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Endogenous horizontal mergers under cost uncertainty

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Abstract

This paper presents a study of endogenous horizontal mergers under cost uncertainty. Before knowing the exact values of their costs, firms decide sequentially whether or not to join a merger. After the merger decision is made, uncertainty is resolved and firms engage in Cournot competition with incomplete information about one another's costs. Due to production rationalization, the merged firms enjoy an advantage over non-merged firms in the sense that the merged firms' expected cost is lower. I show that mergers occur if and only if the uncertainty is large and that the larger the uncertainty, the greater the number of firms that will merge. Although a merger reduces competition and therefore hurts consumers, it improves productivity under cost uncertainty. I find that a merger increases social welfare whenever there are at least four firms in the industry before the merger.

JEL classification: L13; D43

Keywords: Endogenous horizontal mergers; Cost uncertainties; Production rationalization

1. Introduction

In the economics literature, horizontal mergers are typically studied in a deterministic environment. In real life, however, merger decisions are often made with uncertain consequences. One source of such uncertainty is the production costs of participant firms. At the time of the merger, firms may not have perfect knowledge about their future production costs, which could depend on past investments in a stochastic way. For example, oil companies may consider mergers when they are still exploring for oil in their oil fields, the productivity of which is unknown at the time of the merger. Pharmaceutical companies may need to make merger decisions without knowing whether their R&D efforts are successful. In general, any investment could result in indeterminate costs in the future, and sometimes a merger occurs before the uncertainty is resolved.[1]

In this paper, I study endogenous mergers under cost uncertainty. Ex ante homogeneous firms play a two-stage game. In the first stage, all firms sequentially vote Yes/No on a merger proposal that would combine those who vote Yes into one merged firm. Those who vote No will remain

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[1] Alternatively, the uncertainty may come from exogenous demand or supply shocks. For example, firms may be scattered in different locations. If the location of an emerging market is random, firms' transportation costs will be uncertain. Or, if firms face capacity constraints, they may not be certain that their capacities will be fully utilized in the face of demand shocks.

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independent. In the second stage, firms simultaneously compete in quantities. The merged entity, if it exists, acts as a single firm managing the members' production facilities and maximizing their joint profits, which will be shared equally among all members. I assume that costs are uncertain at the time of the vote and that firms learn their costs privately before production starts.

Because of the uncertainty, costs are almost always heterogenous ex post even though the firms are ex ante homogeneous. The merged entity, with multiple facilities under its control, will allocate its production among the facilities in an optimal way. Such production rationalization gives the merged firms an advantage over non-merged firms in the sense of reduced expected costs. In equilibrium, the advantage is balanced by the benefit of remaining independent to free ride on the reduced competition brought about by the merger. Because the advantage increases with the degree of uncertainty, I find that mergers occur if and only if uncertainty is sufficiently large and that more firms will join the merger as the uncertainty increases.

On the normative side, although the merger reduces competition, it also improves productivity. In fact, the two effects are well-balanced in endogenous mergers because the reduced competition (measured by the number of firms that join the merger) and the improved productivity are both connected to the magnitude of the cost uncertainty. If more firms join the merger (so the competition is reduced more), this means that the uncertainty is larger and therefore the productivity is also improved more. I find that total social welfare is improved by a merger whenever there are at least four firms in the industry before the merger.

This paper is related to two strands of the merger literature. The first strand, typically treating the merger scale (i.e., the number of participants in the merger) as exogenous, investigates the forces that drive mergers. Firms may merge for the benefit of reduced competition and/or costs. Mergers for the sole purpose of reducing competition, however, are rarely profitable. Salant et al. (1983) show that, in the absence of cost savings, a merger is profitable only when it involves more than 80% of the firms in an industry.[2] Mergers can also be driven by cost savings through elimination of duplicated fixed costs (Gaudet and Salant, 1992b; Pepall et al., 2002), information exchange (Banal-Estanol, 2007; Qiu and Zhou, 2006) or transfer of superior technology (Farrell and Shapiro, 1990). In this paper, I suggest that cost savings may also arise from production rationalization when costs are uncertain.

The second strand of literature studies how merger scale is endogenously determined. Kamien and Zang (1990) build a simultaneous-bidding model in which each firm tries to sell itself and buy every other firm at the same time.[3] In the spirit of endogenous mergers, D'Aspremont et al. (1983) study the profitability and stability of cartel formation in a price leadership game. Gowrisankaran (1999) lets firms purchase less-efficient competitors sequentially in a pre-determined order. Horn and Persson (2001) use cooperative game theory to model endogenous mergers. Brito (2003) and Fridolfsson and Stennek (2005) suggest that firms may engage in preemptive mergers. A key message from these models is that there is a tendency for firms to remain independent. The tendency may be so strong that, when costs are deterministic, mergers will never happen among identical firms (Kamien and Zang, 1990). By contrast, I show in this paper that mergers can easily take place if costs are uncertain.

The present paper is most closely related to the work of Rodrigues (2001). The same game (sequential voting on mergers followed by quantity competition) is used to model the endogenous merger process. The key difference is what drives the mergers: production rationalization in my model and savings on fixed costs in Rodrigues's model.[4] Although "to reduce overhead" (i.e., to save fixed costs) is a reason frequently cited by managers who pursue mergers, it could not be the only driving force, as its implications seem to be at odds with general perceptions of what is happening behind mergers. For example, Rodrigues shows that the whole merged entity (consisting of several firms) will produce exactly the same amount of output as a single, non-merged firm, and that a merged firm always earns less profit than does a non-merged firm.[5] In contrast, in my

[2] This is because, under Cournot competition, non-merged firms respond to the merger by producing more, which hurts the merged firms. Therefore, Salant et al.'s (1983) extreme result can be relaxed if the expansion of non-merged firms is restricted through, say, decreasing returns to scale (Perry and Porter, 1985), product differentiation (Qiu and Zhou, 2006), convex demand (Hennessy, 2000) or a disadvantageous position for non-merged firms (Daughety, 1990; Levin, 1990). For mergers under Bertrand competition, see Deneckere and Davidson (1985).

