A Simple Model of Entry That Increases Price Levels and Price Dispersion

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Abstract

A monopolist who originally charges a uniform price across all markets may switch to discriminatory pricing upon the entry of a competitor. As a result, intensified competition may lead to more dispersed prices as well as higher prices for some or all consumers.

KEYWORDS: entry, price competition, price discrimination, price dispersion

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1 Introduction

Consider the following situation: The only movie theatre in town used to charge $6 for a ticket. After a second theatre opened recently, the first theatre started to charge $7 for its tickets except on Tuesdays, when it charged a “discounted” price of $6.50. How could increased competition through the entry of a competitor raise rather than reduce market prices?

In this paper, I explain the phenomenon with a model of price discrimination. There are two types of consumers: the low-elasticity type (called loyals) and the high-elasticity type (called shoppers). A firm would like to discriminate on price, and it does so by charging different prices in different markets that it serves (e.g., on different days). If shoppers are more likely to move between markets than are loyals, a low price in one market may attract shoppers from other markets, while loyals remain and therefore can be charged higher prices. The two types of consumers are thus segregated and the prices discriminate.

Price discrimination is not always desirable, however. When moving between markets, shoppers incur some disutility, which is a deadweight loss that could have been turned into a firm’s profits. Compared with uniform pricing, whether or not price discrimination is desirable depends on whether the loss can be covered by the higher prices charged to loyals. When competition intensifies, each firm retains its loyal customers but loses some shoppers to its competitors. Since fewer shoppers are moving between markets, the deadweight loss becomes smaller and therefore price discrimination becomes less costly. A firm that originally charged a uniform price may then find it desirable to price discriminate. As a result, some or all customers pay higher prices when competition intensifies. Market prices also become more dispersed.

This paper is organized as follows. After setting up the model, I will derive the equilibrium under monopoly and duopoly. A comparison between the equilibria will then reveal that a monopolist who originally uses uniform pricing may switch to price discrimination upon the entry of a competitor. Finally, I will relate the conclusions to the pharmaceutical industry and compare the model with a closely related one by Rosenthal (1980). All proofs are in the Appendix.

2 Setting

Two towns, $A$ and $B$, are connected by a linear road of unit length. Originally there is a single firm located in town $A$. Then, a second firm enters and
is located in town $B$. The two firms are referred to as $A$ and $B$, too. The market structures before and after $B$’s entry are called monopoly and duopoly, respectively. Both firms produce a standard, perishable product at zero cost.

There are two types of consumers. The first type, called *loyals*, live in both towns and buy only from the firm that is located in the same town. The second type, called *shoppers*, live on the road and may choose between the two firms. Shoppers incur a transportation cost of $t$ per unit of distance traveled. Regardless of type, each consumer is willing to pay at most $v$ for one unit of the product within a week, which, for simplicity, is assumed to consist of two days. Consumers have exogenous preferences for their ideal purchasing dates. Among all consumers who prefer to buy on day $i$ ($i = 1, 2$), there are $x_i$ loyals in each town and $y_i$ shoppers distributed uniformly along the road. Loyals are assumed to stick with their ideal purchasing dates, while shoppers have the option of buying on the non-ideal day by incurring a disutility, $s$, which is the same for all shoppers regardless of their location or time preference.

In summary, each consumer is characterized by two parameters: his geographic location and ideal purchasing date. Loyals switch neither the date nor the brand, while shoppers can switch both. A loyal customer who lives in town $j$ ($j = A, B$) and prefers day $i$ ($i = 1, 2$) derives utility $v - p^i_j$ if he buys (on the ideal date), where $p^i_j$ is firm $j$’s price on day $i$. A shopper who is located at $z$ and prefers day $i$ derives utility $v - p^i_j - t z^j - \delta s$ if he buys from firm $j$ on day $k$, and 0 if he does not buy, where $z^j = \begin{cases} z, & \text{if } j = A \\ 1 - z, & \text{if } j = B \end{cases}$ is the distance between location $z$ and firm $j$, and $\delta = \begin{cases} 0, & \text{if } k = i \\ 1, & \text{if } k \neq i \end{cases}$ is a binary variable indicating whether the shopper buys on his ideal date.

