Aggregation Bias in International Business Cycles*

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Abstract

We study an international business cycle model with a global input-output network that encompasses a large class of international business cycle models to quantify the "aggregation bias" in cross-country shock spillover resulting from using an aggregated one-sector model. We show analytically that international shock spillover in a multi-sector model and an aggregated one-sector counterpart are generally different, and only the same under special parameterizations such as when sectoral shocks are identical. Quantitatively, using data for 20 countries between 1990 and 2009, we find that aggregation bias varies over time and, on average, results in about 25% lower international shock spillover, suggesting that an aggregate model tends to underestimate the role of foreign shocks in the domestic economy.

JEL classification: E32, F31.

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*The views expressed in this paper are those of the authors and do not necessarily represent the view of the Federal Reserve Bank of San Francisco, or the Federal Reserve Bank system.

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1 Introduction

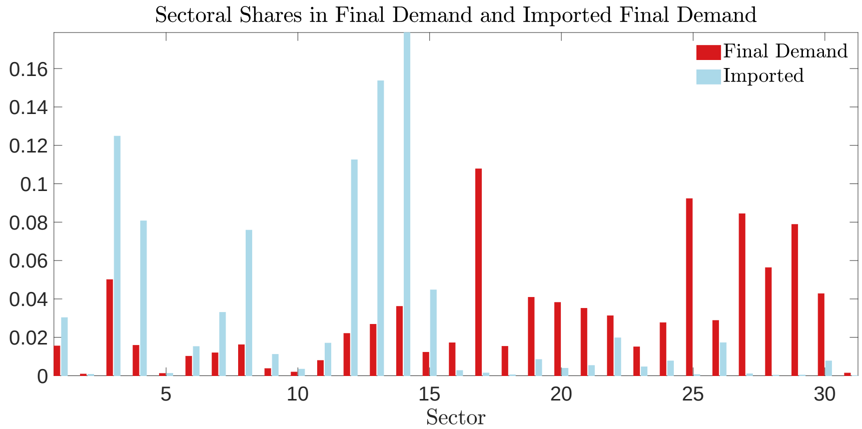
Understanding the international propagation of business cycles across countries is an important topic in international macroeconomics. A large literature starting with Backus, Kehoe, and Kydland (1994), Heathcote and Perri (2002) and subsequent work including a recent influential paper by Itskhoki and Mukhin (2021) often analyze international business cycles in a context of a one-sector model where each country produces one differentiated good. These models are known to have difficulty matching salient features of the data such as the correlations of output and employment across countries and the volatilities of net exports, and exchange rates. In recent years, with the rise of the global value chain, and more available data on input-output tables as well as time series of disaggregated sectoral output, an increasing number of research adopts a multi-sector and multi-country framework. For example, Huo, Levchenko, and Pandalai-Nayar (2023) decompose international comovements due to different shocks in a global economy; our work, Miyamoto and Nguyen (2024), use a multi-country two-sector model to study how trade linkages affect business cycles over time; Bonadio et al. (2021) study the role of global supply chains, and di Giovanni et al. (2023) analyze inflation drivers during COVID–19 on the global economy using a multi-sector model. Since world economies consist of many consumers, firms, sectors, and countries, it is arguable possible to study a further disaggregated model. The question is then whether the aggregate implications on international business cycles, such as shock spillover, are substantially different in a granular model of the world economy such as a multi-sector networked model where countries and sectors are interconnected from those in an aggregated one-sector model counterpart where each country produces one differentiated good and trade with one another. In other words, how important is it to model a granular world economy with heterogeneity for international business cycles?

This paper studies a large class of international business cycle models where countries are connected in a global input-output network and investigates how aggregating a multi-sector networked model into a one-sector model may lead to biased estimates of international shock spillover across countries. We focus on the transmission of shocks across countries in the multi-sector production networked model and compare with that in a one-sector model counterpart where each country produces one differentiated good using a production function and trades with other countries. In both models, countries trade both intermediate and final goods, and the production function for each firm requires endogenous labor input, intermediate goods and capital services with exogenous productivity processes. Households can borrow and save intertemporally. The model also features variable capital utilization and investment adjustment costs, which have been shown to help better match business cycle moments. The main differences between the two models are the existence of more disaggregated sectors in the multi-sector model. In particular, each country's production side consists of S sectors, each of which produces a differentiated good using a production function. These sectors can have different exposure to international trade through the input-output linkages and different productivity shock processes. We use this framework as a laboratory to characterize aggregation bias in a simple version of the model, where we show that aggregation bias arises in most cases due to the different importance of the sectors in the networked economy as well as the heterogeneous nature of sectoral productivity shocks. We then quantify the aggregation bias in the data for 20 countries over time between 1970 and 2009 and find that a one-sector model tends to underpredict shock transmission across countries. In the period between 1970 and 2009, shock spillover is 25% lower in a one-sector model than in the multi-sector model counterpart, on average, over this period.

