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The effects of competition on the R&D portfolios of multiproduct firms ☆

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ABSTRACT

We investigate the R&D portfolio choices of multiproduct firms. When a firm increases cost-reducing R&D investment in a given product, its rivals will modify their entire R&D portfolios by reducing R&D investments in that particular product and increasing R&D investments in other competing products. Our analysis demonstrates that R&D portfolios will be more specialized when firms face greater competition, which will be the case if products become closer substitutes, a monopolist begins to face competition from a rival firm, or firms compete on price rather than quantity. R&D cooperation allows firms to internalize the negative externalities of their R&D investments in two ways: by reducing such investments across all products and by increasingly focusing their R&D portfolios on different products. Firms may completely shut down a subset of their R&D projects under R&D cooperation if the products concerned are sufficiently close substitutes.

1. Introduction

The relationship between market competition and incentives for innovation has been a controversial subject in economics since Schumpeter (1934, 1942) advanced the argument that competitive markets are not necessarily the most effective mechanism of exchange for promoting innovation. Arrow (1962) made the counter-argument that competitive markets provide firms with a stronger incentive for R&D. Numerous theoretical and empirical studies have examined this subject, but as Gilbert (2006a, p. 206) pointed out, economists "remain far from a general theory of innovation competition, although the large body of theoretical and empirical studies is beginning to yield conclusions, however meager."

A distinct feature of the vast R&D literature is that it focuses almost exclusively on single-product firms.[1] In real life, multiproduct firms are ubiquitous, and they face the constant problem of selecting R&D portfolios. A recent case study highlighted the daunting task faced by multiproduct firms[2]: "For Merck, the Research & Development portfolio and associated resource management problem has far-reaching economic implications as indeed it has for the whole pharmaceutical industry. Effective decision-making with respect to development and resource strategies is critical for companies to remain competitive in this challenging and uncertain environment. Two of the most critical decisions are selecting the right projects for the portfolio and allocating resources appropriately to deliver on the portfolio." In reviewing the recent literature on new product development, Ofek (2008, p. 71) pointed out that "while the 'one-dimensional' view of next-generation product development yields excellent insights into the incentives to invest in R&D effort and into the evolution of market structure, it does not capture one of the most important dilemmas confronting firms in the new product planning phase—where should these efforts be directed?"

This paper is among the first to investigate R&D incentives among multiproduct firms. We consider a 2-by-2 symmetric multiproduct duopoly in which two firms invest in cost-reducing R&D for the two products they both offer before competing on quantity in the product markets. Each firm has an initial cost advantage in producing one

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[1] See, for example, the survey articles by Reinganum (1989) and Gilbert (2006a). Vives (2008) recently provided a comprehensive analysis of the effects of competitive pressure on process and product innovation, and generated robust results for a range of market structures and competition modes. That article also considers only single-product firms.

[2] See http://www.aimms.com/aimms/references/case_studies/merck_co_r_d_portfolio_management. Based on an extensive database of research joint ventures registered with the U.S. Department of Justice and the Federal Trade Commission under the National Cooperative Research Act, Vonortas (2000) found overwhelming evidence that large diversified corporations interact in a multiproduct–multiproject environment.

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product, dubbed its core product, and the two firms' core products are different. We are particularly interested in studying the effects of competition on the pattern of equilibrium R&D portfolios. R&D outcomes are assumed to be non-drastic and certain, as D'Aspremont and Jacquemin (1988) had done, but without spillovers.

Our analysis first reveals a coordination effect: After cost-reducing R&D in a given product, a multiproduct firm will lower its output of the other product it offers in order to internalize the negative externality between the two substitutes. Given this coordination effect, when the products become closer substitutes, a firm will choose a more specialized R&D portfolio in favor of its core product. This is true for both monopolies and duopolies.

In addition to the direct effect by which firms benefit from R&D because it leads to lower production costs, two further effects arise when a multiproduct firm faces competition. In response to a rival's cost-reducing R&D for a given product, a firm lowers its output of that product (the business-stealing effect) and fights back in the substitute market by producing more of the substitute product (the cross-market effect). The business-stealing effect turns firms' R&D investments in the same product into strategic substitutes, as is the case with single-product firms. In contrast, the cross-product effect is unique to multiproduct firms and implies that firms' R&D investments in substitute products are strategic complements. Taken together, these effects lead multiproduct firms to concentrate their R&D investments in different products to avoid head-to-head competition. We find that in comparison with monopolists, duopolists choose more specialized R&D portfolios (toward their respective core products) in equilibrium.

When firms cooperate in R&D but continue to compete in product markets, they internalize the negative externalities among their R&D projects by cutting R&D investments in all products and specializing more in their respective core products. If the degree of product substitutability is sufficiently high, firms will choose complete specialization in R&D by closing down their non-core products' R&D labs. Although the possibility of R&D cooperation reducing R&D investments has been well analyzed in existing studies, its effect on the structure of R&D portfolios has not been adequately considered. Our analysis demonstrates that unlike that among single product firms, R&D cooperation among multiproduct firms may lead to less competitive R&D portfolios through more differentiated R&D research projects.

We also extend our model to cases in which firms compete on price in product markets. Under Bertrand competition, all of the main results continue to hold, such that the equilibrium R&D portfolios are more specialized if products become closer substitutes or the market structure changes from a monopoly to a duopoly. In addition, we show that firms choose more specialized R&D portfolios under Bertrand competition than they do under Cournot competition, which further underlines the paper's main message that competition in product markets leads to more specialized R&D portfolios.

Our paper contributes to the vast literature on R&D in two ways. First, we identify a unique channel through which firms' R&D investments in certain products affect rival firms' behavior in other products. This cross-market effect implies that firms' R&D investments are strategic substitutes within the same product, and are strategic complements across products. Second, and perhaps more importantly, we examine how competition in product markets affects equilibrium R&D portfolios, in the spirit of the Schumpeterian approach, and show that market competition, whether measured by an increase in the number of firms (from monopolies to duopolies) or product substitutability, or by a change from quantity to price competition, leads to more specialized R&D portfolios. If firms can coordinate their R&D decisions, they will also choose more (sometimes complete) specialized R&D portfolios.

Some early studies on R&D portfolios in the industrial organization literature have focused on relative risks among different R&D projects (e.g., Bhattacharya and Mookherjee, 1986; Cabral, 1994; Dasgupta and Maskin, 1987). These studies explored single-product firms, and were largely concerned with the divergence between market and socially optimal levels of risk. Following the Schumpeterian approach, recent developments have continued to focus on the relationship between competition and innovation, with the general conclusion being that it depends on many model-specific details (Gilbert, 2006b; Lin and Saggi, 2002; Schmutzler, 2009; Vives, 2008). For example, Aghion et al. (2005) demonstrated an inverted U-shaped relationship between market competition and innovation. Similarly, Sacco and Schmutzler (2011) found a U-shaped relationship between innovation and the degree of product substitutability. All of these studies, however, focused on single-product firms. Our model abstracts away from the risk aspect of R&D and focuses on the composition of R&D investments by multiproduct firms. We investigate the impact of intensified competition with a focus on R&D portfolios of multiproduct firms rather than absolute R&D investments by single-product firms.[3]

A few recent studies have examined R&D incentives in multiproduct firms. Lambertini (2003) used a model with horizontal product differentiation to investigate a monopolist's R&D portfolio in terms of the number of symmetric products and the allocation of R&D resources among them. He showed that monopoly leads to inefficiencies in both product and process R&D investments, and that incentives for process R&D increase with the number of products. Lin (2004) corrected an error in Lambertini's second result. Unlike the traditional Schumpeterian–Arrowian debate, Lambertini (2003) and Lin (2004) did not focus on the effects of competition on R&D portfolios. Chen and Schwartz (forthcoming) recently considered incentives for a monopolist in an existing product to innovate and introduce a new product. Although their model allowed for a multiproduct monopoly, it only considered one R&D project.

The remainder of this paper is organized as follows. Section 2 sets out our basic model. Section 3 presents the benchmark monopoly case before the symmetric duopoly case is analyzed in Section 4. R&D cooperation is considered in Section 5. Section 6 illustrates two extensions (quality differential and Bertrand competition), and Section 7 concludes.

