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Inflexible Repositioning: Commitment in Competition and Uncertainty

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Abstract. We study the value of commitment in a business environment that is both competitive and uncertain, in which two firms face stochastic demands and compete in positioning and repositioning. If the future demand tends to disperse or the demand uncertainty is sufficiently large, one firm chooses rigidity (i.e., commits not to change its positions), and the other chooses flexibility (i.e., to reposition freely). We find that a firm's rigidity can benefit not only itself, but also its flexible rival. When uncertainty is larger, rigidity becomes more valuable relative to flexibility. These results arise because the asymmetric equilibrium generates two collective gains in addition to the usual individual gain (in terms of competitive advantages) accrued to the committing firm. A firm's rigid repositioning can soften competition and generate a commitment value, and the other firm's flexible repositioning generates an option value. Both values then spill over to competitors within the ecosystem. These results suggest that, when firms compete under uncertainty, commitment and options are valuable not only for the party that is making the choice, but also for all competing parties collectively. Commitment value and option value do not have to be mutually exclusive; they can coexist and even strengthen each other through unilateral commitment, which achieves the best of both strategies.

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1. Introduction

It has been well known that commitment is valuable in competition as the inability to back down is an advantage in confrontation (Schelling 1960, Fudenberg and Tirole 1991, Dixit and Nalebuff 2008). It has been equally well known that the opposite strategy, that is, flexibility, is valuable in an uncertain environment as future adjustment remains an option (Jordan and Graves 1995, Sanchez 1995, Johnson et al. 2003). In the literature, these two lines of arguments rarely intersect. Commitment is usually evaluated in a deterministic world, whereas option value is mainly studied for a single decision maker, that is, in a noncompetitive setting. In real life, however, the most common environment in which a business operates is both uncertain and competitive. In that case, should a company commit or not commit? Does commitment become more or less valuable as uncertainty increases? Does a company's commitment always hurt its competitors (Dixit 1980, Fudenberg and Tirole 1984)? More broadly, what is the relationship between commitment value and option value when both are present?

These questions are not only relevant for theoretical interests; they also have practical values. Many corporate decisions have long-term impacts on how costly it may be for a company to adjust its businesses in the future. For example, modularity in product design can help a firm reconfigure its products at minimum cost (Sanchez 1995, Worren et al. 2002). Multipurpose resources, such as cross-trained labor and flexible machines and factories, facilitate customization (Iravani et al. 2005). Corporate restructuring, on the other hand, tends to make adjustment more difficult as internal coordination and information flow are no longer available after a divestiture or spin-off. In these examples, how flexibly a business can be adjusted in the future becomes a choice that must be decided in advance.[1]

To study flexibility choices in competition and uncertainty, we develop a game-theoretical model in which firms compete in positioning and repositioning. Imagine a Hotelling straight line occupied by two firms. The demand is stochastic so that consumer distribution may disperse or concentrate relative to the two firms' initial positions. Before the demand realizes, each firm must choose one of two repositioning strategies: flexibility (meaning that the firm can reposition freely in the future) or rigidity (meaning that the firm cannot change its position). Afterward, the demand realizes, and the two firms compete by adjusting their positions subject to the constraints placed by their respective choices of the repositioning strategy.

The analysis reveals that a firm's optimal strategy depends on its rival's choice as well as how future demands distribute relative to its initial position. If its rival is rigid, then a firm's best choice is always flexibility; this manifests the conventional option value in uncertainty. If its rival is flexible, a firm faces the following trade-off when choosing between flexibility and rigidity.[2] When demand disperses, rigidity commits the firm to its initial position, which is now closer to the center because of demand dispersion. This forces its flexible rival to move away, giving the rigid firm valuable market shares. Therefore, rigidity generates a competitive advantage in demand dispersion; this is the traditional argument of commitment value in competition. When demand concentrates, the opposite happens: rigidity prevents the firm from adjusting to the new distribution of demand, which is now farther from its initial position. In that case, rigidity brings a competitive disadvantage. Such a drawback is a new effect of commitment that arises only when competition is carried out under uncertainty.

Given this trade-off when facing its rival's flexibility, the firm's ex ante optimal choice (i.e., before the uncertainty is resolved) is rigidity if demand dispersion is more likely and flexibility if concentration is more likely. Consequently, there can be two possible outcomes. When demand concentration dominates, both firms choose flexibility; when dispersion dominates, one firm chooses rigidity and the other chooses flexibility, that is, unilateral rigidity. Note that rigidity is valuable only when it can influence its rival's positioning choice, that is, only when the rival is flexible. Interestingly and somewhat surprisingly, we find that a firm's rigidity can benefit its flexible rival, and rigidity is more valuable when demand uncertainty is larger. A flexible rival may gain because the rigid firm will be far away from the center when demand concentrates, which softens the overall price competition. When the demand varies more, a rigid firm's gain (when business is good under demand dispersion) is further expanded and its loss (when business is bad under demand concentration) is further suppressed, so the expected net gain is larger.

These results demonstrate that competition and uncertainty interact in a nontrivial way, giving rise to new features that do not exist when commitment and option value are studied separately. Two features are worth mentioning. First, uncertainty enriches the value of commitment. In a deterministic world, *equilibrium* commitment always hurts competitors. This is because competition is usually a zero-sum game. If a party gains competitive advantages through commitment, its rivals must be facing disadvantages in competition.[3] In some situations, it may be possible for commitment to soften competition and, therefore, benefit all competitors, but then every party would prefer its rivals to commit rather than committing itself.[4] In our model, by contrast, there always exists an area in the parameter space in which one firm chooses rigidity and the other firm benefits from the rigidity. Such a win–win outcome requires coexistence of competitive advantage and softened competition. The former leads to an individual gain that is needed for a party to indeed want to commit, whereas the latter leads to a collective gain that allows competitors to be better off even when the committing party is apparently better off too. These two gains seem to contradict each other, and they coexist only in an uncertain environment as they appear in different realizations of the uncertainty. In our model, the same choice of rigidity leads to aggression when demand disperses and appeasement when demand concentrates; a rigid firm is essentially a cat that is at once "fat" and "lean and hungry" (Fudenberg and Tirole 1984).

Second, uncertainty also enhances the value of commitment. To commit is to give up the opportunity to make adjustments. Because an option is more valuable when uncertainty is larger, one may think that a greater uncertainty would favor flexibility. In fact the opposite is true in our model. Fixing its rival's flexibility, a firm would choose flexibility when uncertainty is small, and rigidity when uncertainty is large. Therefore, uncertainty favors commitment. More precisely, uncertainty increases the value of flexibility, but it increases the value of commitment even more. This is because a committing party does not lose the option value associated with uncertainties. Rather, flexibility by its rival continues to generate an option value, a substantial part of which is now captured by the committing party.

The key message of the research can, therefore, be summarized as follows. When firms compete under uncertainty, commitment and options are valuable not only for the party that is making the choice, but also for all competing parties collectively. For a given decision maker, these two values are indeed mutually exclusive as commitment means giving up options and flexibility means giving up commitment.[5] In the whole ecosystem, however, they can coexist and even strengthen each other. Collective gains are realized through unilateral commitment in which one firm's choice generates a commitment value and the other firm's choice generates an option value, each of which then spills over to its competitor.

In addition to these theoretical insights, the research also develops a series of prescriptions that can directly inform or illuminate practitioners. A company should take a holistic view. Commitment is most valuable when the rival is not committing. A firm does not have to commit itself in order to gain from commitment; it may rely on its rival to provide the benefit of commitment. Conversely, a firm may also enjoy the benefit of flexibility from its rival. In terms of individual strategies, a firm should consider commitment more if uncertainty is larger or future demands tend to disperse. If firms differ in production costs, it is the inefficient firm that is more likely to commit. Regardless of which firm is committing, positioning advantage can overcome cost disadvantage such that the inefficient firm earns a higher profit than the efficient firm. A greater efficiency gap makes it easier for the efficient firm to benefit from the inefficient firm's rigidity but more difficult for the inefficient firm to benefit from the efficient firm's rigidity. Finally, commitment is more likely to be the optimal choice for a new product than for an existing product.

The suggestion that a firm may purposely restrict its business options is not that far stretched. The leader in instant photography, Polaroid, deliberately refused to diversify, leaving itself no route for repositioning. The rigidity strategy paid off in 1976 when Eastman Kodak entered the market. Polaroid responded strongly by suing Kodak for patent infringement and, after 24 years of marathon litigation, forced Kodak to withdraw. Polaroid chairman, Edwin Land, explained his resolve: "This is our very soul and our whole life. For them it's just another field. . .We will stay in our lot and protect that lot" (Dixit and Nalebuff 2008, p. 126).[6]

The value of commitment has long been recognized in game theory and war: "A party can strengthen its position by overtly worsening its own options" (Schelling 1960). In business, strategic investment in extra capacity commits an incumbent to aggressive competition and, therefore, deters entry (Spence 1977). Commitment through loss-leader pricing (Lal and Matutes 1994) or limited capacity in advance selling (Xie and Shugan 2009) can lead to win–win, but that is among business partners, such as a company and its customer or supplier, unlike the two firms in our model, which are competitors. Consumers may become vulnerable if they incur costs before a transaction, and a seller may commit to competitive pricing by colocation (Wernerfelt 1994), limited span of product lines (Villas-Boas 2009), or the introduction of a competitor (Farrell and Gallini 1988). None of these studies involves uncertainty.

The "strategic flexibility" literature emphasizes the value of flexibility and focuses on the means of achieving flexibility through careful planning and arrangements in operation management (Jordan and Graves 1995, Van Mieghem 1998), marketing (Johnson et al. 2003), and strategic management (Sanchez 1995). Most of these studies are carried out in a noncompetitive environment.

A few studies look at settings with both competition and uncertainty and focus on the trade-off between commitment's competitive advantage and flexibility's option values.[7] In some research, the choice between commitment and flexibility is complicated by other decisions involving capacity, production, interlinked markets, and output allocation. The key message is that commitment is still valuable in uncertainty as long as a firm faces competition or entry threat (Anand and Girotra 2007, Anupindi and Jiang 2008), and flexibility is more valuable when uncertainty is larger (Goyal and Netessine 2007). Assuming output as the only choice, Spencer and Brander (1992) studied whether it is advantageous to produce before or after uncertainties are resolved and found that commitment becomes less valuable when uncertainty is larger. This is the opposite of our finding. The reason is that the commitment variable and the competition variable are the same in Spencer and Brander (1992) (i.e., the output) but are different in our model (i.e., positioning and pricing, respectively).[8] This demonstrates that our results are particularly relevant for positioning and repositioning.

Positioning is a core concept in marketing, and repositioning is a commonly observed practice. However, the strategic role of repositioning is not well studied.[9] In a recent paper, Villas-Boas (2018) analyzed a monopolist's optimal repositioning strategy under changing consumer preferences. The major trade-off is between an exogenous repositioning cost and a better match with evolving preferences. Because there is no competition, there is no room for commitment.

The remainder of the paper is organized as follows. After setting up the model in Section 2 with preliminary analysis, we establish in Section 3 the conditions under which rigidity may arise in equilibrium and demonstrate that a flexible firm may also benefit from its rival's rigidity. Section 4 considers seven extensions, including sequential move, asymmetric firms, alternative ways of modeling consumer preference change, mild rigidity, demand link, new products, and vertical product differentiation. Finally, Section 5 concludes.

2. Model Setting and Preliminary Analysis

Two firms, A and B, compete on a Hotelling straight line stretching from negative infinity to positive infinity. Marginal cost of production is constant and identical for both firms and is, therefore, normalized to zero. A total mass of $m>0$ consumers distribute uniformly on $[-\mu,\mu]$ for $\mu>0$. Demand is uncertain in that m and μ are both random variables such that

$$\begin{cases} \text{with probability } p_h \in [0,1]: & \mu = h \text{ and } m = m_h \\ \text{with probability } 1-p_h: & \mu = l \text{ and } m = m_l \end{cases},$$

where $h \geq l > 0$ with the interpretation that h means more dispersed consumer tastes than the l state of the world. The demand parameters, p_h, h, l, m_h, and m_l, are common knowledge. Consumers have unit demand (i.e., each purchasing at most one unit of either product) with valuation $v > 0$ and incur quadratic transportation costs when purchasing a product. In particular, if a consumer located at x purchases a product located at y and priced at p, the consumer's net utility is $v - p - t(x-y)^2$. We assume that v is sufficiently high such that all consumers make positive purchase for all the relevant positions of the two firms.

The game proceeds in three stages. In stage one, the two firms simultaneously choose their repositioning strategies. There are two possible choices at equal cost (which is normalized to zero): *flexibility*, meaning that a firm can freely change its position in the future in response to any demand realization, and *rigidity*, meaning that a firm has to stay at its original position regardless of the demand realization. Assume that the two firms locate at $a_0 = -3/2$ and $b_0 = 3/2$ initially.[10] We denote flexibility by zero and rigidity by ∞, reflecting the interpretation that flexibility means repositioning at zero cost and rigidity means repositioning at infinite cost. In stage two, one of the two possible states of the world is realized, and the two firms simultaneously choose their positions on the straight line subject to their respective repositioning constraints. We assume that a firm's position can be outside the range of consumer location distribution.[11] Without loss of generality, assume firm A is on the left and firm B is on the right. In stage three, after observing each other's positions, the two firms simultaneously choose prices. Facing the two firms' positions and prices, consumers choose which product to purchase, and the two firms receive their profits.