[3] Kamien and Zang's (1990) basic model has been applied to other situations: increasing marginal costs (Kamien and Zang, 1991), sequential bidding (Kamien and Zang, 1993), price competition with perfect complements (Gaudet and Salant, 1992a), savings of fixed costs (Gaudet and Salant, 1992b), and delegation (Gonzalez-Maestre and Lopez-Cunat, 2001).

[4] "[E]xcluding fixed costs, the only mergers that will occur endogenously will be mergers to monopoly." (Rodrigues, 2001, pp. 1255–56).

[5] This is because the fixed cost is an additive term that is independent of the firms' competition choices, which means that the profitability of a given merger is directly determined by the magnitude of the fixed cost without any reference to market competition. In my model, by contrast, the merged entity's cost advantage is an endogenous term that directly affects firms' competition choices.

model, the expected output of the merged entity is much greater than that of a non-merged firm, the more so the more firms join the merger. And the expected profit of a merged firm can be higher than that of a non-merged firm.

2. The model

To begin, $n \geq 3$ risk-neutral firms produce a homogeneous good and compete in quantities. Demand is given as $p = \max\{0, a - X\}$, where X is the total output of the n firms. Firms have constant marginal costs and zero fixed cost. A firm's marginal cost is a random variable independently drawn from an identical, uniform distribution over $[c_0 - \tau, c_0 + \tau]$, with $c_0 \geq \tau \geq 0$ and $a \geq c_0 + \tau$. Therefore, τ parameterizes the degree of uncertainty, with $\tau = 0$ representing deterministic (and identical) costs.

Firms play a two-stage game. In the first stage, before the costs are realized, firms sequentially vote Yes or No to a proposal of joining a merger. If a firm is indifferent between Yes and No, I assume that it votes Yes. After the voting is finished, the costs are privately revealed. Each firm learns the exact value of its own cost but not the costs of other firms. If a merger takes place (when at least two firms vote Yes), the participants share their technologies (and therefore the cost information) and seek to maximize their joint profits, which will be divided equally among the participants regardless of their cost realizations. In the second stage, Cournot competition is carried out under incomplete information about costs.

3. The cost advantage of the merged firms

Because marginal costs are constant, when a group of firms merges, they will use only the most-efficient technology that they have. Therefore, if $k \in [2, n]$ firms merge, the merged entity's marginal cost will be the lowest value of k independent draws from the uniform distribution. Call the merged entity firm z and its marginal cost c_z. It can be shown (Balakrishnan and Cohen, 1991) that c_z follows the probability density function $g(c_z) = \frac{k(c_0 + \tau - c_z)^{k-1}}{(2\tau)^k}$. As a result, the expected cost of the merged entity is

$$\overline{c}_z = c_0 - \frac{k-1}{k+1}\tau,$$

which is smaller than the expected cost of an independent firm, $\overline{c} = c_0$. We say that the merged firms enjoy an advantage over non-merged firms in the sense that the merged firms' expected cost is lower. Notice that the advantage,

$$\delta \equiv \overline{c} - \overline{c}_z = \frac{k-1}{k+1}\tau \geq 0,$$

exists only when costs are uncertain ($\tau > 0$). Furthermore, the advantage increases in both k and τ: it is greater when more firms merge or when the costs are more uncertain.

4. The equilibrium when the uncertainty is not very large

In the second-stage Cournot competition, the cost information remains private. One complication in calculating the equilibrium is that, if the uncertainty is very large, a firm's cost may turn out to be so high that its optimal choice is to shut down (i.e., produce zero). The calculation would then be very complicated. In this section, I exclude this possibility by assuming that the uncertainty is not very large, i.e.,

$$\tau \leq \tau^* \equiv \frac{2(a - c_0)}{n+1}.$$

All results are then proved analytically. In Section 5, I verify numerically that these results continue to hold when the uncertainty is very large ($\tau > \tau^*$).

As is customary in the literature, a firm that votes Yes is called an *insider* and a firm that votes No is called an *outsider*. Suppose that $k \in [2, n]$ firms have voted Yes in the first stage. By assumption, each insider's expected profit is identical and is denoted as $\pi^I(k)$. Since all outsiders draw their costs from the same distribution, each outsider's ex ante expected profit is also identical and is denoted as $\pi^O(k)$. Under the assumption that $\tau \leq \tau^*$, analytical expressions of the two profits are obtained as (see Appendix A):

$$\pi^I(k) = \frac{[(a - c_0) + \delta(n - k + 1)]^2}{k(n - k + 2)^2} + \frac{\tau^2}{(k+1)^2(k+2)}, \tag{1}$$

$$\pi^O(k) = \frac{[(a - c_0) - \delta]^2}{(n - k + 2)^2} + \frac{\tau^2}{12}. \tag{2}$$

Obviously, insiders benefit from the cost advantage, δ, while outsiders are hurt. Since the two expressions are both homogenous of degree two in τ and $a-c_0$, I define $t\equiv\frac{\tau}{a-c_0}$ and discuss the outcomes in terms of t. The upper bound on τ is then translated into $t\leq t^*\equiv\frac{2}{n+1}$.

4.1. The first-stage merger decision

If $k^*$is a unique integer that satisfies $\pi^I(k^*)\geq\pi^O(k^*-1)$ and $\pi^O(k^*)>\pi^I(k^*+1)$, then we can show that the merger game has a unique equilibrium in which exactly k^* firms vote Yes. Furthermore, firms' equilibrium merger decisions must be the following (see Rodrigues, 2001). If $\pi^I(k^*)\geq\pi^O(k^*)$, the first k^* firms vote Yes while the remaining $n-k^*$ firms vote No. If $\pi^I(k^*)<\pi^O(k^*)$, the first $n-k^*$ firms vote No while the last k^* firms vote Yes.

