)}{dv} = -\lambda \frac{1}{2y}$  measure the within-firm decline of output across varieties. After globalization,  $L$  increases, so the decline is faster, meaning that the outputs of more-efficient varieties increase relative to the outputs of less-efficient varieties.

<sup>26</sup> In the formula,  $\frac{1}{v^*(x)}$  is the density of  $v$ , which distributes uniformly between 0 and  $v^*(x)$ . The density must be included in the calculation for the same reason as in calculating the output variance within a firm:  $\sigma^2(x) \equiv \frac{1}{v^*(x)} \int_0^{v^*(x)} (q(x, v) - \bar{q}(x))^2 dv$ . A HHI (which focuses on more-efficient varieties) is more informative than the variance (which measures deviation from the average output) because the aim is to capture how a firm shifts its total output toward its more efficient varieties. To see how the two measures may differ, consider the example of  $\lambda = 0$ , in which case  $\eta(x) = \frac{1}{v^*(x)}$ , but  $\sigma^2(x) = 0$ . When varieties are produced with equal efficiency within each firm, within-firm output concentration should drop with increasing firm productivity, because more productive firms maintain larger scopes. What matters for within-firm concentration is not how varieties differ (as measured by  $\lambda$ , representing  $\frac{dc(x, v)}{dv}$  or  $\frac{dq(x, v)}{dv}$ ), but how widely the total output is spread over different varieties. By this reasoning, a HHI indeed reflects such a property:  $\eta(x) = \frac{1}{v^*(x)}$  drops as  $v^*(x)$  rises (i.e., a firm becomes more productive and consequently maintains larger scope). By contrast,  $\sigma^2(x) = 0$  merely shows that all varieties have identical output without saying anything about how the total output is spread among different products, which is the focus of this investigation. Another way to look at the deficiency of variance is the following. It can be shown that globalization raises the mean and variance of a firm's output distribution if and only if the firm is sufficiently productive. While the larger average output of more productive firms seems to indicate more efficient production, a greater variance has ambiguous implications: a greater deviation from the mean may indicate redistribution to either the high or the low end.

where  $Q(x) \equiv \int_0^{v^*(x)} q(x, v) dv$  is firm  $x$ 's total output. A greater HHI indicates greater concentration of output within a firm. The second variable is the firm-level average cost:

$$\bar{c}(x) \equiv \frac{\int_0^{v^*(x)} c(x, v) q(x, v) dv}{Q(x)}.$$

Both the HHI and average cost take into consideration the product-mix effect (between existing varieties) and the scope-adjustment effect (which pertains to new varieties). The following proposition shows that it is possible for the scope-adjustment effect to dominate so that a firm's output becomes less concentrated and its average efficiency declines:

**Proposition 6.**

- (i)  $\frac{dH(x)}{dx} < 0$  if and only if  $\frac{x}{x_n} < r_q \equiv \frac{1}{k+3} \left( k - \frac{(3\gamma+1)(6\gamma^2+9\gamma+4)}{\gamma(12\gamma^2+18\gamma^2+9\gamma+1)} \right)$ .
- (ii)  $\frac{d\bar{c}(x)}{dx} > 0$  if and only if  $\frac{x}{x_n} < r_c \equiv \frac{1}{k+3} \left( k - \frac{1+6\gamma+6\gamma^2}{\gamma^2(2+3\gamma)} \right)$ .

Part (i) of the proposition says that a firm's output will be less concentrated if and only if the firm is sufficiently productive, and part (ii) says that a firm's average production cost rises if and only if the firm is sufficiently productive (with a different cutoff level). Globalization may therefore reduce a productive firm's output concentration and efficiency. In fact the changes in the two performance variables are connected, and they are both related to scope expansion. It can easily be shown that  $r_c < r_q < r_v$  (see Fig. 3 for the case where  $k = 1$ ), which implies that scope expansion is necessary (but not sufficient) for output to become less concentrated, and lower concentration in turn is necessary (but not sufficient) for average cost to rise.