The model setting is fairly standard. Later, in a series of extensions, we examine the impacts of many alternative assumptions.

2.1. Stage Three: Price Competition

The analysis starts with backward induction. In stage three, given the two firms' positions (a and b) and prices (p^A and p^B), a consumer is indifferent between buying from either firm if the consumer's location, $\tilde{x}$, is such that $v - p^A - t(\tilde{x}-a)^2 = v - p^B - t(b-\tilde{x})^2$, which means $\tilde{x} = [p^B - p^A + t(b^2 - a^2)]/[2t(b-a)]$. Consumers with location $x < \tilde{x}$ buy from firm A, and consumers with location $x > \tilde{x}$ buy from firm B. The sales for firm A is, therefore,

$$q^A(p^A, p^B) = \begin{cases} m & \text{if } \tilde{x} > \mu \\ (\frac{1}{2} + \frac{\tilde{x}}{2\mu})m & \text{if } -\mu \le \tilde{x} \le \mu \\ 0 & \text{if } \tilde{x} < -\mu, \end{cases}$$

and the sales for firm B is $q^B(p^A, p^B) = m - q^A(p^A, p^B)$. Price competition leads to the following equilibrium profits:

$$\pi^A(a,b) = \frac{tm}{36\mu}(b-a)(6\mu + a + b)^2, \text{ and}$$
$$\pi^B(a,b) = \frac{tm}{36\mu}(b-a)(6\mu - a - b)^2. \quad (1)$$

2.2. Stage Two: Positioning Competition

In stage two, the two firms choose their positions simultaneously subject to their repositioning constraints. Let $f^i \in \{0, \infty\}$ denote firm i's repositioning choice, and $\pi(f^j, f^k|\mu)$ denote a firm's equilibrium profit when its own repositioning strategy is f^j while its rival's repositioning strategy is f^k given the realized μ.

If $f^A = f^B = 0$, both firms can freely move. Facing any realized μ, they choose their positions a and b to maximize their respective profits as expressed in (1). The best responses are $a^* = -2\mu + b/3$ and $b^* = 2\mu + a/3$. Fixing the rival's position, a firm's optimal position balances the following trade-off: moving closer toward the rival increases the firm's own market share and reduces the mismatch between the product attribute and consumer tastes, but it intensifies price competition. For given μ, positions are strategic substitutes in terms of closeness to the center: when a firm is closer to the center, its rival's best response is to move away from the center. The equilibrium positions are

$$a = -\frac{3}{2}\mu \text{ and } b = \frac{3}{2}\mu.$$

Note that the equilibrium positions are proportional to μ but independent of t and m. Also note that the two positions move apart as μ increases. The resulting equilibrium profit is $\pi(0, 0|\mu) = 3tm\mu^2$.

Lemma 1. *When both firms can move freely, for any realization of the demand,*

i. *positions are strategic substitutes in the sense that when a firm is closer to the center, its rival's best response is to move away from the center, albeit by a shorter distance; and*

ii. *the firms move apart when consumer distribution disperses and move toward each other when the distribution concentrates.*

If $f^A = f^B = \infty$, neither firm can move, so $a = a_0 = -3/2$ and $b = b_0 = 3/2$. In equilibrium, $\pi(\infty, \infty|\mu) = 3tm\mu$.

If $f^A = 0$ and $f^B = \infty$ (the case of $f^A = \infty$ and $f^B = 0$ can be derived symmetrically), firm B stays at its initial position: $b = b_0 = 3/2$. Firm A's optimal position is then determined from its best response in positioning: $a = b/3 - 2\mu = 1/2 - 2\mu$. In equilibrium, the two firms' profits are $\pi(0, \infty|\mu) = tm(1+2\mu)^3/9\mu$ and $\pi(\infty, 0|\mu) = tm(1+2\mu)(4\mu-1)^2/9\mu$. To avoid corner solutions, we assume throughout the paper that $l \geq 1/4$, that is, consumer distribution does not shrink too much in the second period.

The value of rigidity vis-*à*-vis flexibility can now be understood by comparing the relevant profit, which is captured by two profit rankings. First, $\pi(0, \infty|\mu) \geq \pi(\infty, \infty|\mu)$ (with equality if and only if $\mu = 1$), meaning that, for any realized μ, if the rival is not moving at all, a firm always benefits from flexibility. This validates the conventional argument that flexibility allows a firm to adjust to changing conditions (i.e., different demand realizations) and is, therefore, valuable. Second, $\pi(\infty, 0|\mu) > \pi(0, 0|\mu)$ if and only if $\mu > 1$, meaning that, if its rival can freely move, a firm gains from rigidity if consumer taste is more dispersed but loses otherwise. When consumers disperse, the two firms would have moved apart symmetrically if both are flexible (recall Lemma 1(ii)). However, if B cannot move, it stays at a position that is close to the center relative to the new distribution (because of consumer dispersion), which forces A to move further away (Lemma 1(i)). This increases B's market share and profit at the expense of A and, therefore, benefits B. The opposite happens when consumers become more concentrated: B is stuck at its initial position, which is now rather far from the center because of consumer concentration. Its rival takes advantage by moving even closer toward the center, which hurts B.

2.3. Stage One: Repositioning Competition

In stage one, each firm chooses its repositioning strategy to maximize the expected profit:

$$E\pi(f^j, f^k) = p_h \pi(f^j, f^k|h) + (1 - p_h)\pi(f^j, f^k|l).$$

We have established that, if its rival is rigid, a firm benefits from flexibility for any demand realization (i.e., $\pi(0, \infty|\mu) \geq \pi(\infty, \infty|\mu)$ for any μ). Before the realization, then, flexibility's optimality is preserved: $E\pi(0, \infty) > E\pi(\infty, \infty)$ holds unconditionally. If the rival is flexible, however, the optimality of a firm's own flexibility is conditional: it is advantageous if and only if the demand is more likely to concentrate. Ex ante, then, the firm's best response is flexibility if the demand is *expected* to concentrate and rigidity if the demand is expected to disperse. More precisely, rigidity is a best response to flexibility if and only if $E\pi(\infty, 0) > E\pi(0, 0)$, which can be rewritten as

$$p_h m_h \frac{(h-1)(5h^2 + 5h - 1)}{h} > -(1 - p_h) m_l \frac{(l-1)(5l^2 + 5l - 1)}{l}. \qquad (2)$$

Lemma 2. *Facing uncertain demand,*

i. *if its rival is rigid, then a firm's best response is flexibility unconditionally;*

ii. *if its rival is flexible, then a firm's best response is rigidity if (2) holds and flexibility if (2) fails.*

When choosing its repositioning strategy, a firm's major consideration is how the choice would influence its rival's positioning. A firm always wants its rival to be as far away as possible from the center, and the way to achieve the goal is to position itself as close as possible to the center (recall that positions are strategic substitutes). Because the two firms choose their positions simultaneously in stage two, the commitment power comes only from a high degree of immobility committed in stage one. It now becomes clear that commitment is useful only when two conditions are satisfied simultaneously: the rival's position is changeable (meaning that the rival is flexible in repositioning), and the committing firm's initial position is advantageous relative to the new demand, that is, the new demand is more dispersed on average.

3. Equilibrium Rigidity and Its Impacts

To better interpret condition (2), it helps to characterize the demand uncertainty in terms of some mean and variance rather than the raw parameters. Let $p' \equiv p_h m_h / [p_h m_h + (1 - p_h) m_l]$ denote the probability of demand dispersion weighted by consumer mass, $z \equiv p_h m_h / [(1 - p_h) m_l]$ the mass-adjusted relative probability of demand dispersion, $\bar{m} \equiv p_h m_h + (1 - p_h) m_l$ the mean mass of consumers, $\theta \equiv p'h + (1 - p')l$ the mean of demand spread, and $\sigma^2 \equiv p'(h - \theta)^2 + (1 - p')(l - \theta)^2$ the variance of demand spread. Then the four expected profits can be expressed in terms of z, θ, and σ^2:

$$E\pi(\infty, \infty) = 3t\bar{m}\theta, \text{ and } E\pi(0, 0) = 3t\bar{m}(\sigma^2 + \theta^2),$$

$$E\pi(0, \infty) = \frac{t\bar{m}}{9}\left[\frac{\sigma(1 - z) + \theta\sqrt{z}}{(\theta\sqrt{z} + \sigma)(\theta - \sigma\sqrt{z})} + 6 + 2\theta + 8(\sigma^2 + \theta^2)\right],$$

$$E\pi(\infty, 0) = \frac{t\bar{m}}{9}\left[\frac{\sigma(1 - z) + \theta\sqrt{z}}{(\theta\sqrt{z} + \sigma)(\theta - \sigma\sqrt{z})} - 6 + 32(\sigma^2 + \theta^2)\right].$$

As a result, condition (2) can be rewritten as

$$\frac{\sigma(1 - z) + \theta\sqrt{z}}{(\theta\sqrt{z} + \sigma)(\theta - \sigma\sqrt{z})} > 6 - 5(\sigma^2 + \theta^2). \qquad (3)$$

3.1. Equilibrium Rigidity

Given the best responses as specified in Lemma 2, it is straightforward to determine the equilibrium combination of repositioning strategies. Because our focus is whether rigidity can appear in equilibrium, we treat $(0, \infty)$ and $(\infty, 0)$ as the same equilibrium. Then we have the following.[12]

Proposition 1 (Equilibrium Rigidity). *A pure strategy equilibrium in repositioning exists and is unique. In addition,*

i. *Unilateral rigidity (i.e., $(0, \infty)$ or $(\infty, 0)$) is a Nash equilibrium if and only if (3) holds.*

ii. *Bilateral flexibility (i.e., $(0, 0)$) is a Nash equilibrium if and only if (3) fails.*

iii. *Bilateral rigidity (i.e., (∞, ∞)) is never an equilibrium.*

The two firms never choose rigidity simultaneously.[13] In equilibrium, one firm chooses flexibility, and the other chooses flexibility or rigidity depending on (3), which involves only z, σ, and θ. Denote the solution to (3) (in its equality) for σ by $\sigma_u(z, \theta)$. The left panel of Figure 1 shows the equilibrium in the space of θ and σ, where $\sigma_u(z, \theta)$ always goes through the point $(\theta, \sigma) = (1, 0)$. Note that the requirement of $l \geq 1/4$ imposes a lower bound on θ ($\theta \geq 1/4$) and an upper bound on σ: $\sigma \leq (\theta - 1/4)/\sqrt{z} \equiv \sigma_{max}$. For any given z, rigidity appears in equilibrium if the demand is more dispersed on average (i.e., $\theta \geq 1$ for any σ) or if the demand variance is sufficiently large (i.e., $\sigma > \sigma_u(z, \theta)$ when $\theta < 1$). When z increases, $\sigma_u(z, \theta)$ rotates counterclockwise around the point $(\theta, \sigma) = (1, 0)$, meaning that rigidity becomes more likely.[14]

Corollary 1. *Equilibrium rigidity is more likely if*

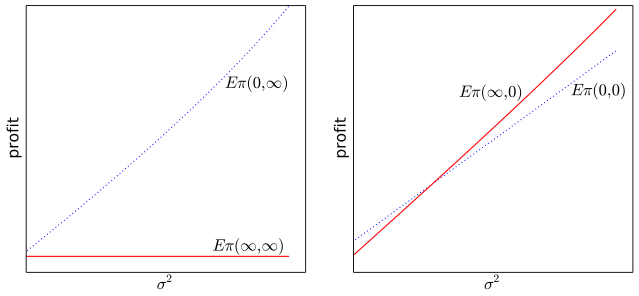
i. *the average demand is more dispersed (i.e., θ is larger), or*

ii. *demand dispersion is more likely (i.e., z is larger), or*

iii. *demand is more uncertain (i.e., σ is larger).*

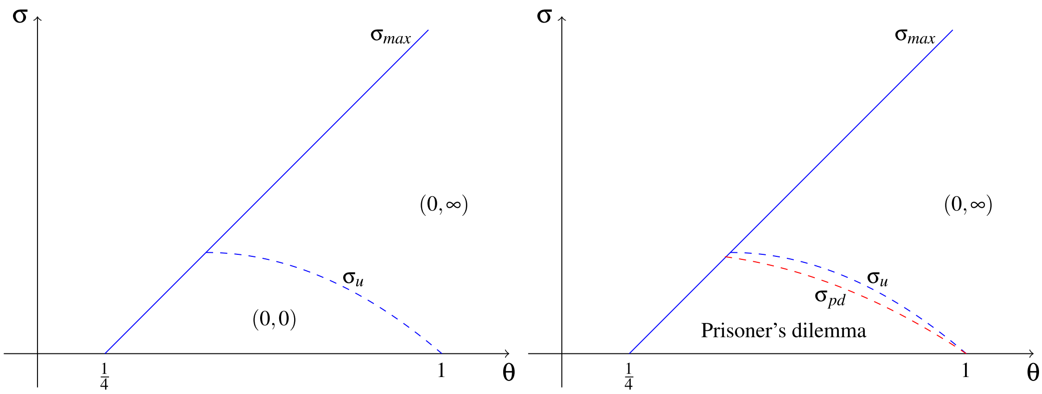
The role of θ and z is easy to understand as we have explained that rigidity is advantageous only when the realized demand is more dispersed. The role of uncertainty (i.e., σ), by contrast, is not so straightforward. Figure 2 shows how σ affects a firm's expected profit. In the left panel, facing a rigid rival, a firm's profit from flexibility is always greater than that from rigidity. Moreover, flexibility is more valuable, that is, $E\pi(0, \infty) - E\pi(\infty, \infty)$ increases, if the uncertainty becomes larger. In the right panel, $E\pi(\infty, 0)$ increases with σ, meaning that a rigid firm benefits from its rival's flexibility even though the firm itself is not moving. This is because the rigid firm gains (i.e., achieving a greater market share) when business is good (i.e., more dispersed demand) and loses when business is bad (i.e., more concentrated demand). On average, then, the rigid firm gains from demand fluctuation, more so when σ is larger. In addition, the value of rigidity vis-à-vis flexibility, that is, $E\pi(\infty, 0) - E\pi(0, 0)$, also increases with σ. Intuitively, in bilateral flexibility, the two firms are symmetric, but in unilateral rigidity, the rigid firm has an advantage over its rival when business is good and a disadvantage when business is bad. Such variation further increases the rigid firm's profits.