Each firm chooses its prices on the two days to maximize its weekly profit. The game is played as follows. At the beginning of the week, each firm announces the prices that it plans to charge on the two days. After learning all prices (two in the case of monopoly and four in the case of duopoly), consumers decide whether to buy and, if so, from which firm on which day. Finally, the firm(s) charge the prices as announced, and consumers carry out the purchases as planned. I look for pure strategy Nash equilibria.

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1In this model, a day is simply a convenient way to label a market, so there is no time discounting. In fact, the model does not involve time in any particular sense. The two days may as well represent two geographic markets.
3 Before the Entry

Before $B$’s entry, firm $A$ is the only supplier, so loyals in town $B$ will not purchase anything. Since all consumers have the same reservation price, $v$, firm $A$ will never charge a price that is higher than $v$. As a result, loyals in $A$ always buy (on their ideal dates). Shoppers, conditional on buying, have an additional choice of the purchasing date. A shopper buys on his non-ideal date only when that day’s price is lower than the ideal date’s price by more than $s$, the switching disutility. Consequently, $A$ has two strategies. $A$ can make the two days’ prices differ by less than $s$. Because one day’s price does not affect the sales on the other day, the two prices will be chosen independently, so the strategy is called independent pricing (IP). Alternatively, $A$ can make the two prices differ by more than $s$. The lower price, also called the discount price, will serve shoppers from both days as well as loyals of that day, while the non-discount price will serve loyals of the non-discount day. This strategy is called discount pricing.

Consider independent pricing first. Let $A$ charge $p_i (p_i \leq v)$ on day $i$ ($i = 1, 2$). The shopper who is indifferent between buying and not is located at $z_i \ (0 \leq z_i \leq 1)$ where $v - p_i - tz_i = 0$, which means that $A$’s sales to shoppers is $z_i = \frac{v-p_i}{t}$. $A$’s profit on that day is $\pi_i = p_i(x_i + y_i z_i)$, leading to an optimal price of $p_i = \frac{1}{2} \left( v + t \frac{z_i}{y_i} \right)$ for $i = 1, 2$. Given $v$ and $t$, the optimal price depends only on the ratio $\frac{x_i}{y_i}$. When the ratio is higher, there are relatively more loyals, whose demand elasticity is low, so the firm will charge a higher price.

Notice that the two days’ prices differ whenever $\frac{x_1}{y_1} \neq \frac{x_2}{y_2}$, so there may be price discrimination even if the two days are treated independently. In that case, the discrimination is third degree. In this paper, my purpose is to show that the entry of a competitor may raise market prices. Rosenthal (1980) has offered an explanation why this may be true. His logic can also be found in my model if firms use independent pricing and each day is considered separately. I would like to focus on a different explanation, which involves the transfer of one day’s demand to the other day, i.e., second-degree price discrimination. For simplicity, I assume throughout the paper that $\frac{x_1}{y_1} = \frac{x_2}{y_2}$. The major finding of the paper continues to hold if the assumption is relaxed.

**Assumption 1:** $\frac{x_i}{y_i} = r$ for $i = 1, 2$.

Under Assumption 1, if more loyals prefer to buy on one day, more shoppers will also prefer to buy on that day. The two days are consequently called
the high- and low-demand days and re-labelled as $H$ and $L$. Furthermore, normalize $y_H + y_L = 1$, which implies that $y_H \geq \frac{1}{2}$ and $y_L \leq \frac{1}{2}$. Given that $\frac{y_H}{y_L} = r$, the two days’ prices under $IP$ are identical:

$$p_i = \frac{v + rt}{2}, \quad i = H, L.$$ 

For this reason, independent pricing is also referred to as “uniform pricing”. $A$’s weekly equilibrium profit is $\pi^{IP} = \frac{(v+rt)^2}{4t}$.