Given these merger decisions, we now focus on the determination of k^*. The two conditions that k^* must satisfy can be written as:

$$\Delta^I(k^*)=\pi^I(k^*)-\pi^O(k^*-1)\geq 0 \text{ if } k^*\in[2,n], \text{and} \tag{3}$$

$$\Delta^O(k^*)=\pi^O(k^*)-\pi^I(k^*+1)>0 \text{ if } k^*\in[2,n-1]. \tag{4}$$

The first condition says that, in an equilibrium, an insider should not want to break away from the merger (internal stability, as identified by D'Aspremont et al., 1983). The second condition says that an outsider should not want to join the merger (external stability). It turns out that they are the flip sides of the same coin—an outsider's incentive to join a coalition of k firms is exactly the opposite of an insider's incentive to leave a coalition of $k+1$ firms: $\Delta^O(k)=-\Delta^I(k+1)$. As a result, we need only care about the requirement of Δ^I. An equilibrium is reached when $\Delta^I(k^*)\geq 0$ while $\Delta^I(k^*+1)<0$.

Consider an outsider's incentive to join the merger when $k-1$ firms have already done so. This will reduce the number of competitors in the Cournot game by one, but, more importantly, the firm is joining a coalition that enjoys a cost advantage. The downside is that it has to share the advantage with all other insiders. For given τ, the advantage, $\delta\equiv\frac{k-1}{k+1}\tau$, is an increasing, concave function in k. When k is larger, more firms will share the advantage, which is increased by the larger k only marginally. As a result, joining the merger will become increasingly less attractive when k grows larger. That is, $\Delta^I(k)$ is strictly decreasing in k. If $\Delta^I(2)\geq 0$, then k^* exists and is uniquely determined as the largest integer, k, such that $\Delta^I(k)\geq 0$.

We now argue that $\Delta^I(2)\geq 0$ is true if and only if the cost uncertainty is sufficiently large. When a merger takes place, it generates two effects: reduced competition that benefits all firms including outsiders, and a cost advantage that accrues only to insiders. The reduced competition can be viewed as a public good. Insiders provide the public good while outsiders free ride on the reduced competition. As a result, firms prefer to stay outside the merger (and hope that other firms join the merger).[6] The cost advantage, on the other hand, encourages firms to join the merger. Because the advantage increases with the degree of cost uncertainty, when the uncertainty is small, the free-riding effect dominates and no firm will join the merger. In other words, mergers occur only when the uncertainty is large. We have[7] (see Appendix B):

Proposition 1. *When $t\leq t^*$, mergers occur if and only if*

$$t\geq\frac{3(n^2-1)-3n\sqrt{n^2-2n+3}}{2n^2+n-1}\equiv t_2>0.$$

As a corollary to Proposition 1, when costs are deterministic ($t=0$), all firms will remain independent.[8] This may seem surprising because, if all firms merge into a single firm, each will receive a higher payoff than before the merger. In endogenous mergers, however, a firm compares its profit from joining the merger with that of remaining independent, not with its profit before the merger (when all firms were independent). Sometimes, staying outside the merger may be more profitable for a firm than joining it. Consider the example of a three-firm industry. Absent cost uncertainties, a firm's profit is $\frac{1}{(n+1)^2}$ when n firms compete, so a monopolist's profit is $\frac{1}{4}$, a duopolist's profit is $\frac{1}{9}$, and a triopolist's profit is $\frac{1}{16}$. If all three firms merge into a monopoly, each receives $\frac{1}{12}$, which is higher than the pre-merger profit of $\frac{1}{16}$. After the first two firms have agreed to join the merger, however, the third firm will choose not to join, as $\frac{1}{9}>\frac{1}{12}$. This means that monopolization will not occur. In fact, given the anticipation that the third firm will not join, the second firm will also decide not to join, because $\frac{1}{18}<\frac{1}{16}$.

The critical value of uncertainty above which mergers take place, t_2, is inversely U-shaped in n. Therefore, the likelihood of mergers depends on n non-monotonically.[9]

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[6] To use the now well-known quotation of Stigler (1950, p. 25): "the major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant… the promoter of a merger is likely to receive much encouragement from each firm—almost every encouragement, in fact, except participation."

[7] It can be shown that $t_2<t^*\equiv\frac{2}{n+1}$ so $[t, t^*]$ is always nonempty.

[8] This is a repeated theme in endogenous merger models. Kamien and Zang (1990) and Thoron (1998) obtain the same result. In contrast, the conclusion from exogenous mergers is very different: merging into a monopoly is always an equilibrium (Salant et al., 1983).

[9] By contrast, Rodrigues (2001) shows that the likelihood depends on n in a monotonic way: whenever mergers occur at n, they also occur at any $n'<n$.

For example, at $t=0.14$, mergers occur for $n \leq 3$ or $n \geq 7$ but not for $n=4$, 5 or 6. When n is small, mergers are mainly driven by the pursuit of market power (a relatively large reduction in the number of firms); when n is large, mergers are driven by the pursuit of cost advantages, which, not relying on n, become more significant for a larger n.

4.2. The scale of the merger

We have seen that the driving force for joining the merger is the cost advantage, which increases with t. As a result, the equilibrium merger scale, k^*, should also increase with t. Furthermore, if t is bounded from above, k^* is also bounded from above.

Proposition 2. *For $t \in [t_2, t^*]$,*

(i) k^* is weakly increasing in t (weakly because k^* must be an integer).
(ii) When $n=4$, 5, 6, at most four firms merge. When $n=3$ or $n \geq 7$, at most three firms merge.

The proof is provided in Appendix C. Since $t \equiv \frac{\tau}{a-c_0}$, a higher t may come from a higher τ, a higher c_0 or a lower a. Therefore, more firms will merge if costs are more variable, demand is lower, or production becomes more costly on average.

4.3. Welfare

For any given cost realization, the social welfare is

$$W = \int_0^X (a-X)\mathrm{d}X - zc_z - \sum\nolimits_{i=1}^{n-k} x_i c_i,$$

where $X=z+\sum_{i=1}^{n-k} x_i$ is the total output and $p=a-X$ is the market price. Appendix D provides the proof of the following results:

Proposition 3. *For $t \in [t_2, t^*]$, at the equilibrium k^*, ex ante in the expected sense,*

(i) both insiders and outsiders benefit from the merger;
(ii) the market price rises and consumer surplus drops;
(iii) social welfare is always improved by the merger when $n \geq 4$.