As is clear from Fig. 3,  $r_c$  increases with  $y$  and so decreases with  $L$ , with  $r_c < 0$  when  $L$  is very large and  $r_c > 0$  when  $L$  is very small. Globalization may therefore reduce a productive firm's production efficiency initially, but it will eventually improve the average efficiency of all firms. This result is stated in the following corollary:

**Corollary 4.** *When  $L$  is very small, globalization reduces the more-productive firms' production efficiency. When  $L$  is sufficiently large, globalization improves all firms' efficiencies.*

Proposition 6 and Corollary 4 explain how globalization can reduce some firms' average efficiency, and they highlight the role of scope expansion in such situations. This finding is in sharp contrast with all existing models, in which scope never expands and as a result, firm-level efficiency always improves and output always becomes more concentrated within each firm. Existing models may have exaggerated the role of within-firm rationalization in the early stages of globalization.

**5. CES demand**

So far the analyses have assumed quasi-linear quadratic preferences, which generate linear demand. The major conclusion is that in response to globalization, all firms reduce their scope if the variety-introduction fee (i.e., the marginal cost of scope) is constant, but more-productive firms may expand their scope if the variety-introduction fee increases with the number of varieties. Many previous studies of multiproduct firms (e.g., Bernard et al., 2011) have concluded that firms unambiguously reduce scope based on models which assume CES preferences and have no variety-introduction fee. Is scope expansion still possible under CES preferences if the variety-introduction fee is allowed to rise?

Melitz's (2003) CES model can be adapted to multiproduct firms to answer this question. The framework is similar to the main model except that there are  $L$  identical countries where consumers have CES preferences among all varieties supplied in their country. After paying  $h(v)$  to introduce the  $v$ th variety, a firm needs to incur an additional fixed cost  $f_D \geq 0$  per variety to sell to the domestic

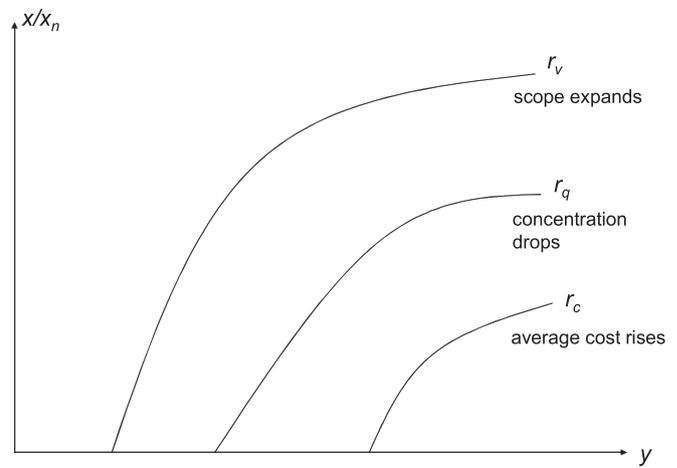


Fig. 3. The impact of globalization (for the case of  $k = 1$ ).

market, and another additional fixed cost  $f_E$  per variety to export to any foreign market. As Bernard et al. (2011) assumed, these fixed costs,  $f_D$  and  $f_E$ , are market specific and are constant for all varieties. There is also a variable exporting cost in the form of an iceberg cost captured by  $\tau > 1$ . As in all CES models, assume that  $\tau^{\sigma-1} f_E > f_D$ . Further globalization can be represented by a decrease in  $f_E$ , a decrease in  $\tau$ , or an increase in  $L$ . As in the main model, the key difference between this setting and standard CES models is the inclusion of the variety-introduction fee. It must be emphasized that  $h(v)$  is specific to a variety, not to any market.<sup>27</sup>

As is well understood from models of CES preferences and heterogeneous single-product firms, the presence of trade costs induces a sorting between exporters, which are more productive, and non-exporters which are less productive. Trade liberalization then raises the profits of every exporter and decreases the profits of every non-exporter. Mapping such a dichotomy to multiproduct firms, the different variety-level fixed costs ( $\tau^{\sigma-1} f_E > f_D$ ) induce a similar sorting within a firm among its varieties: inefficient varieties are supplied only to the domestic market while more-efficient varieties are both sold domestically and exported. Trade liberalization will then raise the profit of each exported variety and reduce the profit of each variety not exported. As has been established with the main model, a multiproduct firm expands its scope after trade liberalization if and only if its marginal variety's profit increases. The question of how a multiproduct firm adjusts its scope thus boils down to whether or not the firm's marginal variety is exported.