For the above price to be indeed optimal, some regularity conditions must be satisfied. In particular, $z_i \geq 0$ (which is equivalent to $p_i \leq v$) requires $v \geq rt$, while $z_i \leq 1$ requires $v \leq 2+rt$. That is, $v$ must take some intermediate values. If $v$ is too small, the firm would serve no shoppers; if $v$ is too large, the firm would serve all shoppers. In either case, the optimal price would be a corner solution and $p_i = \frac{v + rt}{2}$ is no longer optimal. Throughout the paper, I assume that $v$ indeed falls between the two boundaries:

**Assumption 2:** $r \leq \frac{v}{t} \leq 2 + r$.

Now consider discount pricing. As the two days’ demands are different, discount pricing can be further classified as discounting on the high-demand day ($DH$) and the low-demand day ($DL$). Under $DL$, the non-discount price on day-$H$ serves only loyals, so $A$’s optimal choice is $p_H = v$. Let the discount price on day-$L$ be $p_L$. Then, the shopper on day $L$ who is indifferent between buying and not will have a location $z_L$ where $v - p_L - tz_L = 0$. A similar shopper on day $H$ will have a location $z_H$ where $v - p_L - tz_H - s = 0$. As a result, the monopolist sells $z_L = \frac{v - p_L}{t}$ to shoppers on day $L$ and $z_H = \frac{v - p_L - s}{t}$ on day $H$. Given the weekly profit of $x_H p_H + p_L (x_L + y_H z_H + y_L z_L)$, the optimal discount price is

$$p_L = \frac{v + rt y_L - sy_H}{2}.$$ 

$A$’s weekly payoff is $\pi^{DL} = rvy_H + \frac{(v+rt y_L - sy_H)^2}{4t}$. By symmetry, $A$’s weekly profit from discount pricing on day $H$ is $\pi^{DH} = rvy_L + \frac{(v+rt y_H - sy_L)^2}{4t}$.

**Proposition 1:** Before $B$’s entry, $A$ never discounts on the high-demand day. It discounts on the low-demand day when $s \leq s_m$ and charges a uniform price otherwise, where $s_m = \frac{1}{y_H} \left( v + y_L rt - \sqrt{(v + rt)^2 - 4vry_H} \right)$.

The intuition of the proposition can be understood as follows. The purpose of discount is price discrimination. Between the two types of consumers, loyals

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have a lower elasticity of demand and can therefore be discriminated against. Although discrimination within a single day is not feasible, the two types can be partially separated if shoppers can be induced to buy on a non-ideal date. Compared with independent pricing, discount pricing is beneficial because the firm can charge a higher price to loyals on the non-discount day. There is also a cost, though. To attract switching shoppers, who incur a disutility, $s$, the firm has to further lower its price on the discount day, which gives a free ride to that day’s consumers. To the firm, the cost of inducing shopper switching is proportional to $s$. A small $s$ favors discount pricing. As $s$ increases, discount pricing is increasingly costly and, beyond a certain point, independent pricing becomes the optimal choice.

When using discount pricing, the firm faces the following tradeoff in choosing the discount date. Discount on the high-demand day gives a free ride to more loyals, who would be willing to pay a higher price. Discount on the low-demand day, however, means that switching shoppers are drawn from the high-demand day, and therefore, there are more of them. Since the switching cost is a deadweight loss, the more shoppers who switch, the less profit the firm makes from shoppers. The second effect is proportional to $s$; but this is not true with the first effect. The discount should therefore be on the high-demand day when $s$ is large and on the low-demand day when $s$ is small.

Finally, when all three strategies are considered together, it turns out that whenever discount pricing is more profitable than independent pricing, $s$ must be so small that discount on the low-demand day is more profitable than discount on the high-demand day. The latter therefore is never optimal.

4 After the Entry

Now suppose that firm $B$ enters the market in town $B$. I will first show that an asymmetric equilibrium does not exist.

**Proposition 2**: In duopoly, there is never an equilibrium at which one firm uses discount pricing and the other uses independent pricing.

As discussed above, if there is only one firm, its optimal strategy is $IP$ for a large $s$ and $DL$ for a small $s$. When two firms compete, this is also the pattern

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\(^2\)Consider the case of $s = 0$. Starting with independent pricing, if the firm raises one day’s price to $v$ and keeps the other day’s price unchanged, shoppers from the first day will purchase the product on the second day. The total sales to each type does not change, but loyals of the first day pay a higher price, thereby increasing the firm’s profit.
of each firm’s optimal strategy. If \( s \) is small, both firms want to use \( DL \); if \( s \) is large, both want to use \( IP \). There does not exist an \( s \) such that it is small enough for one firm to use \( DL \) and, at the same time, large enough for the other firm to use \( IP \). No asymmetric equilibrium exists in which one firm uses \( IP \) and the other uses \( DL \). By the same token, we can show there are no other asymmetric equilibria (in which one firm uses \( DH \) and the other uses \( IP \) or \( DL \)). As a result, a pure strategy equilibrium must be symmetric: both firms must use the same strategy, \( IP \), \( DL \) or \( DH \).