For an outsider, the merger is beneficial because it reduces competition, but it is also harmful because it creates a strong competitor. It turns out that the first effect dominates: an outsider's profit always increases with k, so an outsider's profit at $k^* \geq 2$ is higher than its profit at $k=1$, which corresponds to the pre-merger situation. Given this, an insider will also benefit from the merger, because it could have chosen to be an outsider. Because insiders' profits are higher than their pre-merger levels, the merger is collectively profitable for its participants. In other words, the equilibrium in an endogenous merger game is also an equilibrium in the corresponding exogenous merger game.

An insider's expected profit can be lower or higher than an outsider's profit. For example, at $n=10$, if $t=0.13$, two firms will merge, and $\pi^I(2) < \pi^O(2)$. If $t=0.15$, still, two firms will merge, but $\pi^I(2) > \pi^O(2)$.

The welfare improvement comes from the efficiency gain in the production technology. Consumers are actually hurt by the reduced competition, but producers gain from the merger. For fixed n, when t increases, the social welfare will improve for fixed k; but if t increases too much, more firms will merge, which will hurt the social welfare. It turns out that for $n \geq 4$, the total social welfare is always improved by the merger. For $n=3$, when two firms join the merger (which occurs for a certain range of t), the social welfare is hurt with small t and is improved with large t. A similar pattern exists when all three firms merge (which occurs for another range of t).[10]

5. The equilibrium when the uncertainty is very large

The analysis in the previous section is based on the assumption that the cost uncertainty is relatively small so that no firm shuts down. With larger uncertainties, some firms may shutdown when they draw very high costs. This possibility complicates the calculation of the second-stage Cournot equilibrium, making analytical solutions extremely tedious. I therefore utilize numerical simulations to verify whether the major conclusions in the previous section continue to hold.

For a given t, the probability of shutdown decreases with k. When more firms join the merger, fewer are left as independent competitors. The less-intense competition makes it more likely for a weak firm (i.e., a firm that draws a high cost) to survive. As mentioned above, shutdown occurs only when t is large. When t increases, however, the equilibrium merger scale will also increase, which reduces the likelihood of shutdown. It may happen that, for t that is not too large, no firm shuts down at the equilibrium even though $t > t^*$. In that case, the expressions of (1) and (2) are still applicable.

[10] $k^*=2$ if $t \in (0.098, 0.191)$. For that range of t, the welfare is hurt for $t<0.188$ and is improved for $t>0.188$. If $t>0.191$, then $k^*=3$. The welfare is hurt for $t<0.306$ and is improved for $t>0.306$.

Fig. 1 shows how the equilibrium merger scale depends on the only two parameters, n and t. Notice that $t \equiv \frac{\tau}{a-c_0} \leq 1$ because $a \geq c_0+\tau$. The area below the t^* line represents the case discussed in Section 4. Numerical calculations (see Appendix E) have verified that, for $t \in (t^*, 1]$, the conclusions in all three propositions continue to hold except for Proposition 2 (ii). That is:

Proposition 4. *When firms are uncertain about their production costs at the time of the merger decision,*

(i) *mergers occur if and only if the uncertainty is large, i.e.,* $\frac{\tau}{a-c_0} \geq t_2$*;*
(ii) *the equilibrium merger scale increases with the degree of the uncertainty;*
(iii) *social welfare is improved by the merger whenever the industry had at least four firms before the merger.*

6. Concluding remarks

In this paper, I show that firms may merge for the benefit of a more efficient production technology if, at the time of the merger, they are uncertain about their production costs. The larger the uncertainty, the greater the number of firms that merge. Due to the improvement in productivity, mergers are not as damaging as people tend to think. In fact, I show that social welfare is improved by mergers in most cases.

In this model, the merger decision is made before firms learn their costs. Admittedly, things could very well happen in the reverse order: firms learn their private information about their costs before attempting to merge. In that case, an entirely different problem may arise: firms in general have an incentive to misrepresent their costs. Then, the focus of study may shift to how asymmetric information affects firms' merger decisions (Cramton and Palfrey, 1990) and how mergers can be used as a strategic tool for signalling or screening. This is obviously an interesting research direction. In the present paper, however, I have chosen to focus on the impact of uncertainties and ignore information issues.

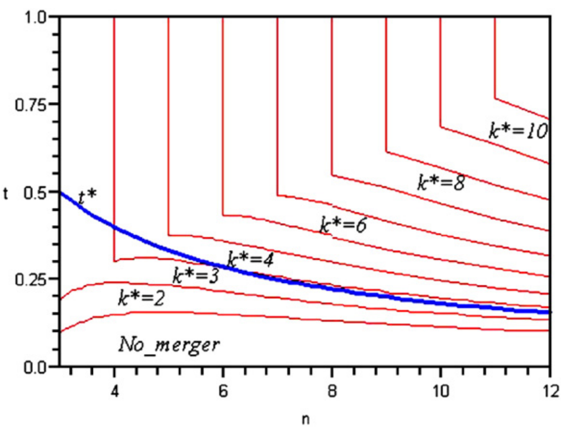

Fig. 1. The equilibrium merger scale as a function of n and t.

Given that firms possess private information about their own costs, when some firms merge, they will know each other's costs. This looks very much like information sharing (Li, 1985; Gal-Or, 1986; Vives, 1990; Raith, 1996). Mergers, however, differ from information sharing in two aspects. First, in information sharing, firms usually have a dominant strategy (to share cost information under Cournot competition or demand information under Bertrand competition, and not to share information otherwise). As a result, the equilibrium is either the status quo or the grand coalition. By contrast, in endogenous mergers, a firm's best response depends on other firms' choices; the equilibrium is most often a partial coalition. In fact, the coalition size itself becomes the focus of most merger studies. Second, when firms share information, they remain independent in production. Joining the coalition means adjusting production to market conditions more accurately. When firms merge, the merged entity can directly move its production to the most-efficient firm. Joining a merger therefore means producing at a lower cost. The efficiency gain is consequently much larger in a merger than under information sharing.