Fig. 4 shows the marginal benefits and marginal costs of scope for three representative firms, 1, 2 and 3 in the order of descending productivity, in the case when the variety-introduction fee increases ( $h'(v) > 0$ ). Firm 1's variety-level profit curve (i.e., the marginal benefit of scope),  $\pi(x_1, v)$ , has a kink where it stops exporting. To the left of the kink the varieties are sufficiently efficient to be exported, so the variety-level profit is the sum of the profits from the domestic and foreign markets (net of the fixed costs in the respective markets). To the right of the kink, the varieties are too inefficient to be exported, so the profit comes only from the domestic market (net of the fixed costs  $f_D$ ). Firm 2 has a similar variety-level profit curve,  $\pi(x_2, v)$ , which lies everywhere below that of firm 1. Note that the two firms' kinks correspond to different variety indices due to the difference in their productivity levels, but they have the same height, which is their common (adjusted) fixed export cost  $\tau^{\sigma-1} f_E$ . Firm 3's productivity is so low that even

<sup>27</sup> By contrast, Arkolakis and Muendler's (2010) "incremental entry cost" is product-destination specific. Using the notations of this paper, they essentially allow  $f_D$  and  $f_E$  to vary with  $v$  while assuming  $h(v) = 0$ . As is explained later in this section, allowing  $f_D$  and  $f_E$  to vary with  $v$  will not change the results as long as  $h(v) > 0$ .

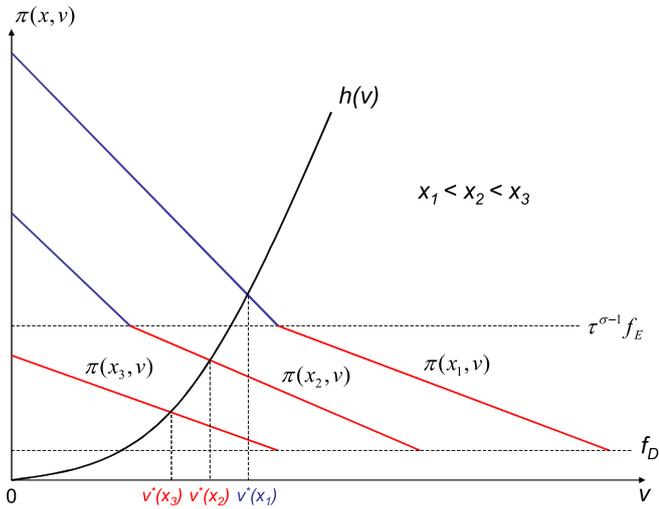


Fig. 4. The determination of scope for CES preferences.

its core product is not efficient enough to be exported. Its profit curve intercepts the vertical axis at a point below  $\tau^{\sigma-1} f_E$ , and has no kink as  $v$  increases. All three firms face the same fee schedule  $h(v)$  for introducing new varieties, but because  $h(v)$  increases with  $v$ , it crosses the three profit curves at increasing variety indices. For firm 1, the crossing is to the left of the kink, so its marginal variety is exported. For firm 2, the crossing is to the right of the kink, so its marginal variety is not exported. Finally, for firm 3, the crossing corresponds to a marginal variety that is supplied only to the domestic market.

These three representative cases show that when  $h'(v) > 0$ , more-productive firms (such as  $x_1$ ) export all their varieties including their marginal varieties, moderately productive firms (such as  $x_2$ ) export their more-efficient varieties but sell their less-efficient varieties including their marginal variety only in the home market, while very unproductive firms (such as  $x_3$ ) sell only to the domestic market. Such sorting immediately implies that more-productive firms (such as  $x_1$ ) expand their scope, while less-productive firms (such as  $x_2$  and  $x_3$ ) always reduce scope in response to trade liberalization.

The above analysis and Fig. 4 highlight the role of  $h(v)$  in determining the efficiency level of each firm's marginal variety and consequently the pattern of their scope adjustment in response to trade liberalization. All existing models assume  $h(v) = 0$ , which means that a firm's scope is determined from the intersection between its profit curve and the  $f_D$  line in Fig. 4. A firm's marginal variety, then, will not be exported no matter how productive the firm is. This immediately leads to the conclusion that all firms reduce scope in response to trade liberalization. Such logic drives Bernard et al.'s (2011) Proposition 1 about scope contraction. The result will not change even if  $f_D$  and  $f_E$  are allowed to increase with  $v$  as Arkolakis and Muendler (2010) have assumed, so long as the standard assumptions of  $\tau^{\sigma-1} f_E(v) > f_D(v)$  and  $h(v) = 0$  are maintained.