First, suppose both firms use \( IP \). Let firm \( j \) (\( j = A, B \)) charge \( p^j_i \) on day \( i \) (\( i = H, L \)). The shopper who is indifferent between buying from \( A \) and \( B \) is in location \( z_i \) where \( v - p^A_i - tz_i = v - p^B_i - t(1 - z_i) \), so \( z_i = \frac{p^B_i - p^A_i + t}{2t} \). Then \( A \)'s sales to shoppers on day \( i \) are \( z^A_i = z_i \), while \( B \)'s sales are \( z^B_i = 1 - z_i \). Firm \( j \)’s profit is \( \pi^j_i = p^j_i (x_i + y_i z^j_i) \), which leads to the equilibrium prices

\[
p^j_i = t(1 + 2r) \quad \text{for } i = H, L \text{ and } j = A, B.
\]

Each firm’s weekly profit is \( \pi^{IP} = \frac{r}{2} (1 + 2r)^2 \).

Next, suppose both firms use \( DL \). Then, \( p^A_H = p^B_H = v \). Given the two discount prices \( p^A_L \) and \( p^B_L \), the shopper on day \( L \) who is indifferent between buying from \( A \) and \( B \) has a location \( z_L \) where \( v - p^A_L - tz_L = v - p^B_L - t(1 - z_L) \), while the indifferent shopper on day \( H \) has \( v - p^A_L - tz_H - s = v - p^B_L - t(1 - z_H) - s \), so \( z_L = z_H = \frac{p^B_L - p^A_L + t}{2t} \equiv z \). Then \( A \)'s sales to shoppers on either day are identically \( z^A_i = z \) while \( B \)'s sales are \( z^B_i = 1 - z \) for \( i = H, L \). The two firms’ weekly profits are \( \pi^A_j = p^A_H x_H + p^A_L (x_L + y_L z^A_L + y_H z^A_H) \), which leads to the equilibrium prices

\[
p^A_L = p^B_L = t(1 + 2ry_L).
\]

Each firm’s equilibrium weekly profit is \( \pi^{DL} = rvy_H + \frac{r}{2} (1 + 2ry_L)^2 \). At the equilibrium, all shoppers are served and they all purchase at the discount price. For shoppers to derive non-negative utilities, \( s \) cannot be too large. For both firms to serve some shoppers, \( v \) cannot be too large. More specifically, I assume

**Assumption 3:** (i) \( \frac{s}{t} \leq \frac{v}{t} - 2ry_L - \frac{3}{2} \); (ii) \( \frac{s}{t} \leq \frac{(1 + 2ry_L)^2}{2ry_L} \).

**Proposition 3:** Let \( s_{ip} \equiv \frac{2r}{y_H} \left( 1 + 2r - r y_H - \sqrt{(1 + 2r)^2 - \frac{2r y_H}{t}} \right) \) and \( s_{dl} \equiv 2t \left( r y_H - 1 - 2r + \sqrt{(1 + r y_L)^2 - r^2 y_L + \frac{2r v}{t}} \right) \). Then, after \( B \)'s entry,

(i) both firms charge a uniform price if \( s \geq s_{ip} \);

(ii) both firms discount on the low-demand day if \( s \leq s_{dl} \);

(iii) there is no equilibrium in which both firms discount on the high-demand day.

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5 Comparison

It is possible that $s_{dl} > s_{ip} > s_m$. In that case, for $s \in [s_m, s_{ip}]$, firm $A$ will use independent pricing before $B$’s entry but will start to discount on the low-demand day after $B$’s entry, which means the high-demand day’s price will increase as a result of the entry. For example, when $v = 2$, $t = 1$, $r = \frac{1}{5}$, $y_H = \frac{4}{5}$, we have $s_{dl} = 0.26$, $s_{ip} = 0.23$, and $s_m = 0.14$. If $s = \frac{1}{5}$, $A$ will charge $p_H^A = p_L^A = 1.10$ before $B$’s entry. After the entry, $p_H^A$ increases to 2 while $p_L^A$ drops to 1.08.