2. Model

Consider duopoly competition between two symmetric firms, A and B, each of which produces two products, a and b. The price of product $i \in \{a,b\}$ is given by

$$p_i = v - \left(q_i^A + q_i^B\right) - \gamma\left(q_j^A + q_j^B\right),$$

where $v>0$, $j=a,b$, $j \neq i$, q_i^K is the output of product i produced by firm $K=A,B$, and $\gamma \in [0,1]$ represents the degree of differentiation between products a and b. The firms' products are perfect substitutes within each market (a or b), but are imperfect substitutes across markets.

Initially, the unit cost of producing product i for firm K is $c_i^{K0}>0$. A firm's initial unit cost differs between the two products. The product that can be produced at a lower cost is called the firm's core product, and the other product is called its non-core product. We assume that product a is firm A's core product and that product b is firm B's core product. We further assume that firms A and B are symmetric in their initial costs: $c_a^{A0}=c_b^{B0}=c^l$ and $c_b^{A0}=c_a^{B0}=c^h>c^l$.

Each firm K can reduce its unit cost of production to $c_i^K \equiv c_i^{K0} - x_i^K$ through process R&D, where x_i^K is the reduction in the production

[3] R&D portfolio management has recently received much attention in the management science literature. See, for example, Graves et al. (2000) and Chien (2002). However, these studies focus on the operational details of R&D projects. They are silent on how the degree of competition and strategic interaction among rival firms affects R&D portfolios.

cost for product i, which requires R&D expenditure $\beta(x_i^K)^2$, $\beta>0$. We refer to x_i^K as firm K's R&D investment in product i and to $\{x_a^K, x_b^K\}$ as its R&D portfolio.

The two firms play a two-stage game. In the first stage, they simultaneously and independently choose their R&D portfolios. In the second stage, after observing the first-stage R&D investments, the firms compete in the two product markets in Cournot fashion. We are particularly interested in the degree of R&D specialization in a firm's R&D portfolio, which is represented by the ratio between the firm's equilibrium R&D investments in its core and non-core products: $\frac{x_a^A}{x_b^A}\left(=\frac{x_b^B}{x_a^B}\right)$.

3. Monopoly: a benchmark

As a benchmark, first consider the monopoly case in which a single firm (say firm A) produces the two differentiated products. The demand function for product $i=a,b$ becomes (the superscript A is omitted throughout this section) $p_i=v-q_i-\gamma q_j$. Given $\{x_a,x_b\}$ and consequently $\{c_a,c_b\}$, in the second stage, the firm chooses the output mix that maximizes its total profit from the two products:

$$\begin{aligned}\max_{\{q_a,q_b\}} \pi &= (p_a-c_a)q_a+(p_b-c_b)q_b \\ &= \left(v-q_a-\gamma q_b-c^l+x_a\right)q_a+\left(v-q_b-\gamma q_a-c^h+x_b\right)q_b.\end{aligned} \tag{1}$$

The equilibrium output levels are

$$q_i=\frac{v(1-\gamma)-\left(c_i^0-x_i\right)+\gamma\left(c_j^0-x_j\right)}{2(1-\gamma^2)} \quad \text{for } i,j=a,b,\ i\neq j.$$

From this expression,[4] it is clear that $\frac{\partial q_i}{\partial x_i}>0$ and $\frac{\partial q_j}{\partial x_i}<0$. That is, a monopolist's R&D investment in a product increases the output of that product and decreases the output of the substitute product. The first property is straightforward. The second, which can be called the coordination effect of R&D by a multiproduct firm, can be understood as follows. As the output of a product increases due to the reduced cost of production, cannibalization from the substitute product also increases, leading the firm to scale down production of the substitute.

3.1. Monopoly R&D portfolio

Let $\mathbf{q}$ denote the vector of outputs and $\mathbf{c}$ the vector of post-R&D costs. In the R&D stage, the monopoly's optimal R&D portfolio is the solution to

$$\max_{(x_a,x_b)} \pi(\mathbf{q}(\mathbf{c}),\mathbf{c})-\beta(x_a)^2-\beta(x_b)^2.$$

Using the envelope theorem, we determine that $x_i=\frac{1}{2\beta}\frac{\partial\pi}{\partial x_i}$. From Eq. (1), $\frac{\partial\pi}{\partial x_i}=q_i$, so $x_i=\frac{q_i}{2\beta}$. In the second-stage, the first-order conditions of Eq. (1) lead to

$$\begin{aligned}2q_a+2\gamma q_b&=v-c_a\\ 2\gamma q_a+2q_b&=v-c_b.\end{aligned}$$

Plug $c_a=c^l-x_a=c^l-\frac{q_a}{2\beta}$ and $c_b=c^h-\frac{q_b}{2\beta}$ into the above equation to obtain

$$\begin{cases}\left(2-2\gamma-\dfrac{1}{2\beta}\right)(q_a-q_b)=c^h-c^l\\ \left(2+2\gamma-\dfrac{1}{2\beta}\right)(q_a+q_b)=2v-c^l-c^h.\end{cases} \tag{2}$$

[4] For general demand functions, the properties hold if $\frac{\partial}{\partial q_a}\left(\frac{\partial(p_a q_a)}{\partial q_b}\right)\le 0$ and $\frac{\partial}{\partial q_b}\left(\frac{\partial(p_b q_b)}{\partial q_a}\right)\le 0$.

From Eq. (2), we can easily solve for the equilibrium outputs and thus the R&D portfolio in a monopoly[5]: $x_a=\frac{1}{4\beta}\left[\frac{2v-c^l-c^h}{2+2\gamma-\frac{1}{2\beta}}+\frac{c^h-c^l}{2-2\gamma-\frac{1}{2\beta}}\right]$ and $x_b=\frac{1}{4\beta}\left[\frac{2v-c^l-c^h}{2+2\gamma-\frac{1}{2\beta}}-\frac{c^h-c^l}{2-2\gamma-\frac{1}{2\beta}}\right]$. Direct observation of the two expressions reveals the following properties. First, $x_a>x_b$, i.e., investment in the core product is greater than that in the non-core product. Second, x_a-x_b increases with γ, which means that as the two products become closer substitutes, the monopolist will invest more R&D in its core product, thereby implying more specialized R&D investments. Third, R&D investment in the non-core product (x_b) decreases with γ, while investment in the core product (x_a) is U-shaped in γ: It decreases with γ when γ is small, but increases with γ when γ is large. There are two forces operating on x_a when γ increases. On one hand, to avoid cannibalization between the two products, x_a and x_b should both be reduced. On the other hand, as x_b (and q_b) decreases, the damage x_a caused to product b decreases, which allows x_a to rise. When γ is large such that q_b is very small, the second effect will dominate. Combining the second and third properties, we have that $\frac{x_a}{x_b}\equiv 1+\frac{x_a-x_b}{x_b}$ increases with γ.

The above results are summarized in the following proposition:

Proposition 1. *A monopolist invests more in R&D for its core product than for its non-core product, and its R&D portfolio will be more specialized if the two products become closer substitutes.*

As with single-product firms, the benefits of reducing costs are proportional to the output produced. Due to the initial cost differential, output of the core product is larger than that of the non-core product, which makes R&D investment in the core product larger. As γ rises, each product increasingly cannibalizes the other product. This cross-product damage is proportional to the output level of the affected product and is therefore more significant for the core product, meaning that the monopolist should reduce output (and investment in) the non-core product relative to that of (in) the core product.

3.2. A measure of R&D specialization

We are interested in *changes* in R&D specialization as measured by $\frac{x_a^A}{x_b^A}$. As will be shown later, in all of the scenarios considered in this paper except R&D cooperation, the equilibrium R&D portfolio satisfies $\frac{x_a^A}{x_b^A}=\frac{q_a^A}{q_b^A}$, where q_a^A and q_b^A are firm A's post-R&D equilibrium output levels for the two products. Define

$$s\equiv\frac{1}{\Delta}\frac{q_a^A-q_b^A}{q_a^A+q_b^A},$$

where $\Delta\equiv\frac{c^h-c^l}{2v-c^l-c^h}$. Because $\frac{q_a^A}{q_b^A}$ increases if and only if $\frac{q_a^A-q_b^A}{q_a^A+q_b^A}$ increases, an R&D portfolio is more specialized if and only if s becomes larger.[6] Using s enables us to study changes in R&D portfolios without relying on the solution of x_a^A and x_b^A. This not only avoids tedious algebra, but more importantly, it also provides us with a better understanding of the intuitions of our results.