In this model, the benefit of joining a merger comes from technology sharing: all participants get to use the most-efficient technologies available among themselves. By signing a binding contract, firms may in principle achieve the same efficiency without merging. This has the advantage of avoiding the free-riding problem. There are, however, some drawbacks to this solution. For one thing, some superior technologies, such as managerial talent, cannot be readily copied. For another, firms may not truthfully reveal information on their technologies if the information is not easily verifiable. The incentive-compatibility constraint may be so severe that the coalition breaks down thoroughly (Cramton and Palfrey, 1990).

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Appendix A. The insiders' and outsiders' ex ante expected profits

Let the probability density function of the uniform distribution be denoted as $f(c)=\frac{1}{2\tau}$. Suppose that k firms join the merger. It can be shown (Balakrishnan and Cohen, 1991) that c_z's probability density function is $g(c_z)=\frac{k(c_0+\tau-c_z)^{k-1}}{(2\tau)^k}$. The expected value and variance of an independent firm are $\overline{c}=c_0$ and $\sigma^2=\frac{\tau^2}{3}$, whereas those of the merged entity are $\overline{c}_z=c_0-\frac{k-1}{k+1}\tau$ and $\sigma_z^2=\frac{4k\tau^2}{(k+1)^2(k+2)}$.

In the second-stage Cournot competition, firm z and an independent firm j's first-order conditions are

$$a-(n-k)\,\overline{x}-2x_z-c_z=0, \tag{5}$$

$$a-(n-k-1)\,\overline{x}-2x_j-\overline{x}_z-c_j=0, \tag{6}$$

in which c_z and c_j are the two firms' costs, x_z and x_j are their output quantities, $\bar{x}_z$ and $\bar{x}$ are the expected values of the output quantities, the latter being the same for all independent firms. By taking expectations over quantities and costs, the two equations yield $\bar{x}=\frac{a+\bar{c}_z-2\bar{c}}{n-k+2}$ and $\bar{x}_z=\bar{x}+(\bar{c}-\bar{c}_z)$. Plugging them into (5) and (6), we get the equilibrium quantities:[11]

$$x(c_i)=\overline{x}-\frac{1}{2}(c_i-\overline{c}),\ \text{and}$$

$$x_z(c_z)=\overline{x}_z-\frac{1}{2}(c_z-\overline{c}_z). \tag{7}$$

The merged entity's profit is $\pi_z=x_z(p-c_z)=[\overline{x}_z-\frac{1}{2}(c_z-\overline{c}_z)]\,\{a-\overline{x}_z-(n-k)\,\overline{x}-\overline{c}_z-\frac{1}{2}\,(c_z-\overline{c}_z)+\frac{1}{2}\sum_{i=1}^{n-k}(c_i-\overline{c}\,)\}$. Its expected value is $E(\pi_z)=\overline{x}_z\,[a-\overline{x}_z-(n-k)\,\overline{x}-\overline{c}_z]+\frac{1}{4}E(c_z-\overline{c}_z)^2=\overline{x}_z^2+\frac{\sigma_z^2}{4}$. Similarly, the expected profit of an independent firm is $E(\pi)=\overline{x}^2+\frac{\sigma^2}{4}$. An outsider's expected profit is simply $E(\pi)$, while an insider's profit is $E(\pi_z)/k$. □

Appendix B. Proposition 1 (Existence of the merger equilibrium)

We want to show that there is a unique integer, $k^*\in[2,\ n]$, that satisfies both (3) and (4). The proof consists of four steps.

Step one: Prove that $\Delta^I(k)$ is strictly decreasing in k. The difference, $\Delta^I(k)-\Delta^I(k+1)$, has the same sign as $Q(t)=\varphi(n,\ k)t^2+\lambda(n,\ k)t+\sigma(n,\ k)$, in which φ, λ, and σ are complex expressions of n and k.[12] We will prove that $Q(t)>0$ for any n, $k\in[2,\ n-1]$, and $t\in[0,\ t^*]$.

Prove that $\sigma(n,\ k)>0$: it turns out that $\frac{\partial^2\sigma(n,k)}{\partial n^2}=12(n-k)^2+12k^2(n-k-1)+30k(k-2)+48n+44>0$. Then, $\frac{\partial\sigma(n,k)}{\partial n}$ is increasing in n. Because $n\geq k+1$, $\frac{\partial\sigma(n,k)}{\partial n}>\left.\frac{\partial\sigma(n,k)}{\partial n}\right|_{n=k+1}=36k^2-36k+96>0$. This, in turn, means that $\sigma(n,\ k)$ is increasing in n. Then, $\sigma(n,\ k)>\sigma(k+1,\ k)=28k^2-52k+64>0$.

Prove that $Q(t)>0$: If $\varphi(n,\ k)<0$, then $Q(t)$ is concave in t. We can show that $Q(t^*)>0$ and $Q(0)=\sigma(n,\ k)>0$. Then, $Q(t)>0$ for $t\in[0,\ t^*]$. If, on the other hand, $\varphi(n,\ k)>0$, then we find that $Q\left(-\frac{\lambda(n,k)}{2\varphi(n,k)}\right)>0$. Because $Q(t)$ achieves its minimum at $t=-\frac{\lambda(n,k)}{2\varphi(n,k)}$, we conclude that $Q(t)>0$.

Step two: Observe that $\Delta^I(k+1)+\Delta^O(k)=0$. As a result, $\Delta^I(k)+\Delta^O(k)>\Delta^I(k+1)+\Delta^O(k)=0$.

Step three: The necessary condition for a merger: Suppose a merger occurs at certain $k\geq 2$. Then, by the internal stability, $\Delta^I(k)\geq 0$. Because $\Delta^I(k)$ is decreasing in k, $\Delta^I(2)\geq 0$, which leads to $t\geq t_2$. Notice that $t_2<t^*$, so the threshold for mergers is well-defined.

Step four: The sufficient condition and uniqueness of k^*. Let $t\geq t_2$. Then $\Delta^I(2)\geq 0$. There are two cases:

Case (A): $\Delta^I(n+1)\geq 0$. Then $\Delta^I(n)>\Delta^I(n+1)\geq 0$. Therefore, $k=n$ is an equilibrium. Because $\Delta^I(n+1)+\Delta^O(n)=0$, we have $\Delta^O(n)\leq 0$. As $\Delta^O(k)$ is increasing in k, we know that $\Delta^O(k)<\Delta^O(n)\leq 0$ for any $k\in[2,n-1]$. That is, $k^*=n$ is the only equilibrium.