Figure 2. (Color online) Variance and Expected Profits



As established, if its rival is flexible, then a firm's best response is flexibility if $\sigma < \sigma_u$. Recall the previous result that, if its rival is rigid, then a firm's best response is always flexibility (Lemma 2(i)). Putting the two together, when $\sigma < \sigma_u$, flexibility is a firm's dominant strategy, and the equilibrium is bilateral flexibility, $(0, 0)$, as shown in the left panel of Figure 1. On the other hand, if both firms adopt rigidity, each earns a profit of $E\pi(\infty, \infty)$. It turns out that bilateral rigidity gives both firms a higher profit than bilateral flexibility, that is, $E\pi(\infty, \infty) > E\pi(0, 0)$, if and only if $\sigma < \sqrt{\theta(1 - \theta)} \equiv \sigma_{pd}$. As shown in the right panel of Figure 1, σ_{pd} is slightly below σ_u. Therefore, when $\sigma < \sigma_{pd}$, the two firms face a prisoner's dilemma: each has a dominant strategy in flexibility, and yet both can be better off if they both adopt the dominated strategy of rigidity. By both staying at their initial positions,

Figure 1. (Color online) Equilibrium Repositioning for Given z



the two firms soften price competition when consumer tastes concentrate. This is collectively beneficial but privately suboptimal as each firm has a unilateral, strong incentive to move toward the center.

Corollary 2. *When $\sigma < \sigma_{pd}$, the two firms face a prisoner's dilemma: each has a dominant strategy in flexibility, and yet both can be better off if they adopt rigidity simultaneously.*

3.2. Rigidity Can Benefit the Flexible Rival

We now evaluate the impacts of rigidity by comparing the two firms' profits in equilibrium unilateral rigidity with those in bilateral flexibility as a benchmark. By revealed preference, the rigid firm must be better off in unilateral rigidity as it voluntarily chooses rigidity. What about the flexible firm? Is it hurt by its rival's rigidity?

It is straightforward to show that $E\pi(0,\infty) > E\pi(0,0)$ is equivalent to

$$\frac{\sigma(1-z)+\theta\sqrt{z}}{(\theta\sqrt{z}+\sigma)(\theta-\sigma\sqrt{z})} > 19(\sigma^2+\theta^2) - 12\theta - 6, \qquad (4)$$

which, in turn, translates into $\sigma < \sigma_w(z,\theta)$. In addition, $\sigma_w(z,\theta) > \sigma_u(z,\theta)$ in most cases.[15] In the left panel of Figure 3, the area between σ_u and σ_w represents the situation in which the equilibrium involves unilateral rigidity and both firms are better off than in bilateral flexibility. Such a situation is referred to as a *win–win rigidity equilibrium*. It can be shown that (4) never holds when $\theta > 1$, so win–win happens only for $\theta \le 1$.

Proposition 2 (Win–Win). *Compared with bilateral flexibility, equilibrium (unilateral) rigidity benefits both firms when demand is expected to concentrate (i.e., $\theta \le 1$) and the uncertainty is moderate (i.e., $\sigma_u(z,\theta) < \sigma < \sigma_w(z,\theta)$).*

The driving force for win–win is that rigidity softens price competition in at least some demand realizations. Compared with bilateral flexibility, rigidity intensifies price competition when demand is more dispersed but softens price competition when demand is more concentrated.[16] If the latter dominates (i.e., if $\theta \le 1$), the two firms' joint expected profit increases. This is the basis for an overlap between equilibrium rigidity (so that the rigid firm gains) and win–win (so that the flexible firm also gains).[17]

3.3. Rigidity Can Benefit Consumers

Compared with bilateral flexibility, unilateral rigidity can also benefit consumers. This is because, in the case of demand dispersion, rigidity intensifies price competition, which passes some surplus from the firms to consumers. At the same time, the two firms are collectively closer to consumers, which reduces dead-weight loss in transportation costs. The opposite is true when demand is more concentrated. If demand dispersion is sufficiently likely, then expected consumer surplus can be higher in unilateral rigidity than in bilateral flexibility.

To calculate, note that the consumer surplus for given μ and positions a and b is

$$CS(\mu) = \int_{-\mu}^{\tilde{x}} \left[v - p^A - t(x-a)^2\right]\frac{m}{2\mu}\,dx + \int_{\tilde{x}}^{\mu}\left[v - p^B - t(b-x)^2\right]\frac{m}{2\mu}\,dx, \qquad (5)$$

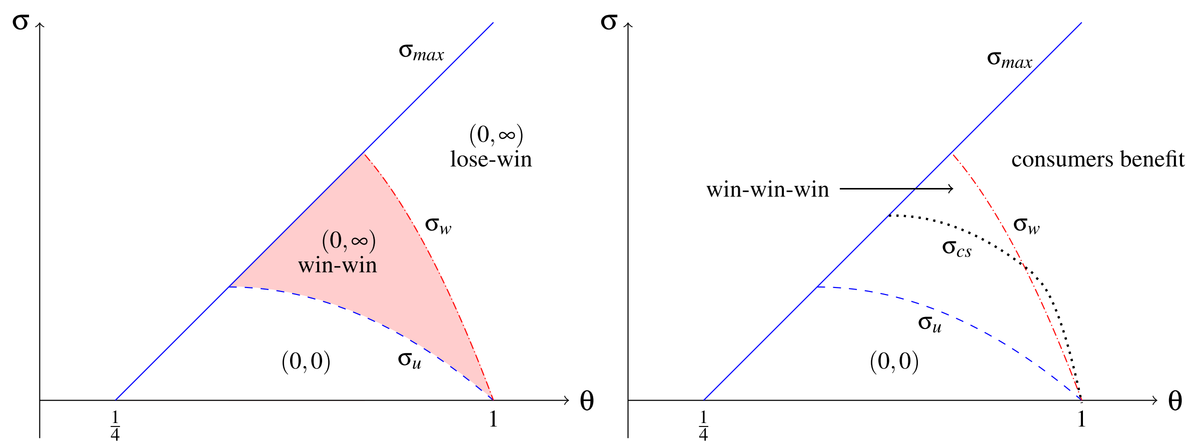
where $\tilde{x}$ is the location of the indifferent consumer. Unilateral rigidity generates a higher expected consumer surplus than bilateral flexibility if and only if

$$\frac{\theta\sqrt{z}+\sigma(1-z)}{(\theta\sqrt{z}+\sigma)(\theta-\sigma\sqrt{z})} > \frac{1}{2}[45 - 67(\sigma^2+\theta^2) + 24\theta], \qquad (6)$$

which, in turn, translates into $\sigma > \sigma_{cs}(z,\theta)$ as shown in the right panel of Figure 3.

Proposition 3 (Consumer Benefit). *Compared with bilateral flexibility, unilateral rigidity benefits consumers if consumer tastes are expected to disperse (i.e., $\theta \ge 1$ for any σ) or the uncertainty is sufficiently large (i.e., $\sigma > \sigma_{cs}(z,\theta)$ when $\theta < 1$).*

Figure 3. (Color online) Rigidity Can Benefit the Rival and Consumers



The conditions that favor equilibrium rigidity (i.e., a large θ or σ) are also conducive to consumer gains. For the three parties (consumers and the two firms) combined, variations in the intensity of price competition is a zero-sum effect, that is, the firms' gain is the consumer's loss, but the reduced transportation cost is a net gain. It is, therefore, possible for all three parties to gain. In the right panel of Figure 3, the area between curves σ_{cs} and σ_w represents the situation of win–win–win. As can be seen from the figure, triple win happens when $\theta < 1$ and σ is moderate.[18]

4. Extensions

So far, we have demonstrated that a firm may benefit from rigidity in the face of competition and uncertain demand, and such commitment may even benefit a rival that chooses the opposite strategy of flexibility. These results are established in the main model with fairly standard assumptions. Nevertheless, one may wonder whether the logic applies to alternative settings. This section presents seven extensions, which cover sequential choices, asymmetric firms, sideways movement of consumer taste distribution, mild rigidity, demand link, new products, and vertical differentiation. The analyses demonstrate that equilibrium rigidity and win–win are the two most salient features when firms compete and the future is uncertain. More usefully, many findings can directly inform managerial decisions as the two firms are usually asymmetric to begin with, making it easier to link a particular strategy and its performance to firm characteristics.

4.1. Sequential Choices

In the main model, the two firms choose their repositioning strategies simultaneously. Now suppose that the choices are made sequentially. If the first mover chooses rigidity, its rival always chooses flexibility, and the first mover's profit is $E\pi(\infty, 0)$. If the first mover chooses flexibility, its rival chooses rigidity if $E\pi(\infty, 0) > E\pi(0, 0)$, in which case the first mover's profit is $E\pi(0, \infty)$. When $E\pi(\infty, 0) > E\pi(0, 0)$, therefore, the first mover may choose flexibility or rigidity depending on how $E\pi(\infty, 0)$ compares with $E\pi(0, \infty)$. Regardless of the first mover's choice, the equilibrium is always unilateral rigidity. On the other hand, when $E\pi(\infty, 0) < E\pi(0, 0)$, the second mover chooses flexibility given the first mover's flexibility, and the first mover's profit is $E\pi(0, 0)$. The first mover's optimal choice is flexibility given that $E\pi(0, 0) > E\pi(\infty, 0)$. As a result, the equilibrium is bilateral flexibility.

In sum, unilateral rigidity appears in sequential choices if and only if $E\pi(\infty, 0) > E\pi(0, 0)$, which is exactly the condition for unilateral rigidity equilibrium in the simultaneous game. In addition, rigidity is chosen by the first mover if $E\pi(\infty, 0) > E\pi(0, \infty)$ or, equivalently,

$$\sigma^2 > \frac{1}{2}(1-\theta)(1+2\theta) \equiv \sigma_h^2. \quad (7)$$

That is, rigidity is chosen by the first mover if the rigid firm earns more than the flexible firm in unilateral rigidity. If (7) fails, that is, if the flexible firm earns more than the rigid firm, the first mover chooses flexibility, and the second mover chooses rigidity. In Figure 4, the first mover chooses flexibility in area D and rigidity in areas G and H. Regardless of the choice, the first mover enjoys an advantage in that its profit is always larger than the second mover's.