There are reasons to believe that heterogeneity matters for the transmission of shocks across countries. Take sectors, for example. Sectors are different in many dimensions. Some sectors are much more exposed to international trade than others. As plotted in Figure 1, in a median country in a sample of 25 OECD countries averaged between 1995 and 2011, the first fifteen sectors that include manufacturing, machinery and equipment account for most of the imported final demand while other sectors such as services contribute little to the total imported final demand. In contrast, the first fifteen sectors are small fractions of total final demand—the last few sectors that include services make up most of the final demand. In addition, shocks driving sectoral fluctuations can also be markedly different. As plotted in Figure 2, according to the EUKLEM's calculation of US TFP, TFP in the Machinery sector is more volatile than that in the Construction sector, and both are more volatile than the aggregate TFP.

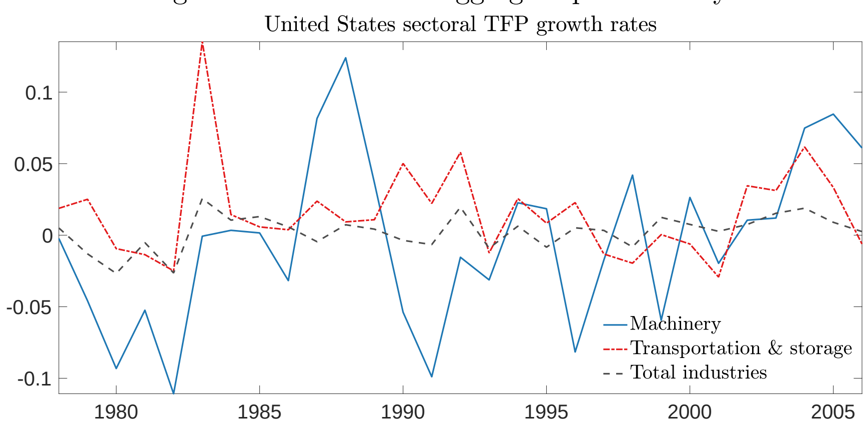
We first describe our full model, which encompasses a large class of international business cycle models in the literature. We then study when aggregation bias arises analytically in a simplified version of the model without investment and input-output linkages. We only consider two countries in the model, the home country (a small open economy) and the foreign country (the rest of the world). Since the home country is small, we approximate the foreign equilibrium as one in a closed economy, and assume that the home country takes foreign prices and expenditure as given. We show that the home country's real GDP response to foreign productivity shocks depends on three

Figure 1: Sectoral Shares in Final Demand



Notes: We compute sectoral shares in final demand and sectoral shares in imported final demand from the world IO table between 1995 and 2011 for 23 countries, then take the median across countries for each sector. The red bars are sectoral shares in final demand, and the grey bars are sectoral shares in final imported.

Figure 2: Sectoral and aggregate productivity



Notes: Productivity growth rates for the United States are computed from the EUKLEMS database.

key variables: the foreign price index, foreign expenditure, and the home country's import prices. A multi-sector model correctly accounts for all three. However, a one-sector model that matches the foreign price index and expenditure will generate different predictions because there is a single price in the foreign country and the model mistakenly uses the foreign price index as a proxy for the home's import prices.

Consider a simple two-sector example with tradable and nontradable goods. A productivity improvement in the foreign tradable sector lowers the home country's import prices, but a productivity improvement in the foreign nontradable sector does not. Therefore, the two shocks affect the home country differently. The multi-sector model captures this sectoral distinction. A one-sector model, calibrated to match the same real GDP change in the foreign country, while the one-sector model does not. We also show when aggregation bias disappears: when foreign sectors have identical export-to-GDP ratios or when all sectors experience the same productivity shock, the one-sector model produces the same predictions as the multi-sector model.