As stated earlier, compared with independent pricing, discount gives a firm one advantage and two disadvantages. The advantage is that the firm can charge a higher price to loyal customers on the non-discount day. The disadvantages are: first, the switching shoppers incur a switching cost, a deadweight loss; and second, the discount price gives a free ride to loyals and shoppers on the discount day. With the entry of firm $B$, $A$ gets only half of the shopper market on both days. Fewer switching shoppers means a smaller deadweight loss, while fewer non-switching shoppers means fewer customers are getting the free ride. Both effects make discount less costly than in the monopoly situation. For some parameter values, a firm that originally chose independent pricing when it had the entire market may find discounting more attractive after the entry of a competitor.

For given $r$ and $y_H$, Figure 1 shows the area in the $\frac{v}{t}$ – $\frac{s}{t}$ space in which the entry raises market prices. The horizontal side of the triangle represents the upper bound on $v$ in Assumption 2 ($\frac{v}{t} \leq 2 + r$). The vertical line and hypotenuse correspond to $IP$ in the monopoly situation ($s > s_m$) and $DL$ under duopoly ($s < s_{dl}$), respectively. That is, for given $r$ and $y_H$, both $\frac{v}{t}$ and $\frac{s}{t}$ should take some intermediate value, and $\frac{v}{t}$ should be close to $r$. When $\frac{s}{t}$ is large, $\frac{v}{t}$ needs to be large, too. For different levels of $r$ and $y_H$, the triangle becomes larger when $r$ is smaller or $y_H$ is larger. Therefore, entry causes prices to rise when consumers’ valuations of the product and shoppers’ switching disutilities are both moderate. A price rise is more likely when there are relatively more shoppers and the imbalance between the two days’ demands is larger.

6 Concluding Remarks

This analysis explains how the entry of a competitor may cause market prices to increase. Competition reduces the number of price-sensitive consumers that the incumbent firm serves, prompting it to price discriminate. This logic
Figure 1: The area in which entry of a competitor raises prices

may be relevant to the “generic competition paradox” (Scherer, 1993) in the pharmaceutical industry. Empirical evidence has shown that after drug patents expire and competition from generic drugs begins, branded drugs are often priced higher than before (Grabowski and Vernon, 1992; Frank and Salkever, 1997). Scherer (1993, p.101) explains that, “once generic substitutes enter at much lower prices, the market is bifurcated, and the incumbent branded seller commonly finds it more profitable to desert the price-sensitive market than to reduce the prices quoted to price-insensitive customers.”

The present model does not apply to the pharmaceutical industry directly. First, the product differentiation between the two days is horizontal, while that between the branded drug and generics is vertical. Second, both firms sell on both days in my model, while patent holders sell only branded drugs and generic competitors sell only substitutes. Despite the differences, the general logic of the model, that competition may intensify price discrimination and

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therefore lead to higher prices, seems to be relevant. The branded seller may earn more profit with a strategy of price discrimination, i.e., a high price for its branded drug and a low price for a generic version of the drug (the so-called authorized generics or pseudo generics). The practice is rare in the US due to regulations, but is very common in Canada, Australia, New Zealand, Germany, the UK and Sweden (Hollis, 2003).

Entry may lead to higher equilibrium prices for other reasons. Compare the monopoly $IP$ price with that under duopoly. The duopoly price $t(1 + 2r)$ may exceed the monopoly price $\frac{v + rt^2}{2}$. Price discrimination must lead to higher price. A uniform price may also rise due to $B$’s entry. The intuition is that each firm’s (single day) price balances two concerns: a higher price extracts more rents from loyals but leads to smaller sales to shoppers. With more competition, shoppers are less numerous, so each firm will focus more on its loyal customers by raising its price. This logic is central to the findings of Rosenthal (1980) and Png and Hirshleifer (1987). This study has emphasized a different effect: After the entry of a competitor, an incumbent firm faces a smaller shopper market, which makes discriminatory pricing less costly than before. Entry may thus prompt the incumbent to switch from uniform pricing to price discrimination. As a result, customers who are discriminated against pay a higher price.