Therefore  $h'(v) > 0$  is necessary for scope expansion, but it is not sufficient. As in the main model, the slope of  $h(v)$  is crucial. Refer again to Fig. 4. If  $h(v)$  is constant and falls between  $\tau^{\sigma-1} f_D$  and  $f_E$ , all marginal varieties are still sold only in the domestic market, and all firms reduce their scope. If  $h(v)$  increases with  $v$  but at a rate so slow that it intersects the industry's most productive firm's profit curve at a point to the right of the firm's kink, all firms' marginal varieties are still not exported, and all firms reduce their scope as a result. Only when  $h(v)$  increases sufficiently fast, as in the case depicted in Fig. 4, will the more-productive firms expand their scope because their marginal varieties are exported.

To summarize, when demand is derived from CES preferences and in response to trade liberalization,  $h'(v) > 0$  is necessary for scope expansion, a large  $h'(v)$  is sufficient for the more-productive firms to

expand scope, and less-productive firms always reduce scope. These conclusions are exactly the same as those derived from the main model where the demand arose from linear-quadratic preferences. The main result is therefore robust to different specifications of consumer preferences.

### 6. Concluding remarks

This analysis has shown that, contrary to the conclusions of most previous studies, globalization may lead more-productive firms to expand their scope and that when that happens, their efficiency may drop even though overall industry efficiency always rises. The results therefore caution about extending findings about cross-firm rationalization among single-product firms to within-firm rationalization among the different varieties that a multiproduct firm produces. The pattern of scope adjustment depends on a number of factors including the distribution of firm productivity, the degree of within-firm heterogeneity, the properties of the variety-introduction fee, and the stage of globalization. The key finding is that a fast-rising variety-introduction fee is necessary and sufficient to induce productive firms to expand their scope in response to globalization. This has been ignored in all previous studies, which has led them to make different predictions regarding scope adjustment by multiproduct firms.

These results complement those prior studies by providing a more complete picture of how globalization affects multiproduct firms and their industries. It clarifies forces driving different patterns of scope adjustment. The rich set of predictions arising from all these studies should prove very useful for future empirical investigations.

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### Appendix A

#### A.1. Proof of Proposition 1

We first establish the existence and uniqueness of  $v^*(x)$ . For a given  $x$ , the optimal  $v^*$  is reached when  $\pi(x, v^*) = h(v^*)$ .  $\pi(x, v)$  is weakly decreasing in  $v$ , while  $h(v)$  is weakly increasing in  $v$ . If  $\pi(x, 0) < h(0)$ , the firm will exit; it cannot be an active firm. If  $\pi(x, v) > h(v)$  for any  $v \geq 0$ ,  $v^*(x) = \infty$  and the firm's profit will be infinite, which again cannot happen because the equilibrium condition (5) requires firm-level profit to be finite. So there is a unique solution of  $v^*(x) \geq 0$  satisfying (3).

- (i) The sign of  $\frac{dv^*(x)}{dx}$ : Total differentiation with respect to  $x$  of both sides of Eq. (3) yields

$$-\frac{L}{2\gamma}(A-c(x, v^*(x))) \left[ c_x + c_v \frac{dv^*(x)}{dx} \right] = h'(v^*(x)) \frac{dv^*(x)}{dx}. \quad (7)$$

If  $A - c(x, v^*(x)) > 0$ , then (7) implies that  $\frac{dv^*(x)}{dx} = -\frac{(A-c(x, v^*(x)))c_x}{(A-c(x, v^*(x)))c_x + 2\gamma h'(v^*(x))} < 0$ . If  $A - c(x, v^*(x)) = 0, \forall x$ , then (3) implies that  $h'(\cdot) \equiv 0$  and, consequently  $c_v > 0$ . A total differentiation with respect to  $x$  on  $A - c(x, v^*(x)) \equiv 0$  yields  $c_x + c_v \frac{dv^*(x)}{dx} = 0$ . Because  $c_x > 0$  and  $c_v > 0$ , we have  $\frac{dv^*(x)}{dx} < 0$ .