In Appendix B, we extend the analysis to incorporate domestic and international input-output linkages. In the more general setting, we find additional sources of the aggregation bias. For example, foreign productivity shocks that are heavily loaded onto domestic sectors with larger employment shares have stronger spillovers, but these differences are missed by a one-sector model. However, in the model-based decomposition in Appendix C, we find these additional biases are quantitatively small compared to the bias due to heterogeneous trade intensities across sectors.

Equipped with the intuition from the simplified model, we quantify the aggregation bias using our full model with data for 20 advanced countries, each vis-à-vis the ROW. For each pair of countries, the steady states input-output linkages in this multi-sector model are calibrated to match with the average of the World Input-Output Table (WIOD) between 1995 and 2011, which include 31 sectors, so as to match the linkages within and across countries. The calibration for the one-sector model counterpart is an aggregate of the multi-sector WIOD. In addition to heterogeneous input-output sectoral characteristics, we also assume that sectors are also driven by different productivity shock volatilities. Focusing on the role of sectoral heterogeneity in the input-output linkages, we find that the responses of value added in the home country are larger in a multi-sector model than that in a one-sector model when the productivity shocks happen in some sectors such as those in manufacturing. In other words, if shocks are skewed toward sectors that trade more extensively, assuming a one-sector model will underpredict the effects of foreign shock spillover to the domestic economy. However, consistent with the simple model analyses, if shocks happen in all sectors at the same time and change all sectors' productivity by the same amount, cross-country transmission in a one-sector model is similar to that in a multi-sector model. Furthermore, as sectoral shock composition changes over time, the aggregation bias can vary over time without any changes in input-output linkages. Indeed, after backing out productivity shocks by observing sectoral value added for these countries in the same period, our analysis suggests that the bias caused by using a one-sector model is more pronounced during the 2008-2009 Great Recession. On average, a one-sector model generates 25% lower shock spillover than a multi-sector model. Overall, our results highlight how taking a granular approach with sectoral heterogeneity seriously can theoretically and quantitatively can change the answers to business cycle comovement across countries. Under plausible parameterizations, one-sector models may underpredict the importance of foreign shocks in driving domestic business cycles.

Related Literature Our paper contributes to three strands of literature. First, our analyses help to better understand different frameworks to study international business cycles. As noted above, international business cycle literature include many different frameworks: aggregated one-sector model such as Backus, Kehoe, and Kydland (1994), Heathcote and Perri (2002) and Itskhoki and Mukhin (2021), two-sector model of tradeable and nontradable goods or durable goods such as Corsetti, Dedola, and Leduc (2008), Engel and Wang (2011), Johnson (2013), and Miyamoto and Nguyen (2024), as well as multi-sector model such as Huo, Levchenko, and Pandalai-Nayar (2023). Our general model encompasses prominent models in this literature featuring fundamentally different sectors helps to shed light on the different aggregate implications of these models, as well as the quantitative importance of modeling more granular aspects of the world economy.

Second, our paper is related to understanding the macroeconomic impact of microeconomic shocks. For example Hulten (1978), Gabaix (2011), Carvalho and Tahbaz-Salehi (2019), Baqaee and Farhi (2019), and Baqaee and Rubbo (2022) concentrate on closed economy implications, our paper provides new insights on the transmission of shocks across countries in a networked economy.

Third, we also relate to the growing literature studying the role of sectoral heterogeneity in business cycles. For example, Pasten, Schoenle, and Weber (2023) highlight the role of sectoral heterogeneity in nominal rigidity in amplifying sectoral shocks in a closed economy setting. Pasten, Schoenle, and Weber (2020), Cox et al. (2023), Bouakez, Rachedi, and Santoro (2023a), and Bouakez, Rachedi, and Santoro (2023b) study the propagation of monetary policy shocks and government spending shocks in a multi-sector closed economy. Unlike these papers, we analyze productivity shocks in an open-economy setting and highlight how a one-sector model may have missed the aggregate implications in the multi-sector model. In this sense, we are related to Devereux, Gente, and Yu (2022) who study fiscal shock spillover in a production network model.

Our paper proceeds as follows. In Section 2, we describe our full model in detail. We build intuition in Section 3. Section 4 presents the quantitative exercises for the aggregation bias. We concluded in Section 5.