[5] The second-order conditions in the product stage are satisfied for all $\gamma\in[0,1]$. Those in the R&D stage hold if $\gamma<1-\frac{1}{4\beta}$ (or put differently, β is sufficiently large for any given γ). Large values of β are needed to ensure that the R&D cost function is sufficiently convex, and hence the profit function is concave. To guarantee that the monopolist produces a positive amount of both products after R&D, we also need $\gamma<\frac{v-c^h}{v-c^l}\left(1-\frac{1}{4\beta}\right)$. Note that when this condition holds, the second-order condition is always satisfied.

[6] When firms cooperate in R&D, it will be shown that $\frac{x_a^A}{x_b^A}=\frac{2q_a^A-q_b^A}{2q_b^A-q_a^A}>\frac{q_a^A}{q_b^A}$. In this case, the corresponding s is still instrumental in studying R&D specialization, as shown in the proof of Proposition 4.

From Eq. (2), R&D specialization by a monopolist can therefore be represented by

$$s^m = \frac{2(1+\gamma)-\frac{1}{2\beta}}{2(1-\gamma)-\frac{1}{2\beta}}. \tag{3}$$

Because s^m increases with γ, Proposition 1 is immediately established. Henceforth, we derive our other main results using R&D specialization as measured by the corresponding s.

4. Duopoly competition

We now turn to the model's original setting of duopoly competition. Given the two firms' R&D portfolios chosen in the first stage, firm A's optimization problem (firm B's problem is symmetric) in the second stage is

$$\begin{aligned} \max_{(q_a^A, q_b^A)} \pi^A &= \left(p_a - c_a^A\right) q_a^A + \left(p_b - c_b^A\right) q_b^A \\ &= \left(v - q_a^A - q_a^B - \gamma q_b^A - \gamma q_b^B - c_a^A\right) q_a^A + \left(v - q_b^A - q_b^B - \gamma q_a^A - \gamma q_a^B - c_b^A\right) q_b^A. \end{aligned} \tag{4}$$

The equilibrium Cournot outputs are easily solved as

$$q_i^A = \frac{v(1-\gamma) - 2c_i^A + 2\gamma c_j^A + c_i^B - \gamma c_j^B}{3(1-\gamma^2)} \quad \text{and} \qquad q_i^B = \frac{v(1-\gamma) - 2c_i^B + 2\gamma c_j^B + c_i^A - \gamma c_j^A}{3(1-\gamma^2)} \tag{5}$$

for $i,j \in \{a,b\}$ and $i \neq j$. It is evident that when a firm increases its R&D investment in a product, its output of that product increases, while its competitor's output of the same product decreases. This is the standard business-stealing effect that also exists for single-product firms. What is unique about multiproduct firms is that when a firm increases its R&D investment in a product, it reduces its output of the substitute product to mitigate within-firm cannibalization, which allows its competitor to expand in the latter market. As the next subsection shows, the above property is related to the slopes of the firms' best response functions in R&D investment.

4.1. R&D stage: three effects

In the first stage of the game, firm A chooses its R&D portfolio $\{x_a^A, x_b^A\}$ to maximize its total profit from the two product markets. Let $\mathbf{q}$ denote the vector of the four output levels and $\mathbf{c}$ the vector of the four post-R&D cost levels. Firm A's second-stage equilibrium profit can be expressed as $\pi^A(\mathbf{q}(\mathbf{c}), \mathbf{c})$. Its first-stage optimization is then

$$\max_{\{x_a^A, x_b^A\}} \pi^A(\mathbf{q}(\mathbf{c}), \mathbf{c}) - \beta\left(x_a^A\right)^2 - \beta\left(x_b^A\right)^2.$$

The first-order conditions lead to $x_i^A = \frac{1}{2\beta}\frac{d\pi^A}{dx_i^A}$. Applying the envelope theorem, we have

$$\frac{d\pi^A}{dx_i^A} = \underbrace{\underset{(+)}{\frac{\partial \pi^A}{\partial x_i^A}}}_{\text{direct effect}} + \underbrace{\underset{(-)}{\frac{\partial \pi^A}{\partial q_i^B}}\underset{(-)}{\frac{\partial q_i^B}{\partial x_i^A}}}_{\text{business-stealing effect}} + \underbrace{\underset{(-)}{\frac{\partial \pi^A}{\partial q_j^B}}\underset{(+)}{\frac{\partial q_j^B}{\partial x_i^A}}}_{\text{cross-market effect}}, \qquad i,j = a,b,\ j \neq i. \tag{6}$$

When firm A's unit cost of producing product i is reduced, its total profits are affected in three ways.[7] The first is the typical *direct effect* of cost-reducing R&D: A reduction in the unit cost of producing a product increases the firm's total profit. From Eq. (4), we see that for linear demand, this direct benefit is proportional to the output level of the product concerned:

$$\frac{\partial \pi^A}{\partial x_i^A} = q_i^A.$$

The second and third effects both pertain to responses from the competing firm, one through its output decision regarding the same product, and the other regarding the substitute product. The *business-stealing effect* captured by $\frac{\partial \pi^A}{\partial q_i^B}\frac{\partial q_i^B}{\partial x_i^A}$ is the product of two terms. The second term measures how much firm B will retreat in a market in which A has reduced its cost, whereas the first term measures how much such a retreat benefits firm A in both markets. Specifically, for linear demand,

$$\frac{\partial \pi^A}{\partial q_i^B} = -\left(q_i^A + \gamma q_j^A\right) \quad \text{and} \quad \frac{\partial q_i^B}{\partial x_i^A} = \frac{\partial q_i^A}{\partial x_i^B} = -\frac{1}{3(1-\gamma^2)}.$$

The third effect captured by $\frac{\partial \pi^A}{\partial q_j^B}\frac{\partial q_j^B}{\partial x_i^A}$ and called the *cross-market effect* is present only in multiproduct firms. When firm A becomes more aggressive in R&D in a given market, firm B will respond via its output decisions in the substitute market, which will again affect A's profits in both markets. The expressions are

$$\frac{\partial \pi^A}{\partial q_j^B} = -\left(q_j^A + \gamma q_i^A\right) \quad \text{and} \quad \frac{\partial q_j^B}{\partial x_i^A} = \frac{\partial q_i^A}{\partial x_j^B} = \frac{\gamma}{3(1-\gamma^2)} > 0.$$

As firm A's unit cost of producing a product declines, it scales down its production of the substitute product to internalize the negative externalities between the two competing products. This enables the rival firm to raise its output of the substitute product, which indirectly hurts firm A.

Thus, a firm's lower production cost affects its profit through three channels: the direct and business-stealing effects are beneficial, whereas the cross-market effect is detrimental. Because the business-stealing effect exists within the same market and the cross-market effect exists across substitute markets, the former dominates the latter. Using the expressions derived above, we obtain the combined effect as

$$\frac{d\pi^A}{dx_i^A} = \frac{4}{3} q_i^A > 0, \quad i = a,\ b. \tag{7}$$

Therefore, the marginal benefit of cost-reducing R&D investment in a product is proportional to the output level for that product.

4.2. Equilibrium R&D portfolio

Given Eq. (7), the R&D-stage optimization by firm A leads to

$$x_i^A = \frac{1}{2\beta}\frac{d\pi^A}{dx_i^A} = \frac{2}{3\beta} q_i^A, \quad i = a,\ b, \tag{8}$$

which implies that $\frac{x_a^A}{x_b^A} = \frac{q_a^A}{q_b^A}$. In the second-stage first-order conditions, let $q_b^B = q_a^A$, $q_a^B = q_b^A$, $c_a^A = c^l - \frac{2q_a^A}{3\beta}$ and $c_b^A = c^h - \frac{2q_b^A}{3\beta}$, and we obtain

$$\begin{cases} \left(1-\gamma-\dfrac{2}{3\beta}\right)\left(q_a^A - q_b^A\right) = c^h - c^l \\ \left(3(1+\gamma)-\dfrac{2}{3\beta}\right)\left(q_a^A + q_b^A\right) = 2v - c^h - c^l. \end{cases} \tag{9}$$

[7] Note that these three effects are present for any general demand and cost functions.