Both effects are captured by the model, and may be present simultaneously. That is, $B$’s entry may push $A$ to switch from uniform pricing to discriminatory pricing, so the non-discount day’s price increases; meanwhile, shoppers are less numerous, so the discount day’s price may also increase. For example, if $y_H$ is $\frac{3}{5}$ instead of $\frac{4}{5}$, the monopoly prices are $p^A_H = p^A_L = 1.10$ while the duopoly prices are $p^A_H = 2$ and $p^A_L = 1.16$. Therefore, $B$’s entry causes both days’ prices to increase, albeit by different amounts.

Another effect of $B$’s entry is that, contrary to the prediction of most theories of competition, market prices become more dispersed. In the above numerical examples, there is only one price before $B$’s entry but there are two prices after the entry. Note that both firms use the same strategy, so the price dispersion is across days, not across firms. In terms of price dispersion across firms, an empirical study by Png and Reitman (1994) found that, in gasoline retailing, more competition leads to larger price differentials. The reason why competition causes larger price dispersion, however, is different in this model from that discussed by Png and Reitman. In this model, competition intensifies price discrimination; Png and Reitman proposed that competition intensifies product differentiation (some stations set higher prices and therefore offer shorter queues than do other stations).
Appendix

Proof of Proposition 1

\[ \pi_{DH} \geq \pi_{DL} \] if and only if \( s \geq rt \), while \( \pi_{DH} > \pi_{IP} \) if and only if \( \beta(s) = yLs^2 - 2(v+y_Hrt)s + rt[2v - rt(1+y_H)] \geq 0 \). \( \beta(s) \) is a convex, quadratic function in \( s \) with \( \beta(rt) = -4y_Hr^2t^2 < 0 \) and \( \beta(v) = -(v - rt)^2 - y_H(v + rt)^2 < 0 \). So for \( rt \leq s \leq v \), \( \pi_{DH} < \pi_{IP} \); for \( s < rt \), \( \pi_{DH} < \pi_{DL} \). Therefore, \( DH \) is never optimal.

\[ \pi_{DL} \geq \pi_{IP} \] if and only if \( yHs^2 - 2(v + y_Hrt)s + rt[2v - rt(1+y_L)] \geq 0 \), which leads to \( s \leq s_m \).

Proof of Proposition 2

Suppose there is an asymmetric equilibrium in which firm \( A \) uses \( DL \) and firm \( B \) uses \( IP \). Then \( p_H^A = v \). For given \( p_L^1, p_H^B, \) and \( p_L^B \), the indifferent shopper has \( v = p_L^A - s - tz = v = p_H^B - t(1 - z_H) \) on day \( H \), and \( v = p_L^A - tz = v = p_L^B - t(1 - z_L) \) on day \( L \). As a result, \( A \)'s market shares of shoppers on the two days are \( z_H = p_H^B - p_L^A + t - s \) and \( z_L = p_L^B - p_L^A + t \). Firm \( A \) chooses \( p_L^A \) to maximize \( \pi_A = x_Hp_H^A + p_L^A(x_L + y_Hz_H + y_Lz_L) \), while firm \( B \) chooses \( p_H^B \) and \( p_L^B \) to maximize \( \pi_B = p_H^B[x_H + y_H(1 - z_H)] + p_L^B[x_L + y_L(1 - z_L)] \). The equilibrium prices are then:

\[ p_L^A = t(1 + 2r) - \frac{yH}{6}(4rt + s), \quad p_L^B = t(1 + 2r) - \frac{yH}{6}(4rt + s) \quad \text{and} \quad p_H^B = p_L^B + \frac{s}{2}. \]

The equilibrium profits are \( \pi_A = rv_H + \frac{(v_H)^2}{24} \) and \( \pi_B = \frac{1}{24}y_H(p_H^B)^2 + y_L(p_L^B)^2 \).