2 The Model

This section describes our model and solution concepts. Our model encompasses several models used in the literature such as Backus, Kehoe, and Kydland (1994), Heathcote and Perri (2002) and Huo, Levchenko, and Pandalai-Nayar (2023) among others. In particular, the model is a general dynamic framework featuring multi-country multi-sector setup with a production network, where countries trade with one another in both intermediate and final goods in each sector. This feature is designed to capture the input-output linkages within and across countries in the data. The world consists of I countries and each country has S sectors. A one-sector model counterpart is when $S = 1$. We denote the set of countries and industries with $\mathcal{I}$ and $\mathcal{S}$, and use i, j to denote individual countries and k, s to denote individual sectors. Each country i has $n(i)$ symmetric representative households. To better match key features of the international business cycle facts, we augment the model with variable capacity utilization and investment adjustment costs. These features have been widely used in the macroeconomic literature as they are minimal departures from standard models, and are supported in the data.

2.1 Households

A representative household in country i maximizes its lifetime expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta_t(i)\, U\left(C_t(i), H_t(i)\right), \tag{1}$$

where $C_t(i)$ denotes consumption in country i at time t, and $H_t(i)$ is hours worked in country i. We do not specify a functional form for the utility function U, following Bilbiie (2009) and Furlanetto and Seneca (2014). We directly impose the values for parameters for the Frisch elasticity of labor supply, the intertemporal elasticity of substitution, and the wealth effects on labor supply in the linearized model. We assume incomplete financial markets, where households can trade non state-contingent bonds, so we use Uzawa-type preferences to induce stationarity. Then, $\beta_t(i)$ evolves as follows:

$$\beta_{t+1}(i) = \beta \left(\frac{C_t(i)}{C_{ss}(i)}\right)^{-\phi_\beta} \beta_t(i), \tag{2}$$

where $\phi_B \geq 0$ is a parameter, $C_{ss}(i)$ is consumption at the steady state, and β is the steady-state discount factor.

The period-by-period budget constraint is given by:

$$C_t(i) + \sum_s p_t^Z(i,s) Z_t(i,s) + \frac{1}{R_t(I)} B_{t+1}(i) ex_t(i) \tag{3}$$
$$= \sum_s W_t(i,s) H_t(i,s) + \sum_s R_t^K(i,s) u_t(i,s) K_t(i,s) + \Pi_t(i) + B_t(i) ex_t(i)$$

where $p_t^Z(i,s)$ is the price of investment goods for sector s, $Z_t(i,s)$ is the investment in sector s, $R_t(I)$ is the real interest rate in country I (the last country), $B_{t+1}(i)$ is the non-contingent bond holding of country i in the unit of consumption in country I, $ex_t(i)$ is the real exchange rate between country i and I, $W_t(i,s)$ is the real wage in sector s, $R_t^K(i,s)$ is the rental rate of capital in sector s, $u_t(i,s)$ is the variable capital utilization rate, $K_t(i,s)$ is capital stock in sector s, and $\Pi_t(i)$ is the sum of profits from firms in country i.

The household allocates hours across sectors, and total hours worked is an aggregator of hours supplied to each sector s with a constant elasticity of substitution as follows:

$$H_t(i) = \left(\sum_s \omega_H(i,s)^{-\frac{1}{\gamma_H}} H_t(i,s)^{\frac{1+\gamma_H}{\gamma_H}} \right)^{\frac{\gamma_H}{1+\gamma_H}}, \tag{4}$$

where γ_H is the elasticity of substitution, $H_t(i,s)$ is hours worked in sector s, and $\omega_H(i,s)$ is the parameter related to the shares of hours worked in each sector at the steady state.

The evolution of capital is standard as follows:

$$K_{t+1}(i,s) = (1 - \delta(u_t(i,s))) K_t(i,s) + Z_t(i,s) \left(1 - S_I \left(\frac{Z_t(i,s)}{Z_{t-1}(i,s)} \right) \right), \tag{5}$$

where $\delta(u_t(i,s))$ is the depreciation rate that depends on the rate of capital utilization rate, and $S_I\left(\frac{Z_t(i,s)}{Z_{t-1}(i,s)}\right)$ is the investment adjustment cost. We assume the following functional form for the capital utilization rate:

$$\delta(u_t(i,s)) = \delta_0(i,s) + \delta_a(i,s)(u_t(i,s) - 1) + \frac{\delta_b(i,s)}{2}(u_t(i,s) - 1)^2, \tag{6}$$

where $\delta_0(i,s)$ is the steady-state depreciation rate in sector s. The model without utilization corresponds to $\frac{\delta''(1)}{\delta'(1)}$ approaching infinity, as it is extremely costly to change the utilization. If $\frac{\delta''(1)}{\delta'(1)}$ goes to zero, the marginal cost of changing utilization is constant. The investment adjustment cost is standard and given by

$$S_I\left(\frac{Z_t(i,s)}{Z_{t-1}(i,s)}\right)=\frac{\phi^I(i,s)}{2}\left(\frac{Z_t(i,s)}{Z_{t-1}(i,s)}-1\right)^2, \tag{7}$$

where $\phi^I \geq 0$ is the parameter that governs the investment adjustment cost.