Case (B): $\Delta^I(n+1)<0$. Because $\Delta^I(k)$ is continuous and strictly decreasing in k and because $\Delta^I(2)\geq 0$, there is a unique solution of $\beta\in[2,n+1)$ such that $\Delta^I(\beta)=0$.

[11] Under the constraint $\tau\leq\frac{2(a-c_0)}{n+1}$, each firms produces non-negative quantities if no merger occurs (i.e., $k=1$). If a merger takes place (i.e., $k\geq 2$), the constraint is sufficient (although not necessary) for non-negative quantities.

[12] Let $\gamma=(k+3)[k(k+1)(k+2)(n-k+1)(n-k+2)(n-k+3)]^2$. Then, $\varphi(n,k)=\gamma\left[\frac{(k-1)^2(n-k+1)^2}{k(k+1)^2(n-k+2)^2}-\frac{(k-2)^2}{k^2(n-k+3)^2}+\frac{1}{(k+1)^2(k+2)}-\frac{k^2(n-k)^2}{(k+1)(k+2)^2(n-k+1)^2}+\frac{(k-1)^2}{(k+1)^2(n-k+2)^2}-\frac{1}{(k+2)^2(k+3)}\right]$, $\lambda(n,k)=2\gamma\left[\frac{(k-1)(n-k+1)}{k(k+1)(n-k+2)^2}+\frac{(k-2)}{k(n-k+3)^2}-\frac{k(n-k)}{(k+1)(k+2)(n-k+1)^2}-\frac{(k-1)}{(k+1)(n-k+2)^2}\right]$, and $\sigma(n,k)=\gamma\left[\frac{1}{k(n-k+2)^2}-\frac{1}{(n-k+3)^2}-\frac{1}{(k+1)(n-k+1)^2}+\frac{1}{(n-k+2)^2}\right]$.

That is, for fixed n and t, $\Delta^I(k) \geq 0$ if and only if $k \leq \beta$. Because $\Delta^I(k+1)+\Delta^O(k)=0$, we also have $\Delta^O(k)>0$ if and only if $k+1>\beta$, or equivalently $k>\beta-1$. Then, both $\Delta^I(k)$ and $\Delta^O(k)$ are non-negative if and only if $k \in (\beta-1, \beta]$. Then, there is a unique integer k^* in the range $(\beta-1, \beta]$ (so k^* is the largest integer of k such that $\Delta^I(k) \geq 0$). As a result, k^* is the equilibrium merger scale. Notice that, because $2<\beta<n+1$, we have $2 \leq k^* \leq n$. □

Appendix C. Proposition 2 (The merger scale)

Proposition 2(i): It suffices to prove that $\Delta^I(k)$ is increasing in t. It turns out that $\frac{\partial \Delta^I(k)}{\partial t}$ has the same sign as a linear expression in t: $F(t)=2u(n,k)t+v(n,k)$. $v(n,k)$ has the same sign as $V(n)=(k-1)n^3-(2k^2-9k+9)n^2+(k^3-11k^2+29k-23)n+(3k^3-16k^2+28k-17)$. We can show that $V''(n)>0$ so $V'(n)$ is increasing in n. Then, $V'(n) \geq V'(k)=4k^2+11k-23>0$. As a result, $V(n) \geq V(k)=4k^2+5k-17>0$, which means $v(n,k)>0$.

Now, if $u(n,k) \geq 0$, the proof is complete. Suppose that $u(n,k)<0$ (this happens when $n \geq 14$ and $3 \leq k \leq 6$). It turns out that $F(t^*)=2u(n,k)t^*+v(n,k)$ is positive for $k \leq 14$. Because we only care about the range $3 \leq k \leq 6$, we conclude that $\frac{\partial \Delta^I(k)}{\partial t}>0$.

Proposition 2(ii): From the previous result, we know that the highest k^* is achieved at the highest t. Let $t=t^*$. Then, $\Delta^I(4) = \frac{-29n^4+168n^3+190n^2-768n+139}{300(n-1)^2(n+1)^2(n-2)^2}<0$ for $n \geq 7$. This means that when $n \geq 7$, at most three firms merge. We also find that $\Delta^I(5) = \frac{-360n^4+1850n^3+1737n^2-13092n+5093}{1575(n-3)^2(n-2)^2(n+1)^2}<0$ for $n \geq 5$. That is, for $n=4$, 5, 6, at most four firms merge. □

Appendix D. Proposition 3 (Welfare)

Proposition 3(i): $\frac{\partial \pi^O(k)}{\partial k} = \frac{2[(k+1)-(k-1)t][(-2n+2k-3-k^2)t+(k+1)^2]}{(k+1)^3(n-k+3)^3}$. Now, $(k+1)>(k-1)t$ and $(-2n+2k-3-k^2)t+(k+1)^2 > (-2n+2k-3-k^2)t^*+(k+1)^2 = \frac{k-1}{n+1}[5+3n+(n-1)k]>0$, so $\frac{\partial \pi^O(k)}{\partial k}>0$. That is, $\pi^O(k)$ increases in k, so $\pi^O(k^*)>\pi^O(1)$. Before the merger, each firm earns an expected profit of $\pi_0=\pi^O(1)=\pi^I(1)$. As a result, $\pi^O(k^*)>\pi_0$, and $\pi^I(k^*) \geq \pi^O(k^*-1) \geq \pi^O(1)=\pi_0$.

Proposition 3(ii): The expected market price is

$$E(p) = a - [\bar{x}_z + (n-k)\bar{x}] = \frac{a+(n-k+1)c_0}{n-k+2} - \frac{(k-1)\tau}{(k+1)(n-k+2)}.$$

The merger changes the expected price by: $E(p)-E(p)|_{k=1} = (a-c_0)\frac{(k-1)[(k+1)-(n+1)t]}{(k+1)(n+1)(n-k+2)} \geq (a-c_0)\frac{(k-1)^2}{(k+1)(n+1)(n-k+2)}>0$.