From Eqs. (9) to (8), we can solve for the equilibrium R&D portfolio,[8] which shows that $x_a^A > x_b^A$ and $x_b^B > x_a^B$: In duopoly competition, each firm invests more in its core product than it does in its non-core product, as in the case of a monopoly.

Before examining the effects of competition on the equilibrium R&D portfolio, we state the following property of the R&D best-response functions in our multiproduct setting (the proof is straightforward and is therefore omitted).

Lemma 1. *Multiproduct firms' R&D investments are strategic substitutes in the same product market and are strategic complements across product markets. That is,* $\frac{\partial x_a^A}{\partial x_a^B} = \frac{\partial x_b^A}{\partial x_b^B} < 0$, *and* $\frac{\partial x_a^A}{\partial x_b^B} = \frac{\partial x_b^A}{\partial x_a^B} > 0$.

The intuition for this result follows from the observation made in the previous subsection regarding the business-stealing and cross-market effects. This property indicates the difference in the ways in which multiproduct and single-product firms respond to their competitor's R&D investments. In particular, a multiproduct firm adjusts its R&D portfolio to avoid competition in the same product market, but fights back by conducting more R&D in the competing product market.

4.3. Effect of product substitutability

The equilibrium level of R&D specialization in a duopoly is obtained from Eq. (9):

$$s^d \equiv \frac{1}{\Delta} \frac{q_a^A - q_b^A}{q_a^A + q_b^A} = \frac{3(1+\gamma) - \frac{2}{3\beta}}{1 - \gamma - \frac{2}{3\beta}}. \tag{10}$$

Because s^d increases with γ, we immediately conclude as follows:

Proposition 2. *In duopoly competition, a multiproduct firm's R&D portfolio will become more specialized toward its core product if the two products become closer substitutes.*

The result and intuition are similar to those found in monopolies. γ captures the degree of the negative cross-market effect of R&D investments. When γ is larger, cross-market damage is intensified, but the damage is asymmetric between the two products. R&D in the non-core product causes a damage to the core product that is proportional to the output of the latter, and vice versa. Because the output of the core product is larger, reducing the cost of the non-core product is more damaging to the firm. As a result, the firm lowers its R&D investment in its non-core product more than it does that in its core product, which leads to a more specialized R&D portfolio in equilibrium.[9]

The property of s also has an interesting implication for market dominance. The ratio s represents not only a firm's R&D specialization ($\frac{x_a^A}{x_b^A}$), but also the relative market share of outputs in any given market, as $\frac{q_a^A}{q_b^A} = \frac{q_a^A}{q_a^B}$. That is, s also represents market dominance. Pre-R&D dominance, $\frac{q_a^{A0}}{q_b^{A0}}$, can be obtained from Eq. (10) by setting $\beta = \infty$: $s^0 = \frac{3(1+\gamma)}{1-\gamma}$. Clearly, $s > s^0$, which means that $\frac{q_a^A}{q_a^B} > \frac{q_a^{A0}}{q_a^{B0}}$. Therefore, R&D investment increases the market dominance of the major firm in each market.[10]

4.4. Comparison between duopoly and monopoly

Comparing Eq. (10) with Eq. (3), we have

$$s^d = \frac{3(1+\gamma) - \frac{2}{3\beta}}{1 - \gamma - \frac{2}{3\beta}} > \frac{2(1+\gamma) - \frac{1}{2\beta}}{2(1-\gamma) - \frac{1}{2\beta}} = s^m.$$

This immediately leads to the following conclusion:

Proposition 3. *The equilibrium R&D portfolio is more specialized under a duopoly than under a monopoly.*

There are two reasons for this comparison. The first pertains to marginal revenue and the second pertains to marginal cost. In comparison with those in a monopoly, A's marginal revenues from both products will drop in a duopoly due to the presence of the competing firm. Because within-market impacts are stronger than cross-market impacts, A's core business is mainly affected by B's non-core product. Because B's non-core product has a smaller output, A's marginal revenue drops by a smaller amount in its core business than it does in its non-core business. On the cost side, note that the equilibrium cost reduction is greater in a duopoly ($x_i^A = \frac{2}{3\beta} q_i^A$) than in a monopoly ($x_i^A = \frac{1}{2\beta} q_i^A$) due to the additional business-stealing and cross-market effects. Therefore, duopolists reduce their costs to a larger extent than do monopolists. This is true for both products, but the extra cost reduction is proportional to the output of each product and is therefore larger for a firm's core product than for its non-core product.

In sum, when a multiproduct monopoly firm starts to face competition from a rival firm, its marginal revenue falls, more so in the non-core product than in the core product. Meanwhile, its marginal cost also drops due to R&D investment, more so in the core product than in the non-core product. Both effects induce a duopolist to focus more on its core product than would a monopolist. As a result, R&D investment is more specialized in a duopoly.

5. R&D cooperation

Suppose the two firms are able to coordinate their R&D choices but remain independent in product competition (perhaps due to antitrust regulation). How will this R&D cooperation affect their R&D portfolios? Given the negative externalities between firms' R&D efforts, one would expect each firm to reduce R&D investments in both products, but the question remains: do they reduce them more for the core product or for the non-core product? Furthermore, will they completely close down some of their R&D activities?

The second-stage Cournot equilibrium is the same as described earlier, so Eq. (5) remains valid. In the first stage, the two firms coordinate their R&D investments to maximize their joint profits:

$$\max_{(x_a^A, x_b^A, x_a^B, x_b^B)} \pi^A(\mathbf{q}(\mathbf{c}), \mathbf{c}) + \pi^B(\mathbf{q}(\mathbf{c}), \mathbf{c}) - \beta\left(x_a^A\right)^2 - \beta\left(x_b^A\right)^2 - \beta\left(x_a^B\right)^2 - \beta\left(x_b^B\right)^2.$$

As Salant and Shaffer (1998, 1999) pointed out in the case of single-product firms, imposing symmetry on cooperative R&D arrangements before a solution is derived may lead to non-optimal outcomes. We thus look at all of the FOCs and SOCs (i.e., the signs of the

[8] As previously noted, γ should be small (or equivalently, β should be large) to ensure that the objective functions in the R&D stage are concave. It can be shown (Lin, 2011) that the second-order conditions for each firm's R&D maximization problem are satisfied for $\gamma < 1 - \frac{4}{9\beta}$, whereas the non-negativity of x_b^A requires that $\gamma < \left(1 - \frac{2}{3\beta}\right) - \frac{9\beta - 4}{3\beta} \frac{c^h - c^l}{v + c^h - 2c^l}$. Note that when this condition holds, the SOC is always satisfied.

[9] We assume throughout this paper that the two products are substitutes, i.e., $\gamma > 0$. If the two products are complements ($\gamma < 0$), $\frac{x_a^A}{x_b^A} = \frac{q_a^A}{q_b^A} > 1$ still holds, and a firm will still invest more in its core product. From Eq. (10), s decreases when complementarity strengthens ($|\gamma|$ increases), which means that the R&D portfolio will be less specialized. When the two products become stronger complements, the marginal revenue from both products increases, but the marginal revenue from the non-core product increases to a greater extent because it is proportional to the output of the core product, which is larger. As a result, the output of the non-core product will increase relative to the output of the core product.

[10] In a setting with competing single-product firms, Athey and Schmutzler (2001) studied whether a market leader can maintain or even increase its market dominance through further investments (in, for example, R&D). They found that under mild conditions, the market leader will invest more and increase its dominance. Our findings echo this conclusion, although in a multiproduct setting.

principal minors of the Hessian matrix) for the above maximization problem without restricting our attention to $x_a^A = x_b^B$ and $x_b^A = x_a^B$, and determine that the equilibrium choices are indeed symmetric (see Lin, 2011).