Proposition 4 (Sequential Game). *Suppose that firms choose repositioning strategies sequentially.*

i. *Unilateral rigidity is an equilibrium if and only if it is an equilibrium in the simultaneous-move game.*

ii. *When the first mover chooses flexibility and allows the second mover to choose rigidity (in area D), the second mover is better off than choosing flexibility, and the first mover is doing even better.*

iii. *When the first mover chooses rigidity and forces the second mover to choose flexibility, the first mover's rigidity benefits the second mover in area G and hurts it in area H.*

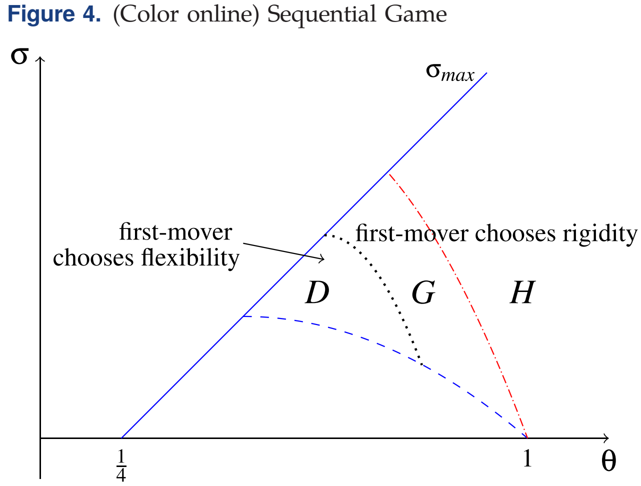
4.2. Asymmetric Firms

In the main model, the two firms are symmetric with identical marginal cost, which is normalized to zero. This extension considers firm asymmetry when their marginal costs, c^A and c^B, differ. To ensure positive market shares by both firms and the existence of pure strategy equilibrium, we assume that the cost differential is not too large (Matsumura and Matsushima 2009):

$$|c^B - c^A| < 9t \min\{1, \mu^2\}.$$

We allow firms' initial positions to reflect their cost differential such that $a_0 = -3/2 + (c^B - c^A)/(6t)$ and

Figure 4. (Color online) Sequential Game



$b_0 = 3/2 + (c^B - c^A)/(6t)$, which would be their equilibrium positions if consumers distribute on $[-1, 1]$. Note that, because of its cost advantage, the efficient firm locates closer to the center than its inefficient rival and gets more than half of the market share.

Let $\sigma_u^i(\theta, z)$ be the solution to $E\pi^i(\infty, 0) = E\pi^i(0, 0)$ and $\sigma_u^e(\theta, z)$ the solution to $E\pi^e(\infty, 0) = E\pi^e(0, 0)$ (the superscripts i and e refer to the inefficient and efficient firms, respectively). It can be shown that $\sigma_u^e(\theta, z) > \sigma_u^i(\theta, z)$. As shown in the left panel of Figure 5, in the area below σ_u^i, the unique equilibrium is for both firms to adopt flexibility; in the area between σ_u^i and σ_u^e, the unique equilibrium is for the inefficient firm to be rigid and the efficient firm to be flexible; finally, in the area above σ_u^e, there can be two unilateral rigidity equilibria in which either firm can choose rigidity. Therefore, the inefficient firm is more likely than the efficient firm to choose rigidity. If the efficient firm is rigid, it gains little when demand disperses because it could have gained competitive advantage through cost efficiency rather than position commitment, but it loses a lot when demand concentrates because it cannot adjust its position to take advantage of its lower cost. The opposite is true for the inefficient firm.

The right panel of Figure 5 shows the win–win outcome. The win–win area for the rigid efficient firm (the shaded area between σ_u^e and σ_w^e) is inside the win–win area for the rigid inefficient firm (bounded by σ_u^i and σ_w^i). As the cost differential increases, it becomes easier for the efficient firm to benefit from the inefficient firm's rigidity, that is, the area between σ_u^i and σ_w^i expands, but more difficult for the inefficient firm to benefit from the efficient firm's rigidity, that is, the shaded area shrinks.

Interestingly, the inefficient firm may earn more profit than the efficient firm. Obviously this happens only in a unilateral rigidity equilibrium as bilateral flexibility would give neither firm any positioning advantage, and the efficient firm's cost advantage must give it a higher profit. In Figure 6, the inefficient firm chooses rigidity in the left panel and flexibility in the right panel. In both cases, the inefficient firm can earn more profit than its efficient rival. This is because, when the cost differential is small, the inefficient firm's cost disadvantage is small, but its advantage in asymmetric positioning can still be substantial and, therefore, dominates its cost disadvantage.

Proposition 5 (Asymmetric Firms). *When the two firms have different marginal costs,*

i. *the inefficient firm is more likely to adopt rigidity;*

ii. *when the cost differential is not very large, the inefficient firm can earn more profit than its efficient rival.*

4.3. Sideways Movement of Consumer Taste Distribution

In the main model, we have assumed that the uncertain demand expands and contracts symmetrically. Because the two firms' initial positions are also symmetric, any realized demand is symmetric to them. In this extension, we consider a sideways movement of taste distribution (see also Villas-Boas 2018), that is, the distribution's center moves to the left or right without changing its width. Specifically, consumers' tastes distribute uniformly on

$$\begin{cases} [-1-\kappa, 1-\kappa] & \text{with probability } p_\kappa \text{ and mass } m_\kappa, \\ [-1+\tau, 1+\tau] & \text{with probability } 1-p_\kappa \text{ and mass } m_\tau, \end{cases}$$

with $\kappa > 0$ and $\tau > 0$ so that the left and right shifts both take place with positive probability and $\kappa < 1$ and $\tau < 1$ so that the original center remains inside any new distribution. Define the mean of the center's location as $\theta \equiv -p'\kappa + (1-p')\tau$ and its variance as $\sigma^2 \equiv p'(-\kappa-\theta)^2 + (1-p')(\tau-\theta)^2$ in which $p' = p_\kappa m_\kappa / [p_\kappa m_\kappa + (1-p_\kappa)m_\tau]$. In Figure 7, the triangle shows feasible combinations of θ and σ given the constraint of $\kappa, \tau \in (0, 1)$.[19]

Figure 5. (Color online) Equilibrium of Asymmetric Firms

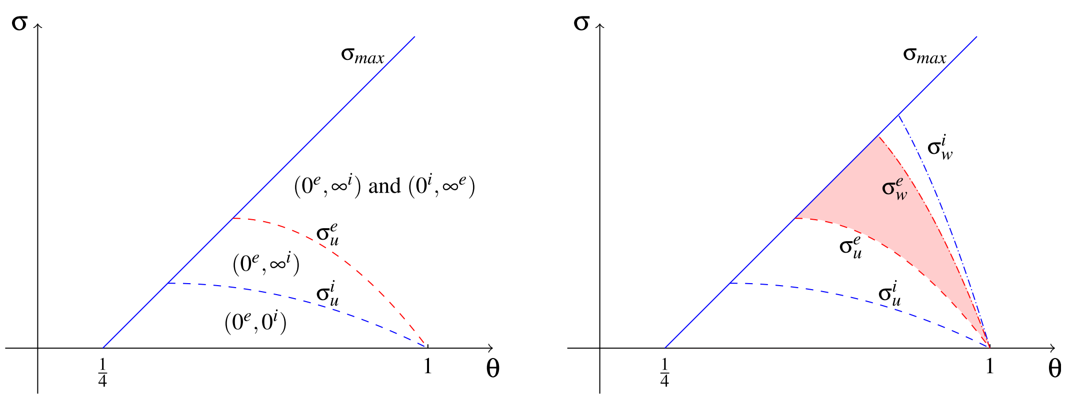
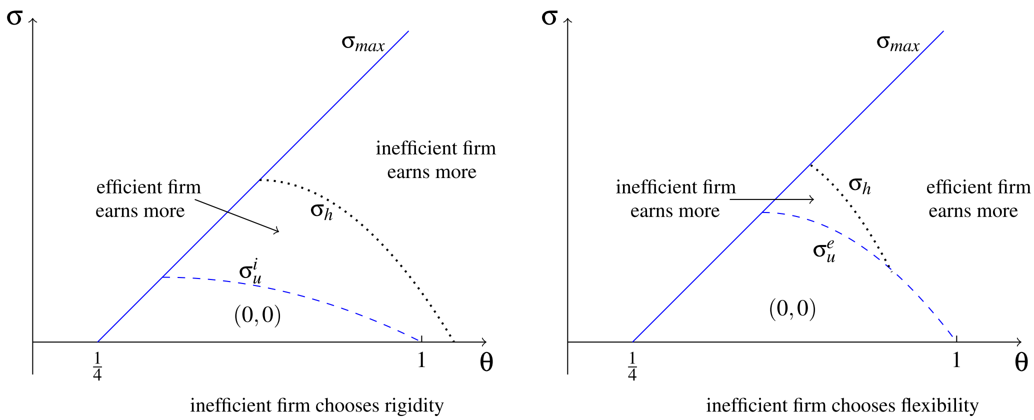


Figure 6. (Color online) The Inefficient Firm May Earn More Than the Efficient Firm



Same as in the main model, if its rival chooses rigidity, a firm's best response is flexibility; if its rival chooses flexibility, the right firm's best response is rigidity (i.e., $(0, \infty)$) if[20]

$$\frac{1-z}{\sqrt{z}} > \frac{\theta}{4\sigma^3}(4\theta^2 + 18\theta - 81) + \frac{3}{2\sigma}(3 + 2\theta), \tag{8}$$

and flexibility otherwise. That is, rigidity is optimal if, on average, consumers move toward this firm and the variance is small. The left panel of Figure 7 shows the equilibrium in the $\theta - \sigma$ space for any given $z \equiv p_\kappa m_\kappa/[(1-p_\kappa)m_\tau] < 1$.[21] When the average change in consumer tastes is small (i.e., θ is not far away from zero), both firms choose flexibility when the demand variance is sufficiently large. When the change is sufficiently large, then the firm toward which consumers are moving chooses rigidity, and the firm from which consumers are moving away chooses flexibility.

As can be seen from the left panel of Figure 7, for any given θ near zero, a large variance hinders the adoption of rigidity. This is the opposite of what is established in the main model, in which a large variance is conducive to rigidity. In sideways movement, uncertainty is not valuable even under maximum flexibility in the sense that, even when both firms adopt flexibility, each firm's profit is independent of the variance. If only one firm chooses flexibility (i.e., in unilateral rigidity), the joint profit is reduced, more so when the uncertainty is larger. By contrast, in the main model with symmetric expansion and contraction of consumer distribution, uncertainty generates value such that the joint profit increases with uncertainty in both bilateral flexibility and unilateral rigidity.

For the equilibrium $(0, \infty)$, win–win happens if

$$\frac{1-z}{\sqrt{z}} > \frac{\theta}{4\sigma^3}(4\theta^2 - 54\theta + 243) + \frac{3}{2\sigma}(2\theta - 9). \tag{9}$$

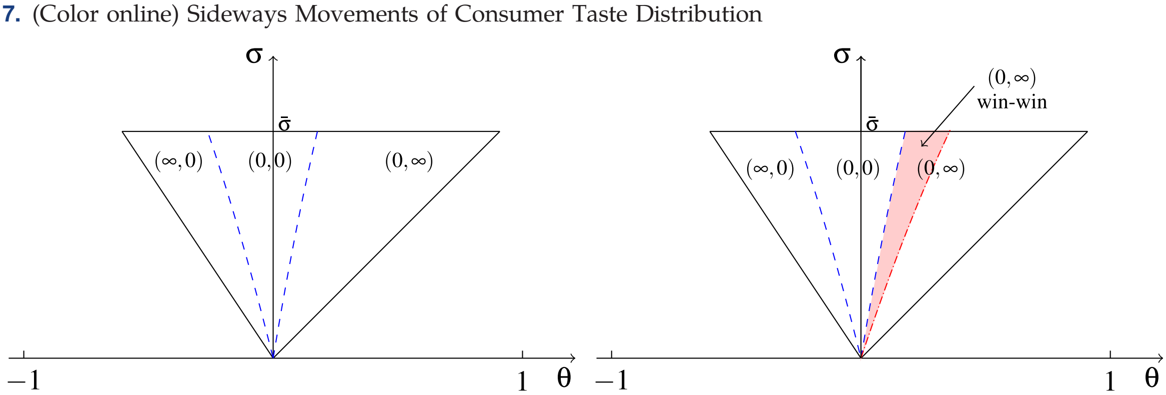
The right panel of Figure 7 shows that, when $z < 1$, only the right rigidity equilibrium (i.e., $(0, \infty)$) can achieve win–win.

Proposition 6 (Sideways Movement). *When consumer tastes move sideways, the equilibrium is unique for any given* θ *and* σ. *In addition,*

i. *If the average taste change is small and the uncertainty is large, then both firms adopt flexibility; otherwise, the firm toward which consumers move adopts rigidity, and the firm from which consumers move away adopts flexibility.*

ii. *Uncertainty hinders rigidity in the sense that, for given* θ, *unilateral rigidity is an equilibrium if and only if* σ *is smaller than some threshold.*

Figure 7. (Color online) Sideways Movements of Consumer Taste Distribution



4.4. Mild Rigidity

In the main model, repositioning strategy takes an extreme form: either complete flexibility (i.e., zero cost in moving) or complete rigidity (i.e., infinite cost in moving). This is the usual assumption in studies of flexible and dedicated technologies (Fine and Freund 1990, Röller and Tombak 1990, Boyabatli and Beril Toktay 2011). Nevertheless, it is conceivable that rigidity may be moderate, that is, repositioning is costly but not infinitely so. This can be modeled such that firm j's ($j = A, B$) repositioning cost takes the form of $f^j(y - y_0)^2$ when moving from y_0 to y (Eaton and Schmitt 1994, Villas-Boas 2018), and a firm's repositioning strategy consists of committing to a particular nonnegative value of the cost coefficient f^j.