The consumer surplus is $CS=\int_0^X(a-X)\mathrm{d}X - pX = \frac{1}{2}X^2$. Let $\bar{X}=\bar{x}_z+(n-k)\bar{x}$. Then, $X=\bar{X}-\frac{1}{2}(c_z-\bar{c}_z)-\frac{1}{2}\sum_{i=1}^{n-k}(c_i-\bar{c})$ and so $E(CS)=\frac{1}{2}E(X^2)=\frac{1}{2}\bar{X}^2+\frac{\sigma_z^2}{8}+\frac{n-k}{8}\sigma^2$. We can show that $\bar{X}$, σ_z^2 and $(n-k)\sigma^2$ are all decreasing in k, so the expected consumer surplus decreases in k. Because the pre-merger case corresponds to $k=1$, whenever a merger takes place, consumers are hurt.

Proposition 3(iii): The expected social welfare is

$$\begin{aligned} EW &= E\left(\int_0^X(a-X)\mathrm{d}X - x_z c_z - \sum_{i=1}^{n-k} x_i c_i\right) \\ &= a\bar{X} - \frac{1}{2}E(X^2) - E(x_z c_z) - (n-k)E(x_i c_i). \end{aligned} \tag{8}$$

We can show that $E(x_z c_z) = \bar{x}_z \bar{c}_z - \frac{\sigma_z^2}{2}$ and $E(x_i ci) = \bar{x}\ \bar{c} - \frac{\sigma^2}{2}$. Then, $EW = \frac{m+2\delta+m\delta^2}{m+1} - \frac{(m+\delta)^2}{2(m+1)^2} + \frac{3}{8}[\sigma_z^2+(m-1)\sigma^2]$, in which $m=n-k+1$. The merger changes the social welfare by $\Delta W = EW - EW|_{k=1}$.

In order to prove that $\Delta W>0$, it suffices to show that $\Delta W>\Delta^I(k^*)$. Define $H(t) = \frac{\Delta W - \Delta^I(k^*)}{(a-c_0)^2}$. From Proposition 2, we know that, for any n, at most four firms merge. Now, suppose that two firms merge ($k=2$). Then, $H(t) = \frac{-(n+1)^2(n^2+2)t^2+12(n+1)^2t+(9n^2-36n-18)}{18n^2(n+1)^2}$. For $n \geq 4$, $H(t_2) = (9n^2)\frac{2(n-2)(n+1)^2\sqrt{n^2-2n+3}-2n^4+2n^3+7n^2-12n-6}{(2n-1)^2}>0$, and $H(t^*)=5n^2-12n-2>0$. Because $t_2 \leq t \leq t^*$ and $H(t)$ is concave in t, we conclude that $H(t)>0$ for $t_2 \leq t \leq t^*$. Using the same method, we can prove that $\Delta W>0$ for $k=3$ and $k=4$. □

Appendix E. The equilibrium when the uncertainty is very large (Section 5)

Firm z's first-order condition takes the form of:

$$x_z(c_z) = \begin{cases} \frac{1}{2}[a-(n-k)\bar{x}-c_z] & \text{if } c_z \leq h_z, \\ 0 & \text{if } c_z > h_z, \end{cases}$$

in which $hz=\min\{a-(n-k)\bar{x}, c_0+\tau\}$. For given hz, let $P_z \equiv Pr(c_z \leq h_z) = \int_{c_0-\tau}^{h_z} g(c)\mathrm{d}c$, $\tilde{c}_z \equiv \frac{1}{P_z}\int_{c_0-\tau}^{h_z} \mathrm{c}g(c)\mathrm{d}c$, and $\tilde{\sigma}_z^2 \equiv \frac{1}{P_z}\int_{c_0-\tau}^{h_z}(c-\tilde{c}_z)^2 g(c)\mathrm{d}c$. Then,

$$\bar{x}_z = \frac{P_z}{2}[a-(n-k)\bar{x}-\tilde{c}_z]. \tag{9}$$

A similar expression can be written for an independent firm:

$$\bar{x} = \frac{P}{2}[a - (n - k - 1)\bar{x} - \bar{x}_z - \tilde{c}], \tag{10}$$

in which $h = \min\{a-(n-k-1)\bar{x}-\bar{x}_z, c_0+\tau\}$, $P \equiv \Pr(c \leq h) = \int_{c_0-\tau}^{h} f(c)\mathrm{d}c$, $\tilde{c} \equiv \frac{1}{P}\int_{c_0-\tau}^{h} cf(c)\mathrm{d}c$, and $\tilde{\sigma}^2 \equiv \frac{1}{P}\int_{c_0-\tau}^{h}(c-\tilde{c})^2 f(c)\mathrm{d}c$.

There are three possible cases: (I) $c_0+\tau \leq a-(n-k-1)\bar{x}-\bar{x}_z \leq a-(n-k)\bar{x}$, which means that $h_z = h = c_0+\tau$ and therefore $P_z = P = 1$; (II) $a-(n-k-1)\bar{x}-\bar{x}_z < c_0+\tau \leq a-(n-k)\bar{x}$, which means that $h_z = c_0+\tau$, $h = a-(n-k-1)\bar{x}-\bar{x}_z$, $P_z = 1$ but $P<1$; (III) $a-(n-k-1)\bar{x}-\bar{x}_z \leq a-(n-k)\bar{x} < c_0+\tau$, which means that $h_z = a-(n-k)\bar{x}$, $\mathrm{h} = a-(n-k-1)\bar{x}-\bar{x}_z$, $P_z<1$ and $P<1$. In each case, express h_z and h properly and then solve the equilibria $\bar{x}_z$ and $\bar{x}$ from Eqs. (9) and (10). These solutions are then plugged back into the expressions of h_z and h to verify that this is indeed the case that has been assumed. After $\bar{x}_z$ and $\bar{x}$ have been determined, firm z's expected profit is $E(\pi_z) = \frac{\bar{x}_z^2}{P_z} + \frac{P_z}{4}\tilde{\sigma}_z^2$ and an independent firm's expected profit is $E(\pi) = \frac{\bar{x}^2}{P} + \frac{P}{4}\tilde{\sigma}^2$. The expected welfare is $EW = a\bar{X} - \frac{1}{2}E(X^2) - E(x_z c_z) - (n-k)E(x_i c_i)$, in which $\bar{X} = \bar{x}_z + (n-k)\bar{x}$, $E(X^2) = E(\pi_z) + (n-k)[E(\pi) + 2\bar{x}_z\bar{x} + (n-k-1)\bar{x}^2]$, $E(x_z c_z) = \bar{x}_z\tilde{c}_z - \frac{P_z}{2}\tilde{\sigma}_z^2$, and $E(x_i c_i) = \bar{x}\tilde{c} - \frac{P}{2}\tilde{\sigma}^2$.