Appendix A shows that R&D specialization when firms cooperate in R&D is represented by

$$\tilde{s} \equiv \frac{1}{\Delta} \frac{\tilde{q}_a^A - \tilde{q}_b^A}{\tilde{q}_a^A + \tilde{q}_b^A} = \frac{3(1+\gamma) - \frac{1}{3\beta}}{1 - \gamma - \frac{1}{\beta}}.$$

The proposition below tells us that two types of symmetric cooperative R&D solutions can occur, depending on the degree of product substitutability. Define $\gamma \equiv \frac{1 - 9\Delta - \frac{1}{\beta}(1-\Delta)}{1+9\Delta}$. Then,

Proposition 4. *Under R&D cooperation,*

- **(i)** *for $\gamma < \gamma$, the equilibrium R&D portfolio is more specialized and entails lower R&D investments than under R&D competition;*
- **(ii)** *the R&D portfolio is more specialized when γ is larger, and becomes completely specialized (i.e., $\tilde{x}_b^A = \tilde{x}_a^B = 0$) if and only if $\gamma \geq \gamma$.*

Part (i) of this proposition informs us that R&D cooperation lowers R&D investments in all products. This is hardly surprising. As the literature on single-product firms confirms, R&D cooperation allows rival firms to internalize the negative externalities among their R&D decisions, which leads to a weaker incentive for conducting R&D. Part (i) also states that R&D cooperation leads to more specialized R&D portfolios. Appendix A shows that $\tilde{x}_i^A = \frac{1}{3\beta}\left(2\tilde{q}_i^A - \tilde{q}_j^A\right)$. Comparing this with what happens in R&D competition: $x_i^A = \frac{2}{3\beta} q_i^A$, one realizes that the extra term in R&D cooperation, $-\frac{1}{3\beta}\tilde{q}_j^A$, represents the negative externality between the two firms' R&D investments. To internalize this externality, each firm reduces its R&D investments in both products, but the reduction should be larger for the non-core product, which has an effect proportional to the output of the core product, which is larger.

There are therefore two ways in which multiproduct firms can internalize negative externalities under R&D cooperation. First, they can lower levels of R&D investment across all R&D projects. Second, they can collectively adjust their R&D portfolios to become more specialized in different products, which minimize the negative effects of such R&D investments on each other's profit.

Part (ii) tells us that similar to the pattern in other cases, a firm's R&D portfolio becomes more specialized as products become closer substitutes. Furthermore, if product substitutability is sufficiently strong, each firm will operate only one research lab and will shut down R&D projects for its non-core product (although it will continue to produce the non-core product). This result of complete R&D specialization highlights firms' incentive to avoid head-to-head competition within the same product market when they are able to coordinate their R&D decisions. The policy implication is clear: R&D cooperation may prompt rival firms to shut down competing research labs, thereby potentially hindering technological progress. Note that our result holds even when $\Delta = 0$. Firms may choose to shut down competing research labs even when a firm's products are symmetric. In that case, R&D coordination leads to a pure incentive to avoid direct R&D competition.

6. Extensions

In this section, we provide two extensions that demonstrate the robustness of the results derived from the basic model.[11] First, we show that nothing changes when the core product's advantage over the non-core product arises from a difference in product quality. Second, we derive R&D portfolios when firms compete on price rather than quantity, and show that the main result (R&D portfolios are more specialized when competition is intensified) continues to hold when competition is measured using other criteria.

6.1. Quality difference

A firm's core business may arise from its cost advantage, consumers' higher valuation, or a first mover advantage that allows the firm to enjoy greater consumer loyalty and/or learning-by-doing benefits. The main model focuses on cost advantage. In this extension, we show that a quality advantage will lead to exactly the same result. In fact, the two approaches are mathematically equivalent. Consider the basic model with the following modification: the demand for product i supplied by firm K is

$$p_i^K = v_i^K - \left(q_i^A + q_i^B\right) - \gamma\left(q_j^A + q_j^B\right),$$

with $v_b^A = v_a^B = v^l > 0$ and $v_a^A = v_b^B = v^h > v^l$. The vertical intercepts of the demand curves can be interpreted as an indication of product quality.[12] The unit cost of producing the high-quality product is higher: $c_a^{A0} > c_b^{A0}$. Product a is firm A's core product in the sense that $v_a^A - c_a^{A0} > v_b^A - c_b^{A0}$, which we assume to hold. Symmetrically, product b is B's core product.

Let $v^l = v$ and $v^h = v + \delta$. Therefore, $v_a^A - c_a^{A0} > v_b^A - c_b^{A0}$ is equivalent to $c_a^{A0} - \delta < c_b^{A0}$. Because the profit function of a given firm is linear in the vertical intercepts of the demand curve and the marginal cost of production, by redefining $c_a^{A0} - \delta$ and c_b^{A0} as c^l and c^h, respectively, the quality-difference model is mathematically identical to the cost-difference model analyzed earlier. Therefore, all of the results obtained in the basic model remain valid in the quality-difference model. This is hardly surprising. In many similar situations, the equilibrium depends solely on the so-called quality-cost margin (Anderson and De Palma, 2001), which may differ due to differences in cost or quality, or any combination of the two.

6.2. Bertrand competition

Now assume that firms compete on price rather than quantity in the second stage of the game. To rule out corner solutions when products are perfect substitutes in a given market, we generalize the demand system to encompass product differentiation both within and across markets:

$$p_i^K = v - \eta\left(q_i^K + \theta q_i^L\right) - \gamma\left(q_j^K + \theta q_j^L\right) \tag{11}$$

where $K, L = A, B$, $K \neq L$, $i, j = a, b$, $i \neq j$; and η, γ, θ are all within [0,1]. We also assume that $\eta > \gamma$, such that firms' products within a given market (a or b) are closer substitutes than those across markets.[13] Inverting the demand system (11), we obtain

$$q_i^A = \rho - \varphi\left(p_i^A - \mu p_i^B\right) + \lambda\left(p_j^A - \mu p_j^B\right), \tag{12}$$

where $\rho = \frac{v(\eta-\gamma)(1-\theta)}{(\eta^2-\gamma^2)(1-\theta^2)}$, $\varphi = \frac{\eta}{(\eta^2-\gamma^2)(1-\theta^2)}$, $\mu = \frac{\theta}{(\eta^2-\gamma^2)(1-\theta^2)}$ and $\lambda = \frac{\gamma}{(\eta^2-\gamma^2)(1-\theta^2)}$.

To study the effects of competition on equilibrium R&D portfolios, we consider the cases of monopoly firms (producing all four products), duopoly price competition and duopoly quantity competition, respectively. Appendix A gives R&D specializations in these three cases as

$$S_{Bert} = \frac{\left(2 + \theta - \theta^2\right)(\eta + \gamma) - \frac{2-\theta^2}{\beta(4-\theta^2)}}{\left(2 - \theta - \theta^2\right)(\eta - \gamma) - \frac{2-\theta^2}{\beta(4-\theta^2)}} \tag{13}$$

[11] We thank an anonymous referee for suggesting these two ways of extending our main model.

[12] See Hackner (2000), who compared price and quantity competition in a differentiated oligopoly producing single products, where the demand functions are of the Singh–Vives type, as in this model.

[13] The formulation in the main text is a special case with $\eta = 1$ and $\theta = 1$.

under Bertrand competition,

$$S^m = \frac{2\left(1-\theta^2\right)(\eta+\gamma)-\frac{1}{2\beta}}{2\left(1-\theta^2\right)(\eta-\gamma)-\frac{1}{2\beta}}$$

under a monopoly, and

$$S_{Cour} = \frac{(2+\theta)(\eta+\gamma)-\frac{2}{\beta\left(4-\theta^2\right)}}{(2-\theta)(\eta-\gamma)-\frac{2}{\beta\left(4-\theta^2\right)}} \tag{14}$$

under a Cournot duopoly.

Proposition 5. *(i) S_{Bert} increases with γ and θ; (ii) $S_{Bert}>S^m$ for all γ and $\theta<\bar{\theta}\equiv\frac{\sqrt{13}-1}{3}$; and (iii) $S_{Bert}>S_{Cour}$.*

Part (i) of the proposition states that R&D portfolios are more specialized if products become closer substitutes either across product markets (i.e., γ goes up) or across firms (i.e., θ goes up). Note that this also holds under Cournot competition in this new setting (S_{Cour}). Part (ii) tells us that R&D portfolios under Bertrand competition are more specialized than those under a monopoly (provided that the two firms' products are not too similar within each market: $\theta<\bar{\theta}\approx0.87$), which is consistent with the corresponding comparison in the basic model (Proposition 3). Part (iii) of the proposition states that R&D portfolios are more specialized under Bertrand competition than under Cournot competition, which is again consistent with the main message of this study that increased competition leads to more specialized R&D portfolios.