Given any demand realization, facing a rival that is extremely rigid or extremely flexible, we can show that a firm's own repositioning strategy has exactly the same impact on its profit as in the main model. This implies that (∞, ∞) is never an equilibrium, whereas $(0, \infty)$ is an equilibrium if $l > 1$ and $(0, 0)$ is an equilibrium if $h < 1$. The qualitative conditions for these equilibria remain unchanged, and win–win continues to hold. All these results are demonstrated in the following numerical example.

Example 1. Let $h = 1.1$, $l = 0.6$, and $m_h = m_l = 1$. Then $(0, \infty)$ is an equilibrium when $p_h \geq 0.725$. Given $p_h = 0.73$, both firms are better off in $(0, \infty)$ than in $(0, 0)$, and the flexible firm receives a higher profit than the rigid firm.

What is new is that equilibrium uniqueness no longer holds. In addition, there can be a symmetric mild rigidity equilibrium in which both firms choose an identical, nonzero repositioning cost as demonstrated by the following example.

Example 2. Let $h = 1.1$, $l = 0.6$, $p_h = 0.9$, and $m_h = m_l = 1$. Then $(0.447, 0.447)$ is an equilibrium. Both firms are worse off in the bilateral mild rigidity equilibrium than in bilateral flexibility.[22] In addition, $(0, \infty)$ is also an equilibrium for this set of parameters.

The intuition behind symmetric, mild rigidity equilibrium is the following. Suppose $\mu > 1$ and a firm's rival adopts a mild repositioning strategy, say $f^* \in (0, \infty)$. If the firm chooses zero, its rival enjoys great commitment power in positioning competition because of its positive repositioning strategy, and the firm is disadvantageous. If it chooses ∞, the firm commits to not moving at all. Because its rival also cannot move too much because of f^*, price competition between these two firms is intense—again, not a good strategy. Therefore, the firm's best response should be some compromise between the two incentives, that is, some moderate value of f. Because the two firms are symmetric, the equilibrium is a symmetric (f^*, f^*).

4.5. Demand Link

In this extension, we assume that some consumers did not buy any product in the past and, therefore, become part of the demand that the two firms face when the game starts. Assume that, with a total mass of past consumers normalized to one, these leftover consumers distribute uniformly on $[-\alpha, \alpha]$ with $\alpha < l$. These consumers, who locate close to the center, join the new consumers who are uniformly distributed to form the total demand that the two firms face. Figure 8 shows consumer distribution in demand dispersion and concentration, respectively, in which the elevated rectangle in the middle represents leftover consumers.

When both firms are flexible, their expected profits are

$$E\pi(0,0) = 3tp_h(m_h + \alpha)\left[\frac{m_h + \alpha}{\frac{m_h}{h} + 1}\right]^2 + 3t(1 - p_h)(m_l + \alpha)\left[\frac{m_l + \alpha}{\frac{m_l}{l} + 1}\right]^2.$$

When one firm is flexible and the other is rigid, their profits can be found in the appendix. The left panel of Figure 9 shows the equilibrium outcome, and the right panel shows win–win. Compared with the main model, the curve σ_u shifts to the right (it now hits the horizontal axis at some $\theta_u > 1$), meaning that unilateral rigidity becomes less likely. Because of leftover consumers, the random total demand is skewed toward consumer concentration. As established earlier, a greater weight of consumer concentration is against rigidity.

Another property to notice is that the σ_u curve becomes upward sloping in θ, meaning that, for a given $\theta > \theta_u$, unilateral rigidity is an equilibrium only when σ is below a certain threshold. In other words, unlike in the main model in which uncertainty facilitates rigidity, here, when leftover consumers are substantial, uncertainty becomes an obstacle to equilibrium rigidity. This is because a greater uncertainty implies the demand concentration and dispersion are both more dramatic. However, the invariant leftover consumers amplify the dramatic concentration but dampen the dramatic dispersion, tilting the average effect toward concentration, which hinders rigidity.

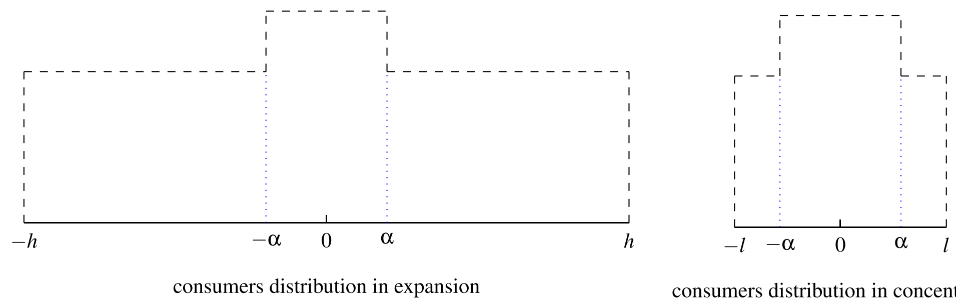
Proposition 7 (Demand Link). *When there are leftover consumers on $[-\alpha, \alpha]$ with $\alpha < l$,*

i. *equilibrium rigidity becomes less likely, and*
ii. *uncertainty hinders rigidity.*

4.6. New Products

In the main model, the two firms' initial positions are given as exogenous. This can be justified as describing existing products, the positioning of which were chosen long before any idea about future demand is

Figure 8. (Color online) Demand Distribution with Leftover Consumers



available. For new products, however, there are no initial positions. When a firm chooses its product's position for the first time, it must consider the need to reposition in the future when the demand changes. Accordingly the game is modified as follows. Without any initial positions, the two firms simultaneously choose repositioning strategies, followed by competition in two periods. Consumers distribute uniformly on $[-\mu_1, \mu_1]$ in period one with mass m_1 and on $[-\mu_2, \mu_2]$ in period two with mass m_2. Within each period, the two firms simultaneously choose positions first and then choose prices. The repositioning constraint applies only in period two. A firm's objective function is its two-period total discounted profit with a common time discount factor δ. For simplicity, we assume that μ_1 and μ_2 are both deterministic.[23]

If $f^A = f^B = 0$, the two periods' positioning competition is independent. In period $i = 1, 2$, the equilibrium positions are $a_i = -3\mu_i/2$ and $b_i = 3\mu_i/2$. The expected total profit of each firm is $\pi(0,0) = 3tm_1\mu_1^2 + 3\delta tm_2\mu_2^2$.

If $f^A = f^B = \infty$, neither firm can move in period two, that is, $a_2 = a_1$ and $b_2 = b_1$. The equilibrium positions can be solved as $b_1 = b_2 = -a_1 = -a_2 = 3(m_1\mu_1 + \delta m_2\mu_2)/[2(m_1 + \delta m_2)]$. The expected profit of each firm is $\pi(\infty,\infty) = 3t(m_1\mu_1 + \delta m_2\mu_2)^2/(m_1 + \delta m_2)$.

If $f^A = 0$ and $f^B = \infty$, then $b_2 = b_1$. The analysis and equilibrium profits can be found in the appendix.

The equilibrium condition is completely characterized by two variables, μ_2/μ_1 and $\delta m_2/m_1$, which capture the relative importance of the two periods in a firm's profit in terms of consumer mass and demand distribution. Figure 10 demonstrates the equilibrium outcomes.

Proposition 8 (New Products). *Suppose the two firms choose repositioning strategies before introducing new products.*

i. *Bilateral rigidity (∞,∞) is the unique equilibrium when μ_2/μ_1 or $\delta m_2/m_1$ is large (i.e., the vast upper right area in Figure 10), but both firms are worse off than in bilateral flexibility.*

ii. *Bilateral flexibility $(0,0)$ is the unique equilibrium when μ_2/μ_1 and $\delta m_2/m_1$ are both very small (i.e., the lower left corner in Figure 10).*

iii. *Unilateral rigidity ($(0,\infty)$ or $(\infty,0)$) is the unique equilibrium when μ_2/μ_1 and $\delta m_2/m_1$ are both relatively small, and win–win arises when μ_2/μ_1 and $\delta m_2/m_1$ are particularly small.*

Unlike in the main model, bilateral rigidity can now be supported as an equilibrium. In the main model, the initial positions are exogenously given. If its rival is rigid, a firm is unable to influence the rival's positioning through its own rigidity. By contrast, here the period-one positions are chosen after firms observe each other's repositioning strategies. Although

Figure 9. (Color online) Demand Link

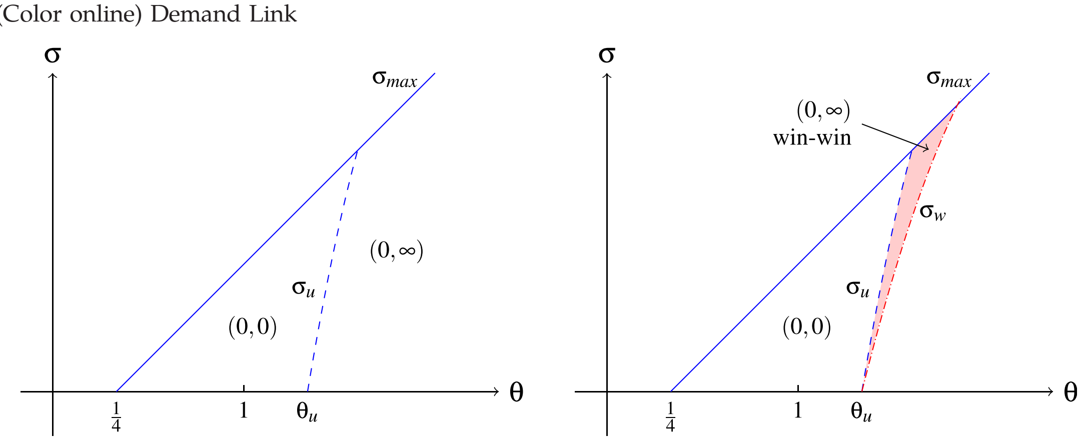
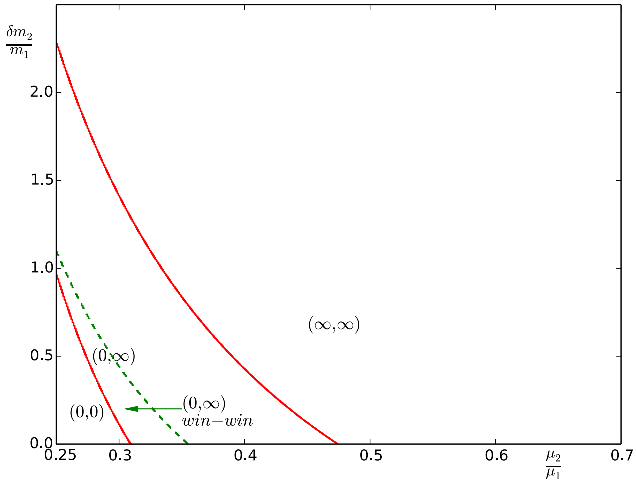


Figure 10. (Color online) Equilibrium Outcome of New Products



a rigid firm's period-two position is not changeable, its period-one position is still influenced by its rival's repositioning strategy. As a result, rigidity can be a best response to rigidity.

In the main model, rigidity appears only when demand is more dispersed. If demand becomes more concentrated, a rigid firm's initial position is too far away from the center, so rigidity is never an optimal choice. By contrast, here, when repositioning is chosen before initial positions, rigidity (unilateral or bilateral) appears even when demand becomes more concentrated (i.e., when $\mu_2/\mu_1 < 1$). This is because a rigid firm's position serves demands in both periods. If the second-period demand becomes more concentrated, the first-period demand appears more dispersed relative to a rigid firm's fixed position, in which case rigidity is advantageous.

4.7. Vertical Product Differentiation

In the main model, firms are differentiated horizontally, but there is no reason why the logic cannot be applied to vertical differentiation, which we consider now. Assume firms A and B sell products with endogenous qualities; that is, they compete by choosing positions on a vertical quality line. The marginal cost of producing a product with quality s is $1/2s^2$. A total mass $m > 0$ of consumers have differentiated tastes for quality, which distribute uniformly on the interval $[1, \lambda']$ for $\lambda' > 1$. The distribution of consumers' quality taste is random such that

$$\begin{cases} \text{with probability } p_h \in [0,1]: & \lambda' = h\lambda_0 \text{ and } m = m_h \\ \text{with probability } 1-p_h: & \lambda' = l\lambda_0 \text{ and } m = m_l \end{cases}.$$

In other words, low-end consumers do not change their preference for quality, but high-end consumers may care more or less about product quality ($h \geq l > 1/\lambda_0$). Consumers have unit demand. A consumer with quality taste x derives a utility $xs - p$ if the consumer buys a product with quality s at price p. We assume that the market is fully covered in both periods for any realization of consumer preference, which requires $h\lambda_0 \leq 9/5$.