- (ii) The marginal variety's unit cost:  $\frac{dc(x, v^*(x))}{dx} = c_x + c_v \frac{dv^*(x)}{dx}$ . If  $h'(\cdot) > 0$ , then (7) implies  $A - c(x, v^*(x)) \neq 0$  so  $A - c(x, v^*(x)) > 0$ . Then (7) again implies  $c_x + c_v \frac{dv^*(x)}{dx} = -\frac{2\gamma h'(v^*(x)) \frac{dv^*(x)}{dx}}{A-c(x, v^*(x))} > 0$ . If  $h'(\cdot) \equiv 0$ , then (7) implies that either  $c_x + c_v \frac{dv^*(x)}{dx} = 0$ , or  $A - c(x, v^*(x)) = 0$ , which in turn implies  $\frac{dc(x, v^*(x))}{dx} = 0$ .

A.2. Proof of Corollary 1

- (i) If  $h'(\cdot) > 0$ , by Proposition 1(ii),  $c(x, v^*(x)) < c(x_n, v^*(x_n)) = c_n$  for all  $x < x_n$ . Since  $c(x, v) \leq c(x, v^*(x))$ , then  $c(x, v) < c_n$  for  $x < x_n$ , and  $c(x, v) = c_n$  if and only if  $x = x_n$ .
- (ii) If  $h'(\cdot) \equiv 0$  (which requires  $c_v > 0$ ), by Proposition 1(ii),  $c(x, v^*(x)) = c(x_n, v^*(x_n)) = c_n$  for all  $x \in [0, x_n]$ . Because  $c_v > 0$ , then  $c(x, v) < c(x, v^*(x)) = c_n$  for  $v < v^*(x)$ .