Investment in the non-core product is always lower under Bertrand competition than under Cournot competition (for demand system (11)). However, investment in the core product can be greater under Bertrand competition for some parameter values. Moving from a Cournot duopoly to Bertrand competition, competition intensifies, so a firm tends to reduce investments in both products. However, as a firm reduces investment in its non-core product, its rival firm's investment in the same product increases, which leads to stronger R&D incentives for the rival firm in its core product. This comparison is different from that in single-product firms,[14] but is consistent with multiproduct firms' behavior when competition is measured by other criteria. Recall that when γ increases, a monopolist's R&D investment in its non-core product always decreases, but investment in its core product is U-shaped such that it may rise with γ when γ is sufficiently large.

7. Conclusion

This paper is among the first to investigate R&D competition among multiproduct firms. It is particularly concerned with firms' R&D portfolio decisions in a setting with two horizontally differentiated products and two (symmetric) firms that compete in both product markets. We consider non-drastic cost-reducing R&D decisions made by Cournot firms and show that they change their entire R&D portfolios in response to increased R&D investment by rival firms in given products. Furthermore, firms modify their R&D investments in different directions. A firm will cut its R&D investment in a product if rivals increase their R&D efforts in that product, but will increase its R&D investment in another competing product. Thus, firms' R&D investments are strategic substitutes for the same product and are strategic complements across (competing) products.

We also show that firms' equilibrium R&D portfolios become more focused on their core products when there is a shift from a monopoly to a duopoly, or as the degree of product substitutability increases. In other words, market competition leads to more specialized R&D portfolios as firms attempt to avoid head-to-head R&D competition within the same product market. These results also hold under price competition.

This paper contributes to the vast innovation literature by demonstrating that increased competition causes firms to adopt increasingly specialized and differentiated R&D portfolios. This finding could also serve as a hypothesis for future empirical study. The study also provides new insights into the effects of R&D cooperation among firms. In a multiproduct setting, there are two ways in which competing firms internalize the negative externalities generated by their R&D investments: by reducing their R&D efforts for all products and by refocusing such efforts on different R&D projects. We show that under R&D cooperation, firms choose more specialized R&D portfolios, with each firm increasingly focusing on its own core product. If their products are sufficiently similar, firms may choose to shut down some of their R&D projects.

Appendix A

A.1. Proof of Proposition 4 (R&D cooperation)

(i) From R&D-stage optimization, we have

$$x_i^A = \frac{1}{2\beta}\left(\frac{d\pi^A}{dx_i^A}+\frac{d\pi^B}{dx_i^A}\right).$$

Because second-stage Cournot competition is the same as described previously, Eq. (7) is still valid, i.e., $\frac{d\pi^A}{dx_i^A}=\frac{4}{3}q_i^A$. However,

$$\begin{aligned}\frac{d\pi^A}{dx_i^B} &= -\frac{\partial\pi^A}{\partial q_i^B}\frac{\partial q_i^B}{\partial x_i^B}-\frac{\partial\pi^A}{\partial q_j^B}\frac{\partial q_j^B}{\partial x_i^B}\\ &= -\frac{\partial\pi^A}{\partial q_i^B}\frac{\partial q_i^A}{\partial x_i^A}-\frac{\partial\pi^A}{\partial q_j^B}\frac{\partial q_i^A}{\partial x_j^A}\quad\text{(by symmetry)}\\ &= \left(q_i^A+\gamma q_j^A\right)\frac{2}{3\left(1-\gamma^2\right)}+\left(\gamma q_i^A+q_j^A\right)\frac{-2\gamma}{3\left(1-\gamma^2\right)}\\ &= -\frac{2}{3}q_i^A.\end{aligned}$$

By symmetry, $\frac{d\pi^B}{dx_i^A}=-\frac{2}{3}q_i^B=-\frac{2}{3}q_j^A$. Therefore,

$$x_i^A = \frac{1}{2\beta}\left(\frac{4}{3}q_i^A-\frac{2}{3}q_j^A\right)=\frac{1}{3\beta}\left(2q_i^A-q_j^A\right).$$

We no longer have $\frac{x_a^A}{x_b^A}=\frac{q_a^A}{q_b^A}$. Rather,

$$\frac{x_a^A}{x_b^A}=\frac{2q_a^A-q_b^A}{2q_b^A-q_a^A}>\frac{q_a^A}{q_b^A}.$$

In the second-stage FOCs, given the symmetry between A and B, and given that $c_a^A=c^l-\frac{1}{3\beta}\left(2q_a^A-q_b^A\right)$ and $c_b^A=c^h-\frac{1}{3\beta}\left(2q_b^A-q_a^A\right)$, we have

$$\begin{cases}\left(2+\gamma-\frac{2}{3\beta}\right)q_a^A+\left(1+2\gamma+\frac{1}{3\beta}\right)q_b^A=v-c^l\\ \left(1+2\gamma+\frac{1}{3\beta}\right)q_a^A+\left(2+\gamma-\frac{2}{3\beta}\right)q_b^A=v-c^h\end{cases} \tag{15}$$

and therefore (the tilde denotes R&D cooperation)

$$\tilde{s}\equiv\frac{1}{\Delta}\frac{\tilde{q}_a^A-\tilde{q}_b^A}{\tilde{q}_a^A+\tilde{q}_b^A}=\frac{3(1+\gamma)-\frac{1}{3\beta}}{1-\gamma-\frac{1}{\beta}}.$$

[14] In a single-product setting, Lin and Saggi (2002) showed that cost-reducing R&D investment is always lower under Bertrand competition than under Cournot competition. Under Bertrand competition, a firm's cost reduction leads to more aggressive pricing by its rival firm, which tends to reduce the incentive for R&D more than in cases under Cournot competition, in which the rival firm is less aggressive in setting outputs in the post-R&D stage.

Because $\tilde{s} > s^d = \frac{3(1+\gamma)-\frac{2}{3\beta}}{1-\gamma-\frac{2}{3\beta}}$, we have $\frac{\tilde{q}_a^A}{\tilde{q}_b^A} > \frac{q_a^A}{q_b^A}$. Therefore,

$$\frac{\tilde{x}_a^A}{\tilde{x}_b^A} > \frac{\tilde{q}_a^A}{\tilde{q}_b^A} > \frac{q_a^A}{q_b^A} = \frac{x_a^A}{x_b^A},$$

which means that cooperative R&D leads to more specialized R&D portfolios.

The above calculation can lead to explicit expressions for equilibrium R&D investments under R&D cooperation: $\tilde{x}_a^A = \frac{1}{6\beta}\left[\frac{2v-c^l-c^h}{3(1+\gamma)-\frac{1}{3\beta}} + \frac{3\left(c^h-c^l\right)}{1-\gamma-\frac{1}{\beta}}\right] = \tilde{x}_b^B$ and $\tilde{x}_b^A = \frac{1}{6\beta}\left[\frac{2v-c^l-c^h}{3(1+\gamma)-\frac{1}{3\beta}} - \frac{3\left(c^h-c^l\right)}{1-\gamma-\frac{1}{\beta}}\right] = \tilde{x}_a^B$. In contrast, when firms compete in R&D (i.e., the case of duopoly competition in Section 4), the equilibrium R&D investments can be solved as $x_a^A = \frac{1}{3\beta}\left[\frac{2v-c^l-c^h}{3(1+\gamma)-\frac{2}{3\beta}} + \frac{c^h-c^l}{1-\gamma-\frac{2}{3\beta}}\right] = x_b^B$ and $x_b^A = \frac{1}{3\beta}\left[\frac{2v-c^l-c^h}{3(1+\gamma)-\frac{2}{3\beta}} - \frac{c^h-c^l}{1-\gamma-\frac{2}{3\beta}}\right] = x_a^B$. Direct comparison shows that $\tilde{x}_i^A < x_i^A$ for $i = a, b$.

(ii) Because $\frac{\tilde{x}_a^A}{\tilde{x}_b^A} = \frac{2\tilde{q}_a^A-\tilde{q}_b^A}{2\tilde{q}_b^A-\tilde{q}_a^A}$, $\frac{\tilde{x}_a^A}{\tilde{x}_b^A}$ increases if and only if $\frac{\tilde{q}_a^A}{\tilde{q}_b^A}$ increases, but $\frac{\tilde{q}_a^A}{\tilde{q}_b^A}$ increases if and only if $\frac{\tilde{q}_a^A-\tilde{q}_b^A}{\tilde{q}_a^A+\tilde{q}_b^A} \propto \tilde{s} = \frac{3(1+\gamma)-\frac{1}{3\beta}}{1-\gamma-\frac{1}{\beta}}$ increases. Because $\tilde{s}$ increases with γ, we conclude that $\frac{\tilde{q}_a^A}{\tilde{x}_b^A}$ increases with γ.