The game is similar to that in the main model: the two firms simultaneously choose repositioning strategies (zero or ∞) in stage one, qualities in stage two subject to repositioning constraints, and prices in stage three. Assume the initial qualities of the two firms are $s_0^A = (5-\lambda_0)/4$ and $s_0^B = (5\lambda_0 - 1)/4$,[24] and firm A is the low-quality firm (we do not consider leapfrogging). Define taste diversities as $\lambda \equiv \lambda_0 - 1$, $\lambda_h \equiv h\lambda_0 - 1$, and $\lambda_l \equiv l\lambda_0 - 1$. Then the mean of taste diversity is $\theta \equiv p'\lambda_h + (1-p')\lambda_l$, and the variance is $\sigma^2 \equiv p'(\lambda_h - \theta)^2 + (1-p')(\lambda_l - \theta)^2$, where $p' \equiv p_h m_h/[p_h m_h + (1-p_h)m_l]$ and $z \equiv p_h m_h/[(1-p_h)m_l]$.

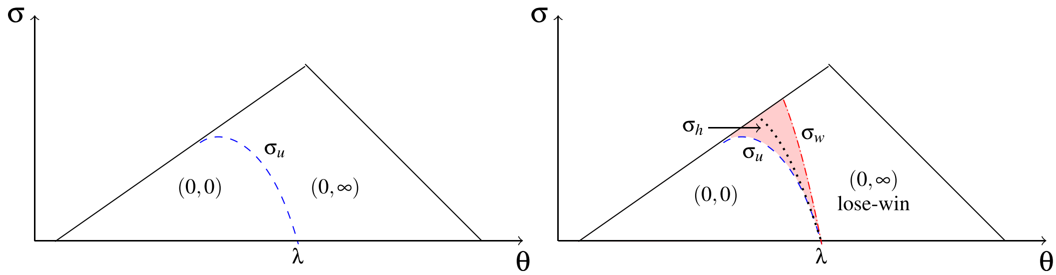
If $f^A = f^B = 0$, the equilibrium qualities are $s^A = (5-\lambda')/4$ and $s^B = (5\lambda' - 1)/4$, and the two firms split the market equally. Profits are the same for the two firms ($3m(\lambda' - 1)^2/8$) even though they have different qualities as the high-quality product is sold at a higher price but costs more to produce. Note that, when quality is more important in consumers' preferences (i.e., when λ' increases), the high-quality firm raises its quality (s^B increases), and the low-quality firm lowers its quality (s^A decreases).

If a firm chooses rigidity, then its rival's best response is always flexibility. This means rigidity can be adopted by at most one firm. Then it is an equilibrium for the high-quality firm to choose rigidity (i.e., $f^A = 0$ and $f^B = \infty$) if and only if

$$\frac{\sqrt{z}\theta + (1-z)\sigma}{(\sigma + \sqrt{z}\theta)(\theta - \sqrt{z}\sigma)} > \frac{120\lambda^2 - 84\lambda\theta - 11(\theta^2 + \sigma^2)}{25\lambda^3}.$$

The left panel of Figure 11 shows the equilibrium in the space of θ and σ for given z.[25] As can be seen from the figure, a sufficient condition for unilateral rigidity to be an equilibrium is that consumers on average care more about quality (i.e., $\theta > \lambda$). If consumers care less about quality (i.e., $\theta < \lambda$), rigidity is still an equilibrium choice if the uncertainty in preference change is large (i.e., $\sigma > \sigma_u$ for given $\theta < \lambda$). The intuition is the following. Suppose the quality preference expands. If both firms are flexible, the high-quality firm B would have increased its quality while the low-quality firm A would have decreased it. However, if firm B commits to not moving, its market share increases at the expense of A. In addition, both firms' quality levels drop as compared with bilateral flexibility, which reduces production costs but intensifies price competition. As a result, both firms' markups are smaller. The combined effect is that B gains, but A is hurt. The opposite is true when quality tastes shrink: B's rigidity softens price competition so that B

Figure 11. (Color online) Equilibrium of Vertical Differentiation



is hurt but A gains. On balance, then, rigidity benefits B if the quality preference is more likely to expand.

When the high-quality firm chooses rigidity, the flexible, low-quality firm benefits from its rival's rigidity if and only if

$$\frac{\sqrt{z}\theta + (1-z)\sigma}{(\sigma + \sqrt{z}\theta)(\theta - \sqrt{z}\sigma)} > \frac{133(\theta^2 + \sigma^2) - 60\lambda^2 - 48\lambda\theta}{25\lambda^3}.$$

The win–win outcome is shown as the shaded area in the right panel of Figure 11.

In unilateral rigidity, either firm can earn higher profits than its rival. As shown in the right panel of Figure 11, the part to the right of σ_h is the space in which the rigid firm earns higher expected profit than its flexible rival. This implies that, when the two firms choose sequentially, the first mover usually chooses rigidity, forcing the second mover to choose flexibility. In the area between σ_u and σ_h, the flexible firm earns higher expected profits than the rigid firm.

So far, we have been discussing the situation in which the high-quality firm chooses rigidity. It is also possible that the low-quality firm chooses rigidity in equilibrium (and, consequently, the high-quality firm chooses flexibility). The results are similar and, therefore, are skipped.

Proposition 9 (Vertical Differentiation). *When firms compete with vertically differentiated products,*

i. *One of the firms (either the high- or low-quality firm) chooses rigidity, and the other chooses flexibility if quality preference disperses (i.e., $\theta \geq \lambda$) or the preference uncertainty is sufficiently large (i.e., $\sigma > \sigma_u$ when $\theta < \lambda$); otherwise, both firms choose flexibility.*

ii. *A flexible firm benefits from its rival's rigidity in equilibrium when the taste change is moderate (i.e., both θ and σ are moderate).*

5. Conclusion

This paper explores whether a firm should commit in a business environment that is both competitive and uncertain. In the main model and most of the extensions, the equilibrium is asymmetric such that one firm adopts rigid repositioning and the other firm adopts flexible repositioning. Rigidity benefits not only the rigid firm itself, but also its flexible rival. When uncertainty is larger, rigidity becomes more valuable relative to flexibility. In such an asymmetric equilibrium, one firm's rigidity provides a commitment value and the other firm's flexibility provides an option value, both of which are then shared within the competitive ecosystem. These results demonstrate that commitment and options are valuable not only for the firm that is making the choice, but also for all competitors collectively. The two values can coexist and even strengthen each other.

We would like to point out that our study is carried out in a stable industry, in which rigidity brings competitive advantages and also softens competition. If a company is fighting disruptive technologies, over which its own inflexibility gesture has no influence, then obviously rigidity can only be damaging. The photography industry provides a perfect example; with the rise of the digital camera, both Polaroid and Eastman Kodak have long faded into history by now.

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Appendix

Here we only report the most important calculations and expressions. Details and other calculations and proofs can be found in the online appendix.

The Main Model

■ Price competition: given positions $a \leq b$, the equilibrium prices are $p^A = t(b-a)(6\mu + b + a)/3$ and $p^B = t(b-a) \cdot (6\mu - b - a)/3$. The corresponding equilibrium sales are $q^A = m[1/2 + (a+b)/(12\mu)]$ and $q^B = m[1/2 - (a+b)/(12\mu)]$.

■ $E\pi(\infty, 0) - E\pi(0,0)$ and $E\pi(0,\infty) - E\pi(\infty,\infty)$ increases with σ

The expected profits expressed in terms of the raw parameters are

$$E\pi(0,0) = 3t\left[p_h m_h h^2 + (1-p_h)m_l l^2\right], \text{and}$$
$$E\pi(\infty,\infty) = 3t\left[p_h m_h h + (1-p_h)m_l l\right],$$
$$E\pi(0,\infty) = \frac{t}{9}\left[\frac{p_h m_h}{h}(1+2h)^3 + \frac{(1-p_h)m_l}{l}(1+2l)^3\right],$$
$$E\pi(\infty,0) = \frac{t}{9}\left[\frac{p_h m_h}{h}(1+2h)(4h-1)^2 + \frac{(1-p_h)m_l}{l}(1+2l)(4l-1)^2\right].$$

Based on $\theta \equiv p'h + (1-p')l$ and $\sigma^2 \equiv p'(h-\theta)^2 + (1-p')\cdot(l-\theta)^2$, we have $h = \theta + \sigma\sqrt{(1-p')}/\sqrt{p'}$ and $l = \theta - \sigma\sqrt{p'}/\sqrt{(1-p')}$. Let $u(x) = (x-1)(5x^2+5x-1)/x$ for $x > 1/4$. Then

$$\frac{\partial[E\pi(\infty,0) - E\pi(0,0)]}{\partial\sigma} = \frac{t}{9}\left[p_h m_h + (1-p_h)m_l\right]\sqrt{p'(1-p')}[u'(h) - u'(l)].$$

Because $u'(x)$ is an increasing function of x, $u'(h) > u'(l)$, indicating $\partial[E\pi(\infty,0) - E\pi(0,0)]/\partial\sigma > 0$. Moreover, we can prove $\partial^2[E\pi(\infty,0) - E\pi(0,0)]/\partial(\sigma)^2 > 0$. Similarly, we can prove $E\pi(0,\infty) - E\pi(\infty,\infty)$ increases with σ.

■ Consumer surplus: Based on (5), the expected consumer surplus is

$$ECS(0,0) = p_h m_h v + (1-p_h)m_l v - \frac{85t}{12}\left[p_h m_h h^2 + (1-p_h)m_l l^2\right],$$
$$ECS(0,\infty) = p_h m_h v + (1-p_h)m_l v - \frac{t}{36}\left[p_h m_h \frac{188h^3 + 24h^2 + 45h - 2}{h} + (1-p_h)m_l \frac{188l^3 + 24l^2 + 45l - 2}{l}\right].$$

Then

$$ECS(0,\infty) - ECS(0,0) = \frac{t}{36}\left[p_h m_h \frac{(h-1)(67h^2 + 43h - 2)}{h} + (1-p_h)m_l \frac{(l-1)(67l^2 + 43l - 2)}{l}\right].$$

Let $s(x) = (x-1)(67x^2 + 43x - 2)/x$ for $x \geq 1/4$. Then $ECS(0,\infty) > ECS(0,0)$ is equivalent to $p_h m_h s(h) + (1-p_h) m_l s(l) > 0$, which holds if and only if (i) $h > l > 1$, (ii) $h > 1 > l$, and $p_h m_h/[(1-p_h)m_l] > -s(l)/s(h)$. We can prove there always exist $p_h m_h$, $(1-p_h)m_l$, h, and l to make win–win–win happen. In other words, there always exist z, θ, and σ to make win–win–win happen.

Asymmetric Firms

If $f^A = 0$ and $f^B = 0$, the equilibrium of price competition is

$$p^A = \frac{1}{3}t(b-a)(6\mu + a + b) + \frac{2}{3}c^A + \frac{1}{3}c^B, \text{and}$$
$$p^B = \frac{1}{3}t(b-a)(6\mu - a - b) + \frac{1}{3}c^A + \frac{2}{3}c^B.$$

The corresponding sales are

$$q^A = \left[\frac{1}{2} + \frac{a+b}{12\mu} + \frac{c^B - c^A}{12t\mu(b-a)}\right]m, \text{and}$$
$$q^B = \left[\frac{1}{2} - \frac{a+b}{12\mu} - \frac{c^B - c^A}{12t\mu(b-a)}\right]m.$$

The profits are $\pi^A = [p^A - c^A]q^A$ and $\pi^B = [p^B - c^B]q^B$. The equilibrium positions are $a = -3\mu/2 + (c^B - c^A)/(6t\mu)$ and $b = 3\mu/2 + (c^B - c^A)/(6t\mu)$ (under the condition $-9t\mu^2 < c^B - c^A < 9t\mu^2$). The profits are

$$\pi^A = \left[6t\mu^2 + \frac{2}{3}(c^B - c^A)\right]\left[\frac{1}{2} + \frac{c^B - c^A}{18t\mu^2}\right]m, \text{and}$$
$$\pi^B = \left[6t\mu^2 - \frac{2}{3}(c^B - c^A)\right]\left[\frac{1}{2} - \frac{c^B - c^A}{18t\mu^2}\right]m.$$

If $f^A = 0$ and $f^B = \infty$, $b = b_0 = 3/2 + (c^B - c^A)/(6t)$. The optimal a is

$$a = \frac{2t(2b - 3\mu) - \sqrt{\Delta}}{6t}, \text{where } \Delta = 4t^2(3\mu + b)^2 - 12t(c^B - c^A).$$

If $f^A = \infty$ and $f^B = 0$, then $a = a_0 = -3/2 + (c^B - c^A)/(6t)$. The optimal b is

$$b = \frac{2t(2a + 3\mu) + \sqrt{\Delta}}{6t}, \text{where } \Delta = 4t^2(3\mu - a)^2 + 12t(c^B - c^A).$$

The equilibrium prices, sales, and profits can all be calculated based on equilibrium positions as derived here.

Sideways Movement

Assume the variance is not too large.[26]

■ Shifting to the left.

If $f^A = f^B = 0$, then $a = -3/2 - \kappa$ and $b = 3/2 - \kappa$. In equilibrium, $p^A = p^B = 6t$, and each firm gets $3tm_\kappa$.