For example, at $a=2$, $c_0=1$, $n=6$ and $\tau=0.45$, all firms draw their costs from the common support [0.55, 1.45] both before and after a merger. If two firms merge, an independent firm will produce $x(c_i) = \frac{1}{2}(1.28 - c_i)$ if $c_i \leq 1.28$ and zero otherwise, while the merged entity will produce $x_z(c_z) = \frac{1}{2}(1.41 - c_z)$ if $c_z \leq 1.41$ and zero otherwise. The probability of shutdown is 19.1% for an independent firm and is 0.2% for the merged entity. Although the merged entity and independent firms both draw their costs from the same support, the merged entity is more likely to draw a low cost and therefore is less likely to shut down.

For a given τ, the probability of shutdown decreases with k. In the above example, if $k=1$ or 2, both the merged entity and independent firms may shut down; if $k=3$ and 4, the merged entity never shuts down; and if $k=5$ and 6, no firm shuts down. It may happen that, for τ that is not too large, no firm shuts down at the equilibrium even though $\tau > \tau^*$. In the above example, the equilibrium merger scale turns out to be $k^*=6$. As a result, the expressions of (1) and (2) are still applicable.

References

Balakrishnan, N., Cohen, A.C., 1991. Order Statistics and Inference. Academic Press, San Diego, CA.

Banal-Estanol, A., 2007. Information-sharing implications of horizontal mergers. International Journal of Industrial Organization 25 (1), 31–49.

Brito, D., 2003. Preemptive mergers under spatial competition. International Journal of Industrial Organization 21 (10), 1601–1622.

Cramton, P.C., Palfrey, T.R., 1990. Cartel enforcement with uncertainty about costs. International Economic Review 31 (1), 17–47.

D'Aspremont, C., Jacquemin, A., Gabszewicz, J.J., Weymark, J., 1983. On the stability of collusive price leadership. Canadian Journal of Economics 16 (1), 17–25.

Daughety, A.F., 1990. Beneficial concentration. American Economic Review 80 (5), 1231–1237.

Deneckere, R., Davidson, C., 1985. Incentive to form coalitions with Bertrand competition. Rand Journal of Economics 16 (4), 473–486.

Farrell, J., Shapiro, C., 1990. Horizontal mergers: an equilibrium analysis. American Economic Review 80 (1), 107–126.

Fridolfsson, S., Stennek, J., 2005. Why mergers reduce profits and raise share prices—a theory of preemptive mergers. Journal of the European Economic Association 3 (5), 1083–1104.

Gal-Or, E., 1986. Information transmission—Cournot and Bertrand equilibria. Review of Economic Studies 53 (1), 85–92.

Gaudet, G., Salant, S.W., 1992a. Mergers of producers of perfect complements competing in price. Economics Letters 39, 359–364.

Gaudet, G., Salant, S.W., 1992b. Towards a theory of horizontal mergers. In: Norman, G., La Manna, M. (Eds.), The New Industrial Economics. Edward Elgar, Brookfield, VT., USA, 1992.

Gonzalez-Maestre, M., Lopez-Cunat, J., 2001. Delegation and mergers in oligopoly. International Journal of Industrial Organization 19 (8), 1263–1279.

Gowrisankaran, G., 1999. A dynamic model of endogenous horizontal mergers. Rand Journal of Economics 30 (1), 56–83.

Hennessy, D.A., 2000. Cournot oligopoly conditions under which any horizontal merger is profitable. Review of Industrial Organization 17 (3), 277–284.

Horn, H., Persson, L., 2001. Endogenous mergers in concentrated markets. International Journal of Industrial Organization 19 (8), 1213–1244.

Kamien, M.I., Zang, I., 1990. The limits of monopolization through acquisition. Quarterly Journal of Economics 105 (2), 465–499.

Kamien, M.I., Zang, I., 1991. Competitively cost advantageous mergers and monopolization. Games and Economic Behavior 3, 323–338.

Kamien, M.I., Zang, I., 1993. Monopolization by sequential acquisition. Journal of Law, Economics, and Organization 9 (2), 205–229.

Levin, D., 1990. Horizontal mergers: the 50-percent benchmark. American Economic Review 80 (5), 1238–1245.

Li, L., 1985. Cournot oligopoly with information sharing. Rand Journal of Economics 16 (4), 521–536.

Pepall, L., Richards, D.J., Norman, G., 2002. Industrial Organization: Contemporary Theory and Practice. South-Western, Cincinnati.

Perry, M.K., Porter, R.H., 1985. Oligopoly and the incentive for horizontal merger. American Economic Review 75 (1), 219–227.

Qiu, L.D., Zhou, W., 2006. International mergers: incentives and welfare. Journal of International Economics 68 (1), 38–58.

Raith, M., 1996. A general model of information sharing in oligopoly. Journal of Economic Theory 71, 260–288.

Rodrigues, V., 2001. Endogenous mergers and market structure. International Journal of Industrial Organization 19 (8), 1245–1261.

Salant, S.W., Switzer, S., Reynolds, R.J., 1983. Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot–Nash equilibrium. Quarterly Journal of Economics 98 (2), 185–199.

Stigler, G.J., 1950. Monopoly and oligopoly by merger. American Economic Review 40 (2), 23–34.

Thoron, S., 1998. Formation of a coalition-proof stable cartel. Canadian Journal of Economics 31 (1), 63–76.

Vives, Xavier, 1990. Trade association disclosure rules, incentives to share information, and welfare. Rand Journal of Economics 21 (3), 409–430.