A.3. Proof of Proposition 2

- (i) Rewrite Eq. (5) as  $\Pi^e(A(L), L) = f$  to indicate that  $A$  is a function of  $L$ . Total differentiation with respect to  $L$  yields  $\frac{\partial \Pi^e}{\partial A} \frac{dA}{dL} + \frac{\partial \Pi^e}{\partial L} = 0$ . We have seen that  $\frac{\partial \Pi^e}{\partial A} > 0$ . Now,  $\frac{\partial \Pi^e}{\partial L} = \Pi(x_n) \frac{\partial x_n(A, L)}{\partial L} + \int_0^{x_n(A, L)} \frac{\partial \Pi(x)}{\partial L} dG(x) = \int_0^{x_n(A, L)} \frac{\partial \Pi(x)}{\partial L} dG(x) > 0$ , as  $\frac{\partial \Pi(x)}{\partial L} = \int_0^{v^*(x)} \frac{\partial \pi(x, v)}{\partial L} dv > 0$ . As a result,  $\frac{dA}{dL} < 0$ .  
 Look at (4) which determines  $x_n(A, L)$ :  $\frac{1}{4\gamma}(A - x_n(A, L))^2 = h(0)$ . Total differentiation with respect to  $L$  yields  $(A - x_n(A, L)) [A - x_n(A, L) + 2L \frac{dA}{dL} - 2L \frac{dx_n(A, L)}{dL}] = 0$ . If  $A - x_n(A, L) = 0$ , then  $\frac{dx_n(A, L)}{dL} = \frac{dA}{dL} < 0$ . If  $A - x_n(A, L) \neq 0$ , then  $\frac{dx_n(A, L)}{dL} = \frac{1}{2L} [A - x_n(A, L) + 2L \frac{dA}{dL}]$ . The proof of (ii) below shows that  $A + 2L \frac{dA}{dL} = \tilde{c}_\pi < c_n = x_n(A, L)$ . Therefore again  $\frac{dx_n(A, L)}{dL} < 0$ .
- (ii) Look at the variety-level profit:  $\pi(x, v) = \frac{1}{4\gamma}(A - c(x, v))^2$ . Let  $c = c(x, v)$  and write the profit as  $\pi(L, A(L), c) = \frac{1}{4\gamma}(A(L) - c)^2$ . Then  $\frac{d\pi(L, A(L), c)}{dL} = \frac{A-c}{4\gamma}(A - c + 2L \frac{dA}{dL})$ . Because  $c(x, v)$  strictly increases in  $x$  for any given  $v$ , it is impossible to have  $A - c(x, v) = 0, \forall x$ . Given that  $A - c > 0$ , it follows that  $\frac{d\pi(L, A(L), c)}{dL} > 0$  if and only if  $c < \tilde{c}_\pi \equiv A + 2L \frac{dA}{dL}$ . By the definition of  $\tilde{c}_\pi$ ,  $\frac{d\pi(L, A(L), c)}{dL} = \frac{A-c}{4\gamma}(\tilde{c}_\pi - c)$ . If  $\tilde{c}_\pi \leq 0$ , then  $\frac{d\pi(L, A(L), c)}{dL} = \frac{A-c}{4\gamma}(\tilde{c}_\pi - c) \leq 0, \forall x$ , with strict inequality for at least some  $v$  for any given  $x$ . Then  $\frac{d\pi(x)}{dL} = \int_0^{v^*(x)} \frac{d\pi(L, A(L), c(x, v))}{dL} dv < 0$ ,  $\forall x$ . As a result  $\frac{d\Pi^e(L, A(L))}{dL} = \Pi(x_n) \frac{dx_n(A, L)}{dL} + \int_0^{x_n(A, L)} \frac{d\pi(x)}{dL} dG(x) = \int_0^{x_n(A, L)} \frac{d\pi(x)}{dL} dG(x) < 0$ , which cannot happen in equilibrium because Eq. (5) requires  $\frac{d\Pi^e(L, A(L))}{dL} = 0$ . We therefore conclude that  $\tilde{c}_\pi > 0$ . If  $\tilde{c}_\pi \geq c_n$ , then  $\frac{d\pi(L, A(L), c)}{dL} = \frac{A-c}{4\gamma}(\tilde{c}_\pi - c) \geq 0, \forall x$ , with strict inequality for at least some  $v$  for any given  $x$ . Then  $\frac{d\pi(x)}{dL} = \int_0^{v^*(x)} \frac{d\pi(L, A(L), c(x, v))}{dL} dv > 0, \forall x$ . Then  $\frac{d\Pi^e(L, A(L))}{dL} = \int_0^{x_n(A, L)} \frac{d\pi(x)}{dL} dG(x) > 0$ , which again cannot happen in equilibrium due to the free entry condition (5). We therefore conclude that  $\tilde{c}_\pi < c_n$ .
- (iii) The variety-level output is  $q(x, v) = \frac{1}{2\gamma}(A - c(x, v))$ . As in (ii), write this output as  $q(L, A(L), c) = \frac{1}{2\gamma}(A(L) - c)$ . Then  $\frac{dq(L, A(L), c)}{dL} = \frac{1}{2\gamma}(A - c + L \frac{dA}{dL})$ , and therefore  $\frac{dq(L, A(L), c)}{dL} > 0$  if and only if  $c < \tilde{c}_q \equiv A + L \frac{dA}{dL}$ . Because  $\frac{dA}{dL} < 0$ , it follows that  $\tilde{c}_q = A + L \frac{dA}{dL} > A + 2L \frac{dA}{dL} = \tilde{c}_\pi$ .

A.4. Proof of Proposition 3

- (i) The equation that determines the optimal scope, (3), is  $\frac{1}{4\gamma}(A - c(x, v^*(x)))^2 = h(v^*(x))$ . Total differentiation with respect to  $L$  yields

$$\frac{A - c(x, v^*(x))}{4\gamma} \left( A - c(x, v^*(x)) + 2L \frac{dA}{dL} - 2c_v L \frac{dv^*(x)}{dL} \right) = h'(v^*(x)) \frac{dv^*(x)}{dL}. \quad (8)$$

If  $A - c(x, v^*(x)) \neq 0$  (which means  $A - c(x, v^*(x)) > 0$ ), then rearrange (8) as

$$\frac{A - c(x, v^*(x))}{4\gamma} \left( A - c(x, v^*(x)) + 2L \frac{dA}{dL} \right) = \left( h'(v^*(x)) + \frac{c_v L}{2\gamma} (A - c(x, v^*(x))) \right) \frac{dv^*(x)}{dL}.$$

Because  $A - c(x, v^*(x)) > 0$  and  $h'(v^*(x)) + \frac{c_v L}{2\gamma}(A - c(x, v^*(x))) > 0$ , then  $\frac{dv^*(x)}{dL} > 0$  if and only if  $A - c(x, v^*(x)) + 2L \frac{dA}{dL} > 0$ , or equivalently  $c(x, v^*(x)) < A + 2L \frac{dA}{dL} = \tilde{c}_\pi$ .