If $f^A = f^B = \infty$, then $a = a_0 = -3/2$ and $b = b_0 = 3/2$. In equilibrium, $p^A = 2t(3+\kappa)$ and $p^B = 2t(3-\kappa)$; $\pi^A(\infty,\infty|\text{left}) = tm_\kappa(3+\kappa)^2/3$ and $\pi^B(\infty,\infty|\text{left}) = tm_\kappa(3-\kappa)^2/3$.

If $f^A = 0$ and $f^B = \infty$, then $b = b_0 = 3/2$ and, as a result, $a = -3/2 - 2\kappa/3$. In equilibrium, $p^A = 2t(3 + 2\kappa/3)^2/3$ and $p^B = 2t(3 + 2\kappa/3)(3 - 2\kappa/3)/3$; $\pi^A(0,\infty|\text{left}) = tm_\kappa(9+2\kappa)^3/243$ and $\pi^B(\infty,0|\text{left}) = tm_\kappa(9+2\kappa)(9-2\kappa)^2/243$.

If $f^A = \infty$ and $f^B = 0$, $a = a_0 = -3/2$, and consequently, $b = 3/2 - 2\kappa/3$. In equilibrium, $p^A = 2t(3 - 2\kappa/3)(3 + 2\kappa/3)/3$ and $p^B = 2t(3 - 2\kappa/3)^2/3$; $\pi^A(\infty,0|\text{left}) = tm_\kappa(9-2\kappa)(9+2\kappa)^2/243$ and $\pi^B(0,\infty|\text{left}) = tm_\kappa(9-2\kappa)^3/243$.

■ Shifting to the right.

If $f^A = f^B = 0$, $\pi^A(0,0|\text{right}) = \pi^B(0,0|\text{right}) = 3tm_\tau$.

If $f^A = f^B = \infty$, $\pi^A(\infty,\infty|\text{right}) = tm_\tau(3-\tau)^2/3$ and $\pi^B(\infty,\infty|\text{right}) = tm_\tau(3+\tau)^2/3$.

If $f^A = 0$ and $f^B = \infty$, $\pi^A(0,\infty|\text{right}) = tm_\tau(9-2\tau)^3/243$ and $\pi^B(\infty,0|\text{right}) = tm_\tau(9-2\tau)(9+2\tau)^2/243$.

If $f^A = \infty$ and $f^B = 0$, $\pi^A(\infty,0|\text{right}) = tm_\tau(9+2\tau)(9-2\tau)^2/243$ and $\pi^B(0,\infty|\text{right}) = tm_\tau(9+2\tau)^3/243$.

■ Expected profits.

Given the definitions of θ and σ^2, we have $\kappa = \sigma/\sqrt{z} - \theta$, $\tau = \sigma\sqrt{z} + \theta$, $p'\kappa - (1-p')\tau = -\theta$, $p'\kappa^2 + (1-p')\tau^2 = \theta^2 + \sigma^2$, and $p'\kappa^3-(1-p')\tau^3=(1-z)\sigma^3/\sqrt{z}-3\theta\sigma^2-\theta^3$. Then the expected profits are

$$E\pi^A(0,0)=E\pi^B(0,0)=3t\bar{m}; \text{and}$$
$$E\pi^A(\infty,\infty)=E\pi^B(\infty,\infty)=\frac{t\bar{m}}{3}[9-6\theta+\theta^2+\sigma^2],$$
$$E\pi^A(0,\infty)=\frac{t\bar{m}}{243}\left[\frac{8(1-z)}{\sqrt{z}}\sigma^3+12(9-2\theta)\sigma^2-8\theta^3+108\theta^2-486\theta+729\right],$$
$$E\pi^B(\infty,0)=\frac{t\bar{m}}{243}\left[\frac{8(1-z)}{\sqrt{z}}\sigma^3-12(3+2\theta)\sigma^2-8\theta^3-36\theta^2+162\theta+729\right],$$
$$E\pi^A(\infty,0)=\frac{t\bar{m}}{243}\left[-\frac{8(1-z)}{\sqrt{z}}\sigma^3+12(2\theta-3)\sigma^2+8\theta^3-36\theta^2-162\theta+729\right],$$
$$E\pi^B(0,\infty)=\frac{t\bar{m}}{243}\left[-\frac{8(1-z)}{\sqrt{z}}\sigma^3+12(2\theta+9)\sigma^2+8\theta^3+108\theta^2+486\theta+729\right].$$

Mild Rigidity

It can be shown that, for any $f^j>0$ and any μ, $\pi(0,\infty|\mu)>\pi(f^j,\infty|\mu)$. This is because any optimal position under cost $f^j>0$ can be chosen by the firm with $f^j=0$. Given the other firm is rigid, j can always get higher profit from $f^j=0$. We can also prove, for any $f^j\geq 0$, $d\pi(f^j,0|\mu)/df^j>0$ if and only if $\mu>1$. In other words, given the rival has chosen the flexible strategy, it is optimal for the firm to choose the rigid strategy if and only if $\mu>1$. So $(0,\infty)$ is the equilibrium if and only if $\mu>1$, and $(0,0)$ is the equilibrium if and only if $\mu<1$.

Now let us focus on the mild rigidity equilibrium. The best response of firm k to f^j is $BR^k(f^j)$, and the best response of firm j to f^k is $BR^j(f^k)$. Because of the ex ante symmetry, functions $BR^k(f^j)$ and $BR^j(f^k)$ are identical. We plot these two best response functions into one coordinate system and choose f^j as the horizontal axis and f^k as the vertical axis. Because we have proved $(0,\infty)$ and $(\infty,0)$ are equilibria when $\mu>1$, $BR^k(f^j)$ and $BR^j(f^k)$ cross at two points $(0,\infty)$ and $(\infty,0)$ when $\mu>1$. These two identical best response functions must intersect at another point, which lies in the 45° line, generating the symmetric equilibrium (f^*,f^*). In this symmetric equilibrium, two firms get equal profit. When $\mu<1$, we know $BR^k(f^j)$ and $BR^j(f^k)$ cross at the point $(0,0)$ because $(0,0)$ is an equilibrium. We can show that repositioning strategies $(0,0)$ is the unique Nash equilibrium. The detailed proof can be found in the online appendix.

Demand Link

The indifferent consumer $\tilde{x}$ is $\tilde{x}=(a+b)/2+(p^B-p^A)/[2t(b-a)]$. We focus on the case in which the indifferent consumer lies in the interval $[-\alpha,\alpha]$ (the alternative cases can be analyzed similarly). Then $q^A=m/2+\alpha/2+(m/(2\mu)+1/2)\tilde{x}$ and $q^B=m/2+\alpha/2-(m/(2\mu)+1/2)\tilde{x}$. Equilibrium prices are

$$p^A=2t(b-a)\frac{m+\alpha}{\frac{m}{\mu}+1}+\frac{t(b-a)(b+a)}{3}, \text{and}$$
$$p^B=2t(b-a)\frac{m+\alpha}{\frac{m}{\mu}+1}-\frac{t(b-a)(b+a)}{3}.$$

When $f^A=0$ and $f^B=0$, $\partial\pi^A/\partial a=0$ and $\partial\pi^B/\partial b=0$ imply the optimal positions are $a=-3(m+\alpha)/(2+2m/\mu)$ and $b=3(m+\alpha)/(2+2m/\mu)$.

When $f^A=0$ and $f^B=\infty$, $b=b_0=3/2$. From $\partial\pi^A/\partial a=0$, the optimal position of A is $a=1/2-2(m+\alpha)/(1+m/\mu)$. Making sure the indifferent consumer lies in $[-\alpha,\alpha]$, we need some extra conditions for parameters. Firms' expected profits are

$$E\pi(0,\infty)=\frac{8tp_h}{9}\left(\frac{m_h}{h}+1\right)\left[\frac{1}{2}+\frac{m_h+\alpha}{\frac{m_h}{h}+1}\right]^3+\frac{8t(1-p_h)}{9}\left(\frac{m_l}{l}+1\right)\left[\frac{1}{2}+\frac{m_l+\alpha}{\frac{m_l}{l}+1}\right]^3,$$
$$E\pi(\infty,0)=\frac{8tp_h}{9}\left(\frac{m_h}{h}+1\right)\left[\frac{1}{2}+\frac{m_h+\alpha}{\frac{m_h}{h}+1}\right]\left[-\frac{1}{2}+2\frac{m_h+\alpha}{\frac{m_h}{h}+1}\right]^2+\frac{8t(1-p_h)}{9}\left(\frac{m_l}{l}+1\right)\left[\frac{1}{2}+\frac{m_l+\alpha}{\frac{m_l}{l}+1}\right]\left[-\frac{1}{2}+2\frac{m_l+\alpha}{\frac{m_l}{l}+1}\right]^2.$$

Using $h=\theta+\sigma/\sqrt{z}$ and $l=\theta-\sigma\sqrt{z}$, we can derive the condition for unilateral rigidity equilibrium in terms of θ and σ.

New Products

■ $f^A=0$ and $f^B=0$: Equilibrium positions are $a_1=-b_1=-3\mu_1/2$, $a_2=-b_2=-3\mu_2/2$. Equilibrium profits are $\pi^A(0,0)=\pi^B(0,0)=3tm_1\mu_1^2+3\delta tm_2\mu_2^2$.

■ $f^A=\infty$ and $f^B=\infty$: Profits are

$$\pi^A(\infty,\infty)=\frac{tm_1}{36\mu_1}(b_1-a_1)(b_1+a_1+6\mu_1)^2+\frac{\delta tm_2}{36\mu_2}(b_1-a_1)(b_1+a_1+6\mu_2)^2,$$
$$\pi^B(\infty,\infty)=\frac{tm_1}{36\mu_1}(b_1-a_1)(b_1+a_1-6\mu_1)^2+\frac{\delta tm_2}{36\mu_2}(b_1-a_1)(b_1+a_1-6\mu_2)^2.$$

Equilibrium positions are $a_1=a_2=-b_1=-b_2=-3(m_1\mu_1+\delta m_2\mu_2)/(2m_1+2\delta m_2)$. Equilibrium profit is $\pi(\infty,\infty)=3t(m_1\mu_1+\delta m_2\mu_2)^2/(m_1+\delta m_2)$.

■ $f^A=0$ and $f^B=\infty$:

Given b_1, A's best position in the second period is $a_2=b_1/3-2\mu_2$. Then, in the second period, A's profit is $8tm_2\cdot(b_1/3+\mu_2)^3/(9\mu_2)$, and B's profit is $8tm_2(b_1/3+\mu_2)(b_1/3-2\mu_2)^2/(9\mu_2)$. Their two-period total profits are

$$\pi^A(0,\infty)=\frac{tm_1}{36\mu_1}(b_1-a_1)(b_1+a_1+6\mu_1)^2+\frac{8\delta tm_2}{9\mu_2}\left(\frac{1}{3}b_1+\mu_2\right)^3,$$
$$\pi^B(\infty,0)=\frac{tm_1}{36\mu_1}(b_1-a_1)(b_1+a_1-6\mu_1)^2+\frac{8\delta tm_2}{9\mu_2}\left(\frac{1}{3}b_1+\mu_2\right)\left(\frac{1}{3}b_1-2\mu_2\right)^2.$$

Based on first-order conditions, we can have the optimal position for firm B is $b_1=(3\mu_1\eta_1-3\mu_1\sqrt{\eta_1^2-16\eta_2})/(4\eta_2)$, where $\eta_1=5+4\delta m_2/m_1$ and $\eta_2=1+\delta m_2\mu_1/(m_1\mu_2)$. Correspondingly, $a_1=b_1/3-2\mu_1$ and $a_2=b_1/3-2\mu_2$. Their equilibrium profits are

$$\pi(0,\infty)=\frac{8tm_1}{9\mu_1}\left(\frac{1}{3}b_1+\mu_1\right)^3+\frac{8\delta tm_2}{9\mu_2}\left(\frac{1}{3}b_1+\mu_2\right)^3,$$
$$\pi(\infty,0)=\frac{8tm_1}{9\mu_1}\left(\frac{1}{3}b_1+\mu_1\right)\left(\frac{1}{3}b_1-2\mu_1\right)^2+\frac{8\delta tm_2}{9\mu_2}\left(\frac{1}{3}b_1+\mu_2\right)\left(\frac{1}{3}b_1-2\mu_2\right)^2.$$

Vertical Differentiation

When $f^A = f^B = 0$, both firms freely move. We use backward induction to solve the quality-then-pricing game. Given any pair of quality levels (s^A, s^B), the demands of A and B in the pricing stage are $q^A = m[(p^B - p^A) - (s^B - s^A)]/[(\lambda' - 1)(s^B - s^A)]$ and $q^B = m[\lambda'(s^B - s^A) - (p^B - p^A)]/[(\lambda' - 1)(s^B - s^A)]$. The profits of A and B are $\pi^A = [p^A - (s^A)^2/2] \cdot q^A$ and $\pi^B = [p^B - (s^B)^2/2]q^B$. Based on first-order conditions, we can have the optimal prices $p^A = (\lambda' - 2)(s^B - s^A)/3 + (s^B)^2/6 + (s^A)^2/3$ and $p^B = (2\lambda' - 1) \cdot (s^B - s^A)/3 + (s^B)^2/3 + (s^A)^2/6$.