Given \( p_L^A \) and \( p_H^A \), if \( B \) deviates to \( DL \), its discount price on day \( L \) will be \( t(1 + 2r) - \frac{yH}{6}(10rt + s) \). The deviation is unprofitable when \( v \leq \frac{s^2}{8rt} + \frac{1}{2r} - \frac{yH}{6}(4rt + s) \). Similarly, given \( p_L^B \) and \( p_H^B \), if \( A \) deviates to \( IP \), its prices will be \( t(1 + 2r) - \frac{yH}{6}(4rt + s) \) on day \( L \), and \( t(1 + 2r) - \frac{yH}{6}(4rt + s) + \frac{s}{4} \) on day \( H \). The deviation is unprofitable when \( v \geq \frac{s^2}{32rt} + \frac{(2r+1)(4rt+s)}{2r} - \frac{yH}{6} \). Therefore, a necessary condition for the equilibrium to exist is \( v_6 \leq v \leq v_5 \). Recall Assumption 2, which requires \( rt \leq v \leq (2 + r)t \). The equilibrium exists only when \( \max\{v_6, rt\} \leq v \leq \min\{v_5, (2 + r)t\} \) for \( \frac{1}{2} \leq y_H \leq 1 \). However, we can show that \( \min\{v_5, (2 + r)t\} < \max\{v_6, rt\} \), which means the equilibrium never exists.

The above proof is about the non-existence of an \( IP-DL \) equilibrium. Similar arguments can be used to prove that neither an \( IP-DH \) nor a \( DL-DH \) equilibrium exists, either.

Proof of Proposition 3

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(i) Given \( p^A_L = p^A_H = t(1+2r) \), \( B \) has two possible deviations: \( DL \) and \( DH \). Under \( DL \), \( p^B_H = v \). Let \( B \)'s sales to shoppers on day \( L \) are \( z_L^B = \frac{v^B_L - p^L + t}{2t} \), and that on day \( H \) is \( z_H^B = \frac{v^B_H - p^H + t - s}{2t} \). With this deviation, \( B \)'s profit is \( p^B_Hx_H + p(x_L + y_Lz_L^B + y_Hz_H^B) \), which leads to the optimal choice of \( p = t(1+2r) - \frac{v^B_H}{2}(s+2rt) \). The deviation is unprofitable if \( v \leq \frac{s+2rt}{8t} [4t(1+2r) - y_H(s+2rt)] \equiv v_1 \). By symmetry, \( B \)'s second deviation, \( DH \), is unprofitable if \( v \leq \frac{s+2rt}{8t} [4t(1+2r) - y_L(s+2rt)] \equiv v'_1 \). Because \( v'_1 \geq v_1 \), we only need to consider the requirement that \( v \leq v_1 \), which leads to \( s \leq s_{ip} \).

(ii) Given \( p^A_L = t(1+2ry_L) \) and \( p^A_H = v \), \( B \) has two possible deviations: \( IP \) and \( DH \). If \( B \) chooses \( IP \), its optimal prices can be solved to be \( t(1+r + ry_L) \) on day \( L \) and \( t(1+r + ry_H) + \frac{s}{2} \) on day \( H \). The deviation is unprofitable if \( v \geq \frac{(s+4rt)^2 + 4t(s+2rt)}{8rt} - \frac{(s+3rt)y_H}{2} \equiv v_3 \). If \( B \) chooses \( DH \), its optimal discount price should be \( t(1+r) + s(y_H - \frac{1}{2}) \). The deviation is unprofitable if \( v \geq \frac{s+2rt}{8t} [4t+6rt - s + 2y_H(s - 2rt)] \equiv v_4 \). Because \( v_3 - v_4 = \frac{yu}{4rt} [r^2 t^2 + (s + rt)^2] > 0 \), we only need to consider the requirement that \( v \geq v_3 \), which leads to \( s \leq s_{dl} \).

(iii) By symmetry with (ii), \( DH \) by both firms constitutes an equilibrium only when \( v \geq \frac{(s+4rt)^2 + 4t(s+2rt)}{8rt} - \frac{(s+3rt)y_L}{2} \equiv v'_3 \) and \( v \leq \frac{s+2rt}{8t} [4t+6rt - s + 2y_H(s - 2rt)] \equiv v'_4 \). Notice that the second inequality is \( v \leq v'_4 \), not \( v \geq v'_4 \) as in (ii). Because \( v'_3 - v'_4 = \frac{yu}{4rt} [r^2 t^2 + (s + rt)^2] > 0 \), the two requirements, \( v \geq v'_3 \) and \( v \leq v'_4 \), cannot both be satisfied.

References


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