If  $A - c(x, v^*(x)) = 0, \forall x$  (which by (3) implies  $h'(\cdot) = h(\cdot) \equiv 0$ , which in turn requires  $c_v > 0$ ), then total differentiation with respect to  $L$  on  $A - c(x, v^*(x)) = 0$  yields  $\frac{dA}{dL} = c_v \frac{dv^*(x)}{dL}$ . Because  $\frac{dA}{dL} < 0$  and  $c_v > 0$ , it must be so that  $\frac{dv^*(x)}{dL} < 0$ . Note that in this case,  $c(x, v^*(x)) = A > A + 2L \frac{dA}{dL} = \tilde{c}_\pi$ . The result of  $\frac{dv^*(x)}{dL} < 0$  is therefore consistent with the statement that  $\frac{dv^*(x)}{dL} > 0$  if and only if  $c(x, v^*(x)) < \tilde{c}_\pi$ .

- (ii) Suppose that  $c(0, v^*(0)) \geq \tilde{c}_\pi$ . By Proposition 1(ii),  $\frac{dc(x, v^*(x))}{dx} > 0$  (when  $h'(\cdot) > 0$ ) or  $\frac{dc(x, v^*(x))}{dx} = 0$  (when  $h'(\cdot) = 0$ ). In either case,  $c(x, v^*(x)) \geq \tilde{c}_\pi, \forall x$ . By Proposition 3(i),  $\frac{dv^*(x)}{dL} < 0, \forall x$ . Suppose that  $c(0, v^*(0)) < \tilde{c}_\pi$ . Because  $\tilde{c}_\pi < c_n = c(x_n, v^*(x_n))$ , it is impossible that  $c(0, v^*(0)) = c(x_n, v^*(x_n))$ . By Proposition 1(ii), this means that  $h'(\cdot) > 0$ . By Proposition 1(ii) again,  $\frac{dc(x, v^*(x))}{dx} > 0$ . But  $c(0, v^*(0)) < \tilde{c}_\pi < c(x_n, v^*(x_n))$ , so there exists a  $\tilde{x} \in (0, x_n)$  such that  $c(\tilde{x}, v^*(\tilde{x})) = \tilde{c}_\pi$ . Further, because  $\frac{dc(x, v^*(x))}{dx} > 0$ ,  $c(x, v^*(x)) < \tilde{c}_\pi$  if and only if  $x < \tilde{x}$ . Combined with Proposition 3(i), this means that  $\frac{dv^*(x)}{dL} > 0$  if and only if  $x < \tilde{x}$ .

A.5. Proof of Proposition 5

In this and the next proof, it is convenient to define  $t \equiv \frac{\sqrt{4\gamma}}{L} > 0$ . Then for any variable  $z$ ,  $\frac{dz}{dt}$  has the opposite sign of  $\frac{dz}{dL}$ . The optimal scope is  $v^*(x) = \frac{A-x}{\lambda+\rho t}$ . Given the equilibrium  $A(t)$ , calculation shows that  $\frac{dv^*(x)}{dL} > 0$  (i.e.,  $\frac{dv^*(x)}{dt} < 0$ ) if and only if  $A[2k\rho^2 t^2 + \rho t \lambda(k-3) - 2\lambda^2] - \rho t(k+3)(\lambda + 2\rho t)x > 0$ , or equivalently

$$\frac{x}{A} < \frac{2k\rho^2 t^2 + \rho t \lambda(k-3) - 2\lambda^2}{\rho t(k+3)(\lambda + 2\rho t)} = \frac{1}{k+3} \left[ k - \frac{2+3\gamma}{y(1+2\gamma)} \right] \equiv r_v. \quad (9)$$