The profits of A and B are

$$\pi^A = \frac{m}{\lambda' - 1}(s^B - s^A)\left[\frac{\lambda' - 2}{3} + \frac{s^B + s^A}{6}\right]^2,$$
$$\pi^B = \frac{m}{\lambda' - 1}(s^B - s^A)\left[\frac{2\lambda' - 1}{3} - \frac{s^B + s^A}{6}\right]^2.$$

Based on first-order conditions, we can have the equilibrium quality levels $s^A = (5 - \lambda')/4$ and $s^B = (5\lambda' - 1)/4$. In equilibrium, $q^A = q^B = m/2$, $p^A = [25(\lambda')^2 - 58\lambda' + 49]/32$, and $p^B = [49(\lambda')^2 - 58\lambda' + 25]/32$. So equilibrium profits of A and B given λ' are $\pi^A(0,0|\lambda') = \pi^B(0,0|\lambda') = 3m(\lambda' - 1)^2/8$.

When $f^A = f^B = \infty$, $s^A = s_0^A = (5 - \lambda_0)/4$ and $s^B = s_0^B = (5\lambda_0 - 1)/4$. We have $\pi^A(\infty,\infty|\lambda') = m(\lambda_0 - 1)(2\lambda' + \lambda_0 - 3)^2/[24(\lambda' - 1)]$ and $\pi^B(\infty,\infty|\lambda') = m(\lambda_0 - 1)(4\lambda' - \lambda_0 - 3)^2/[24(\lambda' - 1)]$. To make the two firms' demands and markups positive, we need $4\lambda' - \lambda_0 - 3 > 0$.

When $f^A = 0$ and $f^B = \infty$, we have $s^A = (5\lambda_0 - 8\lambda' + 15)/12$ and $s^B = (5\lambda_0 - 1)/4$. Profits are $\pi^A(0,\infty|\lambda') = m(5\lambda_0 + 4\lambda' - 9)^3/[1944(\lambda' - 1)]$ and $\pi^B(\infty,0|\lambda') = m(5\lambda_0 + 4\lambda' - 9)(14\lambda' - 5\lambda_0 - 9)^2/[1944(\lambda' - 1)]$. To make the quality levels, demands and markups positive, we need $5\lambda_0 - 8\lambda' + 15 > 0$ and $14\lambda' - 5\lambda_0 - 9 > 0$.

When $f^A = \infty$ and $f^B = 0$, we have $s^A = (-\lambda_0 + 5)/4$ and $s^B = (16\lambda' - \lambda_0 - 3)/12$. Profits are $\pi^A(\infty,0|\lambda') = m(\lambda_0 + 8\lambda' - 9)(10\lambda' - \lambda_0 - 9)^2/[1944(\lambda' - 1)]$ and $\pi^B(0,\infty|\lambda') = m(\lambda_0 + 8\lambda' - 9)^3/[1944(\lambda' - 1)]$. We need conditions $16\lambda' - \lambda_0 - 3 > 0$ and $10\lambda' - \lambda_0 - 9 > 0$.

We also need to impose some additional conditions to make sure the consumer with the lowest quality preference would purchase in equilibrium.

The expected profits are (where $\varphi = \lambda^3[\theta\sqrt{z} + (1 - z)\sigma]/[(\theta\sqrt{z} + \sigma)(\theta - \sigma\sqrt{z})]$):

$$E\pi^A(0,0) = E\pi^B(0,0) = \frac{3\bar{m}}{8}(\theta^2 + \sigma^2),$$
$$E\pi^A(\infty,\infty) = \frac{\bar{m}}{24}[4\lambda\theta + 4\lambda^2 + \varphi], \text{ and}$$
$$E\pi^B(\infty,\infty) = \frac{\bar{m}}{24}[16\lambda\theta - 8\lambda^2 + \varphi],$$
$$E\pi^A(\infty,0) = \frac{\bar{m}}{1944}[800(\theta^2 + \sigma^2) - 60\lambda\theta - 12\lambda^2 + \varphi],$$
$$E\pi^B(0,\infty) = \frac{\bar{m}}{1944}[512(\theta^2 + \sigma^2) + 192\lambda\theta + 24\lambda^2 + \varphi],$$
$$E\pi^A(0,\infty) = \frac{\bar{m}}{1944}[64(\theta^2 + \sigma^2) + 240\lambda\theta + 300\lambda^2 + 125\varphi],$$
$$E\pi^B(\infty,0) = \frac{\bar{m}}{1944}[784(\theta^2 + \sigma^2) + 420\lambda\theta - 600\lambda^2 + 125\varphi].$$

Endnotes

[1] Flexibility can also be controlled through product design, supply chain management (e.g., portfolio of suppliers, long-term contracts, outsourcing, and in-house production), and the specificity of inventories and major equipment. When Dell developed its new laptop, the product was designed such that it was compatible with two possible battery choices (Krishnan and Bhattacharya 2002). The fashionable build-to-order production enables seamless connection between various configurations of the parts. By contrast, supply commitment and long-term contracts make adjustment more costly. When Canon supplied LaserJet printer engines to HP, the contract required HP to place an order six months in advance without any room for modification (Lee 2004).

[2] To focus on the competitive advantage of inflexibility, we stay away from any cost considerations by assuming the two repositioning strategies to be equally costly. This is different from the usual trade-off in operations management, in which a flexible technology is desirable but more costly than a rigid technology (Fine and Freund 1990, Van Mieghem 1998, Goyal and Netessine 2007).

[3] In our model, if the demand is deterministic (i.e., demand variance $\sigma^2 = 0$), whenever a firm chooses rigidity in equilibrium, its flexible rival is worse off.

[4] This again can be seen from our analysis. When demand concentrates, the two firms' joint profit would be maximized if they both choose rigidity. However, each firm's individual optimal strategy is flexibility. In the end, both firms are trapped in a prisoner's dilemma, foregoing the benefits of mutual commitment.

[5] Vives (1989) interprets the two values in a different way. In his model, a lower marginal cost of production represents simultaneously a stronger commitment *and* a more flexible technology. By definition, then, commitment value and option value always coexist.

[6] As mentioned earlier, the degree of a company's repositioning flexibility is partly determined by the width of its product lines. In particular, a wider range of products can better prepare a company for failures in some markets and provide a safe route of retreat in case competition intensifies on some fronts and, therefore, corresponds to a more flexible repositioning strategy. Viewed from this angle, many other examples can be found in which, between two major competitors, one pursues rigidity while the other chooses flexibility. In the commercial drone market, XAG Technology focused on agriculture drones, and the industry leader, DJI, produced all categories in personal, logistics, and aerial photography drones (Tencent Technology 2018). Among the two major foreign brands in the U.S. beer market, the Mexican Corona pursued a consistent and focused positioning of "fun, sun, and beach," while its major rival, Heineken, tried to appeal to consumers' diverse tastes by emphasizing multiple elements, such as sex, humor, friendliness, energy, fashion, humility, product quality, and market share leadership (Deshpandé 2011).

[7] Appelbaum and Lim (1985) show that output commitment is valuable in reducing entry, but the incumbent should commit less when uncertainty is larger. In their model, the trade-off is complicated by cost considerations as they assume that early production (i.e., commitment) has a cost advantage and late production (i.e., flexibility) has option values, so commitment is valuable even for a single decision maker. By contrast, the advantage of commitment in our model is solely the competitive advantage, in the sense that commitment is never optimal for a single decision maker.

[8] To be more specific, in Spencer and Brander (1992), output is the only choice. Once a firm has committed to an output level, it has no chance to respond to its rival's choice or any realization of the uncertainty. By contrast, in our model, positioning competition is followed by price competition. Even if a firm's position is fixed, it still

has a chance to respond to its rival's position and price choices (which, in turn, are responding to the changing market conditions if the rival is flexible). In a sense, output commitment is a "hard" commitment, whereas repositioning commitment is a "soft" commitment, which allows more room for further interactions both between the two firms and in response to stochastic market conditions.

[9] Meagher and Zauner (2004, 2005) find that uncertainty about demand distribution softens spatial competition. Commitment is not an issue because repositioning is assumed to be impossible. Some other research, mostly empirical, study how business shocks affect repositioning, that is, optimal ways to reposition, in a number of choices concerning product variety (Berry and Waldfogel 2001), TV program content (Wang and Shaver 2014), and supermarket pricing formats (Ellickson et al. 2012).

[10] This is a normalization of symmetric initial positions. Rather than the more natural values of $a_0 = -1$ and $b_0 = 1$, these initial positions are assumed because they correspond to equilibrium positions when consumers distribute on $[-1, 1]$. Such normalization makes it easier to interpret demand dispersion ($\mu > 1$) and concentration ($\mu < 1$).

[11] This assumption has been used in other studies, such as asymmetric equilibria in spatial competition (Tabuchi and Thisse 1995), Hotelling model with general consumer distribution (Anderson et al. 1997), positioning with sequential entry and different marginal cost (Tyagi 2000), and demand uncertainty (Meagher and Zauner 2004, 2005). Allowing positioning outside the range of consumer taste distribution is particularly appropriate for studying repositioning. If firms are constrained to locate within the range, they have to reposition if consumer distribution becomes more concentrated. This is an exogenous force that is orthogonal to the central theme of our study, which is voluntary and unconstrained repositioning. In an extension in the online appendix with a slightly different setting of the model, we show that firms often locate inside the range of consumer distribution in equilibrium, and the major results developed in the main model remain valid.

[12] Here we focus on pure strategies. In fact, when (3) holds, there is also a symmetric mixed strategy equilibrium in which each firm randomizes between flexibility and rigidity with positive probabilities.

[13] In an extensions with new products, bilateral rigidity can be an equilibrium.

[14] When z increases, the upper bound σ_{max} also rotates clockwise around the point $(\theta, \sigma) = (1/4, 0)$, which reduces the feasible parameter space. When we say "rigidity becomes more likely," it means that, for a given point in the (θ, σ) space that is feasible for both z_1 and z_2 ($z_2 > z_1$), if it is unilateral rigidity at z_1, it continues to be unilateral rigidity at z_2, but if it is bilateral flexibility at z_1, it may become unilateral rigidity at z_2.

[15] When $z \geq 1/2$, this inequality holds for any θ; when $z < 1/2$, it holds for most θ except those very close to one.

[16] The intensity of price competition can be measured by the distance between the two firms' equilibrium positions, $b - a$. A greater distance means softened competition. It can be shown that $b - a$ equals 3μ in bilateral flexibility, and $1 + 2\mu$ in unilateral rigidity. Therefore, rigidity intensifies competition when $\mu > 1$ and softens competition when $\mu < 1$.

[17] Between the two firms, either firm can earn more profit than its rival in equilibrium unilateral rigidity. This leads to some interesting properties when the two firms choose repositioning strategy sequentially as shown in an extension.

[18] So far, the analysis is about consumer surplus. It can also be shown that social welfare, which is the sum of consumer surplus and the two firms' profits, also follow the same pattern: it is higher under unilateral rigidity than under bilateral flexibility when $\theta > 1$ and σ is above a certain threshold.

[19] For a given θ, σ cannot be too small; otherwise (when, for example, $\sigma = 0$), both realizations of the taste shift would have to move in the same direction, which is prohibited by the setting.

[20] The condition for the left firm's unilateral rigidity $(\infty, 0)$ is $(1 - z)/\sqrt{z} < [\theta(4\theta^2 - 18\theta - 81)]/(4\sigma^3) + 3(2\theta - 3)/(2\sigma)$.

[21] The figure of $z > 1$ can be derived similarly and can be found in the online appendix.

[22] In the main model, firms actually do not incur any repositioning cost in the unilateral rigidity equilibrium because the rigid firm does not move at all and the flexible firm moves at no cost. In the mild rigidity equilibrium, both firms incur moderate repositioning costs in equilibrium. That's one reason why they are worse off than in bilateral flexibility.

[23] This is equivalent to the stochastic demand in the main model. In both cases, a fixed position needs to serve two different demand distributions.

[24] As is clear later, these initial qualities can be endogenized when the two firms compete for consumers whose quality tastes distribute uniformly on the interval $[1, \lambda_0]$.

[25] The feasible combination of θ and σ is a triangle. When θ is small, σ cannot be too large, otherwise the high-quality firm may lose its market share completely (if it chooses rigidity). When θ is large, σ cannot be too large, either; otherwise, the low-quality firm would give up consumers who care about quality the least, which is against the assumption of full market coverage.

[26] More specifically, we assume $\sigma < \bar{\sigma} \equiv \sqrt{z}/(1 + z)$. If this constraint is removed, the area of feasible (θ, σ) in Figure 7 would double; the triangle is flipped upward to form a parallelogram. This does not affect our conclusions about equilibrium rigidity.

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