Our calculations thus far assume that each firm invests in both products ($\tilde{x}_i^A > 0$). The binding condition for non-negative investment should fall on the non-core product. Because $\tilde{x}_b^A = \frac{1}{3\beta}\left(2\tilde{q}_b^A - \tilde{q}_a^A\right)$, we have $\tilde{q}_b^A > 0$ if and only if $2\tilde{q}_b^A > \tilde{q}_a^A$ or, equivalently, $\tilde{s}\Delta = \frac{\tilde{q}_a^A-\tilde{q}_b^A}{\tilde{q}_a^A+\tilde{q}_b^A} < \frac{1}{3}$, which means that $\gamma < \frac{1-9\Delta-\frac{1}{\beta}(1-\Delta)}{1+9\Delta} \equiv \tilde{\gamma}$. Therefore, $\gamma \geq \tilde{\gamma}$ is sufficient for complete R&D specialization.[15] Now we establish that the condition is also necessary. Under complete specialization, $\tilde{x}_b^A = \tilde{x}_a^B = 0$. Therefore Eq. (15) becomes

$$\begin{cases} \left(2+\gamma-\dfrac{2}{3\beta}\right)\tilde{q}_a^A + \left(1+2\gamma+\dfrac{1}{3\beta}\right)\tilde{q}_b^A = v-c^l \\ (1+2\gamma)\tilde{q}_a^A + (2+\gamma)\tilde{q}_b^A = v-c^h, \end{cases}$$

which leads to the following equilibrium solutions:

$$\tilde{q}_a^A = \frac{[1-3\beta(1-\gamma)]\left(v-c^h\right) - 3\beta(2+\gamma)\left(c^h-c^l\right)}{(4\gamma+5)-9\beta(1-\gamma^2)},$$

$$\tilde{q}_b^A = \frac{[2-3\beta(1-\gamma)]\left(v-c^h\right) + 3\beta(1+2\gamma)\left(c^h-c^l\right)}{(4\gamma+5)-9\beta(1-\gamma^2)}.$$

The Kuhn–Tucker condition for $\tilde{x}_b^A = 0$ is $\frac{4}{3}\tilde{q}_b^A - \frac{2}{3}\tilde{q}_a^A \leq 0$, which is satisfied if and only if $\gamma \geq \tilde{\gamma}$, given the above expressions. Note that the above calculation assumes that $\tilde{q}_i^A \geq 0$. The binding constraint, $\tilde{q}_b^A \geq 0$, can be translated into $\frac{q_a^A-q_b^A}{q_a^A+q_b^A} \leq 1$. However, $\frac{q_a^A-q_b^A}{q_a^A+q_b^A} = s\Delta$. Given $s < \infty$, if Δ is sufficiently small (i.e., v is sufficiently large), the condition $s\Delta \leq 1$ will always be satisfied. Q.E.D.

A.2. Derivation of R&D specialization under Bertrand competition

Under the second-stage Bertrand competition, firm A's optimization problem is

$$\begin{aligned} \max_{\left(p_a^A, p_b^A\right)} \pi^A &= \left(p_a^A - c_a^A\right)q_a^A + \left(p_b^A - c_b^A\right)q_b^A \\ &= \left(p_a^A - c_a^A\right)\left(\rho - \varphi p_a^A + \varphi\mu p_a^B + \lambda p_b^A - \lambda\mu p_b^B\right) \\ &\quad + \left(p_b^A - c_b^A\right)\left(\rho - \varphi p_b^A + \varphi\mu p_b^B + \lambda p_a^A - \lambda\mu p_a^B\right). \end{aligned}$$

[15] Similar to previous cases, the condition $\gamma < \tilde{\gamma}$ guarantees that $\tilde{x}_b^A > 0$, which in turn ensures that all the second-order conditions are satisfied (see Lin, 2011).

The equilibrium price (after substituting the expressions for the demand parameters ρ, φ, μ and λ) is

$$p_i^A = \frac{v\left(2-\theta-\theta^2\right) + 2c_i^A + \theta c_i^B}{4-\theta^2}. \tag{16}$$

In the second-stage, the first-order conditions $\frac{\partial \pi^A}{\partial p_a^A} = \frac{\partial \pi^A}{\partial p_b^A} = 0$ lead to

$$\begin{cases} \left(2-\theta^2\right)q_a^A + \theta q_b^A = \dfrac{v(\eta-\gamma) - \eta c_a^A + \gamma c_b^A}{\eta^2-\gamma^2} \\ \theta q_a^A + \left(2-\theta^2\right)q_b^A = \dfrac{v(\eta-\gamma) - \eta c_b^A + \gamma c_a^A}{\eta^2-\gamma^2} \end{cases} \tag{17}$$

where we make use of the symmetry between A and B and Eq. (11) to turn equilibrium prices into outputs.

Let $\mathbf{p}$ denote the vector of the four prices and $\mathbf{c}$ denote the vector of the post-R&D costs. Firm A's first-stage optimization problem then becomes

$$\max_{\left(x_a^A, x_b^A\right)} \pi^A(\mathbf{p}(\mathbf{c}), \mathbf{c}) - \beta\left(x_a^A\right)^2 - \beta\left(x_b^A\right)^2.$$

The first-order condition leads to $x_i^A = \frac{1}{2\beta}\frac{d\pi^A}{dx_i^A}$, while

$$\begin{aligned} \frac{d\pi^A}{dx_i^A} &= \frac{\partial \pi^A}{\partial x_i^A} + \frac{\partial \pi^A}{\partial p_i^B}\frac{\partial p_i^B}{\partial x_i^A} \\ &= q_i^A + \left[\varphi\mu\left(p_i^A - c_i^A\right) - \lambda\mu\left(p_j^A - c_j^A\right)\right]\frac{-\theta}{4-\theta^2} \\ &= \frac{4-2\theta^2}{4-\theta^2}q_i^A. \end{aligned} \tag{18}$$

As a result, in the second stage, $c_a^A = c^l - \frac{2-\theta^2}{\beta(4-\theta^2)}q_a^A$ and $c_b^A = c^h - \frac{2-\theta^2}{\beta(4-\theta^2)}q_b^A$. Thus, Eq. (17) becomes

$$\begin{cases} \left[\left(2-\theta-\theta^2\right)(\eta-\gamma) - \dfrac{2-\theta^2}{\beta(4-\theta^2)}\right]\left(q_a^A - q_b^A\right) = c^h - c^l \\ \left[\left(2+\theta-\theta^2\right)(\eta+\gamma) - \dfrac{2-\theta^2}{\beta(4-\theta^2)}\right]\left(q_a^A + q_b^A\right) = 2v - c^h - c^l \end{cases} \tag{19}$$

which leads to the expression of S_{Bert} in the text. Q.E.D.

A.3. Monopoly under generalized demand

Given its R&D decisions, in the second stage, the monopolist maximizes the total profit earned from all four products:

$$\max_{\left(q_a^A, q_b^A, q_a^B, q_b^B\right)} \pi = \left(p_a^A - c_a^A\right)q_a^A + \left(p_b^A - c_b^A\right)q_b^A + \left(p_a^B - c_a^B\right)q_a^B + \left(p_b^B - c_b^B\right)q_b^B.$$

The first-order conditions, $\frac{\partial \pi^A}{\partial q_a^A} = \frac{\partial \pi^A}{\partial q_b^A} = 0$ (for a monopolist, choosing quantities is equivalent to choosing prices), lead to

$$\begin{cases} 2\left(1-\theta^2\right)\left(\eta^2-\gamma^2\right)q_a^A = v(\eta-\gamma)(1-\theta) - \eta c_a^A + \gamma c_b^A \\ 2\left(1-\theta^2\right)\left(\eta^2-\gamma^2\right)q_b^A = v(\eta-\gamma)(1-\theta) - \eta c_b^A + \gamma c_a^A. \end{cases}$$

As described previously, from the R&D-stage optimization we obtain $x_i^A = \frac{1}{2\beta}\frac{d\pi}{dx_i^A}$, with $\frac{d\pi}{dx_i^A} = \frac{\partial \pi^A}{\partial x_i^A} = q_i^A$, and thus $x_i^A = \frac{q_i^A}{2\beta}$. Therefore, in the second stage, $c_a^A = c^l - \frac{q_a^A}{2\beta}$ and $c_b^A = c^h - \frac{q_b^A}{2\beta}$. Plugging these into the above expressions, we obtain

$$\begin{cases} \left[2\left(1-\theta^2\right)(\eta-\gamma)-\dfrac{1}{2\beta}\right]\left(q_a^A-q_b^A\right)=c^h-c^l \\ \left[2\left(1-\theta^2\right)(\eta+\gamma)-\dfrac{1}{2\beta}\right]\left(q_a^A+q_b^A\right)=2v-c^h-c^l, \end{cases} \tag{20}$$

which leads to the expression of S^m in the text. Q.E.D.