- (i) There is scope expansion (at least for  $x = 0$ ) if and only if the right-hand side of the above inequality is positive.
- (ii) Given that  $r_v > 0$  and that  $x_n = A$ , the scope-expanding firms are those with  $x \in [0, x_n r_v]$ , so  $r_v$  is the percentage of firms that expand their scope. It is straightforward to verify that  $\frac{\partial r_v}{\partial k} > 0$  and  $\frac{\partial r_v}{\partial \gamma} > 0$ .
- (iii) From (9),  $\frac{dv^*(x)}{dL} > 0$  if and only if  $x < \frac{A}{k+3} \left[ k - \frac{2+3\gamma}{y(1+2\gamma)} \right] \equiv \tilde{x}$ . Note that  $A$  is an endogenous variable depending on  $t, \rho$  and  $\lambda$ :  $A = \frac{\phi t^2(\lambda + \rho t)}{\lambda + 2\rho t}$ . Direct calculation shows that  $\frac{\partial \tilde{x}}{\partial t} = \frac{2A}{t(k+3)^2} \frac{(6\gamma^4 + 12\gamma^3 + 12\gamma^2 + 6\gamma + 1)k + (6\gamma^2 + 6\gamma + 1)}{y(\gamma+1)(2\gamma+1)^2} > 0$ . As a result,  $\frac{\partial \tilde{x}}{\partial t} < 0$ . Also,  $\frac{\partial \tilde{x}}{\partial \lambda} = -\frac{A}{(k+3)^2} \frac{(3\gamma+1)(3\gamma+2)k + (6\gamma^2 + 15\gamma + 8)}{\rho t(\gamma+1)(2\gamma+1)^2} < 0$ .  $\frac{\partial \tilde{x}}{\partial \rho} = \frac{2A}{\rho(k+3)^2} \frac{(2\gamma^4 + 4\gamma^3 + 7\gamma^2 + 5\gamma + 1)k + (6\gamma^2 + 19\gamma^2 + 15\gamma + 3)}{y(\gamma+1)(2\gamma+1)^2} > 0$ .

A.6. Proof of Proposition 6

$$\text{Industry average cost is } \bar{c} = \frac{\int_0^A \int_0^{v^*(x)} c(x,v)q(x,v)dv dG(x)}{\int_0^A \int_0^{v^*(x)} q(x,v)dv dG(x)} = \frac{A(t)[\lambda(\lambda+3\rho t)+k(\lambda+\rho t)(\lambda+2\rho t)]}{(k+3)(\lambda+\rho t)(\lambda+2\rho t)}$$

Given that  $y = \frac{\rho t}{\lambda}$ , calculation shows that  $\frac{d\bar{c}}{dt} < 0$  (or equivalently  $\frac{d\bar{c}}{dt} > 0$ ) always holds.

(i) The within-firm HHI is  $\eta(x) = \frac{1}{v^*(x)} \int_0^{v^*(x)} \left(\frac{q(x,v)}{Q(x)}\right)^2 dv = \frac{4(\lambda+\rho t)^2(\lambda^2+3\lambda\rho t+3\rho^2 t^2)}{3(\lambda+2\rho t)^2(A(t)-x)^2}$ . It can be shown that  $\frac{d\eta(x)}{dt} < 0$  (i.e.,  $\frac{d\eta(x)}{dt} > 0$ ) if and only if  $\frac{x}{A} < \frac{1}{k+3} \left(k - \frac{(3y+1)(6y^2+9y+4)}{y(12y^3+18y^2+9y+1)}\right) = r_q$ .

(ii) Firm-level average cost is  $\bar{c}(x) = \frac{\int_0^{v^*(x)} c(x,v)q(x,v)dv}{\int_0^{v^*(x)} q(x,v)dv} = \frac{A(t)\lambda(\lambda+3\rho t)+2x(\lambda^2+3\lambda\rho t+3\rho^2 t^2)}{3(\lambda+\rho t)(\lambda+2\rho t)}$ . It can be shown that  $\frac{d\bar{c}(x)}{dt} > 0$  (i.e.,  $\frac{d\bar{c}(x)}{dt} < 0$ ) if and only if  $\frac{x}{A} < \frac{1}{k+3} \left(k - \frac{1+6y+6y^2}{y^2(2+3y)}\right) = r_c$ .

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