A.4. Cournot competition with generalized demand

In second-stage Cournot competition, firm A's optimization problem is

$$\max_{q_a^A,q_b^A} \pi^A = \left(v-\eta q_a^A-\eta\theta q_a^B-\gamma q_b^A-\gamma\theta q_b^B-c_a^A\right)q_a^A + \left(v-\eta q_b^A-\eta\theta q_b^B-\gamma q_a^A-\gamma\theta q_a^B-c_b^A\right)q_b^A.$$

The first-order conditions, $\frac{\partial \pi^A}{\partial q_a^A}=\frac{\partial \pi^A}{\partial q_b^A}=0$, lead to

$$\begin{cases} 2\eta q_a^A+\eta\theta q_a^B+2\gamma q_b^A+\gamma\theta q_b^B=v-c_a^A \\ 2\eta q_b^A+\eta\theta q_b^B+2\gamma q_a^A+\gamma\theta q_a^B=v-c_b^A. \end{cases} \tag{21}$$

Given the symmetry, $q_b^B=q_a^A$ and $q_a^B=q_b^A$. The equilibrium output is

$$q_i^A=\frac{v(\eta-\gamma)(2-\theta)-\eta\left(2c_i^A-\theta c_i^B\right)+\gamma\left(2c_j^A-\theta c_j^B\right)}{\left(\eta^2-\gamma^2\right)\left(4-\theta^2\right)}.$$

R&D stage optimization still leads to $x_i^A=\frac{1}{2\beta}\frac{d\pi^A}{dx_i^A}$, but the three effects (expression (6)) become

$$\begin{aligned} \frac{d\pi^A}{dx_i^A} &= \frac{\partial \pi^A}{\partial x_i^A}+\frac{\partial \pi^A}{\partial q_i^B}\frac{\partial q_i^B}{\partial x_i^A}+\frac{\partial \pi^A}{\partial q_j^B}\frac{\partial q_j^B}{\partial x_i^A} \\ &= q_i^A-\theta\left(\eta q_i^A+\gamma q_j^A\right)\frac{-\eta\theta}{\left(\eta^2-\gamma^2\right)\left(4-\theta^2\right)}-\theta\left(\gamma q_i^A+\eta q_j^A\right)\frac{\gamma\theta}{\left(\eta^2-\gamma^2\right)\left(4-\theta^2\right)} \\ &= \frac{4}{4-\theta^2}q_i^A. \end{aligned}$$

Therefore, $x_i^A=\frac{1}{2\beta}\frac{d\pi^A}{dx_i^A}=\frac{2}{\beta(4-\theta^2)}q_i^A$. Thus, $\frac{x_a^A}{x_b^A}=\frac{q_a^A}{q_b^A}$ continues to hold. In the second stage, $c_a^A=c^l-x_a^A=c^l-\frac{2}{\beta(4-\theta^2)}q_a^A$. Similarly, $c_b^A=c^h-\frac{2}{\beta(4-\theta^2)}q_b^A$. Therefore, Eq. (21) becomes

$$\begin{cases} \left[(\eta-\gamma)(2-\theta)-\dfrac{2}{\beta(4-\theta^2)}\right]\left(q_a^A-q_b^A\right)=c^h-c^l \\ \left[(\eta+\gamma)(2+\theta)-\dfrac{2}{\beta(4-\theta^2)}\right]\left(q_a^A+q_b^A\right)=2v-c^h-c^l, \end{cases}$$

which leads to the expression of S_{Cour} in the text. Q.E.D.

A.5. Proof of Proposition 5

(i) By examining the expression of S_{Bert}, it is clear that $\frac{dS_{Bert}}{d\gamma}>0$ and $\frac{dS_{Bert}}{d\eta}<0$, while $\frac{dS_{Bert}}{d\theta}>0$ if and only if $\beta>\frac{\left(8-2\theta^2+\theta^4\right)\eta-2\theta\left(2-\theta^2\right)^2\gamma}{\left(\eta^2-\gamma^2\right)\left(2+\theta^2\right)\left(4-\theta^2\right)^2}\equiv\beta_t$. Note that $q_a^A>q_b^A$ under Bertrand competition (the first equation in Eq. (19)) requires $\beta>\frac{2-\theta^2}{\left(4-\theta^2\right)\left(2-\theta-\theta^2\right)(\eta-\gamma)}\equiv\beta_0$. Because $\beta_0-\beta_t=\frac{\left(2\theta^4-3\theta^3-2\theta^2+2\theta+4\right)\left[\theta\eta+\left(2-\theta^2\right)\gamma\right]}{\left(\eta^2-\gamma^2\right)\left(2+\theta^2\right)\left(4-\theta^2\right)^2(1-\theta)}>0$, we have $\beta>\beta_0>\beta_t$, which means that $\frac{dS_{Bert}}{d\theta}>0$.

(ii) $S_{Bert}>S^m$ if and only if $\beta>\frac{3\theta\left(2-\theta^2\right)\gamma+(4-\theta)^2\eta}{4\left(1-\theta^2\right)\left(4-\theta^2\right)\left(\eta^2-\gamma^2\right)}\equiv\beta_1$. Part (i) reveals that $\beta>\beta_0$, but $\beta_0-\beta_1=-\frac{\left(3\theta^2+2\theta-4\right)\left[\theta\eta+\left(2-\theta^2\right)\gamma\right]}{4\left(\eta^2-\gamma^2\right)(2+\theta)^2(2-\theta)\left(1-\theta^2\right)}$. When $\theta<\bar{\theta}$, we have $3\theta^2+2\theta-4<0$, and therefore $\beta_0>\beta_1$. This means that $\beta>\beta_0>\beta_1$, and hence $S_{Bert}>S^m$.
Now consider what happens when $\theta>\bar{\theta}$. The requirement that $q_a^A>q_b^A$ in a monopoly (the first equation in Eq. (20)) leads to $\beta>\frac{1}{4\left(1-\theta^2\right)(\eta-\gamma)}\equiv\beta_m$. When $\theta>\bar{\theta}$, $\beta_1>\beta_m>\beta_0$, then $S_{Bert}>S^m$ for $\beta>\beta_1$ and $S_{Bert}<S^m$ for $\beta\in[\beta_m,\beta_1]$. Note that the range $[\beta_m,\beta_1]$ is very narrow. For example, when $\eta=1$, $\gamma=0.5$ and $\theta=0.9$, we have $\beta_m=2.632$ and $\beta_1=2.638$. Thus, for most parameter values, $S_{Bert}>S^m$. In addition, note that $q_b^A\geq 0$ will never be a concern as long as Δ is sufficiently small.

(iii) $S_{Bert}>S_{Cour}$ if and only if $\beta\left(4-\theta^2\right)>\frac{\eta}{\eta^2-\gamma^2}$, which is always satisfied because $\beta>\beta_0$ implies that $\beta\left(4-\theta^2\right)>\frac{2-\theta^2}{\left(2-\theta-\theta^2\right)(\eta-\gamma)}$, but $\frac{2-\theta^2}{2-\theta-\theta^2}>1>\frac{\eta}{\eta+\gamma}$, and therefore $\frac{2-\theta^2}{\left(2-\theta-\theta^2\right)(\eta-\gamma)}>\frac{\eta}{\eta^2-\gamma^2}$. Q.E.D.

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