2.2 Firms

There are S sectors in each country. Each firm in each sector produces a differentiated good using capital, labor and a composite intermediate good, which is an aggregate of intermediate goods produced by all countries and sectors. Final consumption and investment are an aggregate of the sectoral final composite consumption and investment goods. These sectoral final composite goods are an aggregate of goods of the same sector with origins from both domestic and foreign countries. Specifically, firms producing the final composite consumption goods in country i use sectoral goods $f_C(i,s)$ with the elasticity of substitution γ_{FC} across sectors as follows:

$$C_t(i)=\left[\sum_{s=1}^{S}\left(\omega_{FC}(i,s)\right)^{\frac{1}{\gamma_{FC}}}\left(f_{C,t}(i,s)\right)^{\frac{\gamma_{FC}-1}{\gamma_{FC}}}\right]^{\frac{\gamma_{FC}}{\gamma_{FC}-1}}, \tag{8}$$

where $\omega_{FC}(i,s)$ is the share of sectoral s goods in final consumption. The final sectoral firms produce sectoral goods $f_{C,t}(i,s)$ by combining goods of the same sector with origins from both domestic and foreign countries, $f_{C,t}(i,(j,s))$, given by:

$$f_{C,t}(i,s)=\left[\sum_{j=1}^{I}\left(\omega_{fC}(i,(j,s))\right)^{\frac{1}{\gamma_{fC}}}\left(f_{C,t}(i,(j,s))\right)^{\frac{\gamma_{fC}-1}{\gamma_{fC}}}\right]^{\frac{\gamma_{fC}}{\gamma_{fC}-1}}, \tag{9}$$

where γ_{fC} is the elasticity of substitution of goods across countries, and $\omega_{fC}(i,(j,s))$ is the share of $f_{C,t}(i,(j,s))$ in final sectoral goods for consumption. The production of the sectoral final investment goods is analogous to that of the sectoral final consumption goods, as follows. First, sectoral investment good, $z_t((i,s),k)$, is an aggregate of goods for investment bought by country i sector s from country j sector k, $z_t((i,s),(j,k))$, over seller country j:

$$z_t((i,s),k)=\left[\sum_{j=1}^{I}\left(\omega_z((i,s),(j,k))\right)^{\frac{1}{\gamma_z}}\left(z_t((i,s),(j,k))\right)^{\frac{\gamma_z-1}{\gamma_z}}\right]^{\frac{\gamma_z}{\gamma_z-1}}, \tag{10}$$

where $\omega_z((i,s),(j,k))$ is the share of the investment goods from country j sector k. Then, the sectoral final composite investment goods is an aggregate of sectoral investment good, $z_t((i,s),k)$, over all sectors k with a sector-specific investment good share, $\omega_Z((i,s),k)$, as follows:

$$Z_t(i,s) = \left[\sum_{k=1}^{S} (\omega_Z((i,s),k))^{\frac{1}{\gamma_Z}} (z_t((i,s),k))^{\frac{\gamma_Z-1}{\gamma_Z}}\right]^{\frac{\gamma_Z}{\gamma_Z-1}}. \quad (11)$$

Each firm in country i in sector s produces goods using the following production function:

$$Q_t(i,s) = \left[\omega_Q(i,s)^{\frac{1}{\gamma_Q}} \left[A_t(i,s)\widetilde{K}_t(i,s)^{\alpha_K} H(i,s)^{\alpha_H}\right]^{\frac{\gamma_Q-1}{\gamma_Q}} + (1-\omega_Q(i,s))^{\frac{1}{\gamma_Q}} M_t(i,s)^{\frac{\gamma_Q-1}{\gamma_Q}}\right]^{\frac{\gamma_Q}{\gamma_Q-1}}, \quad (12)$$

where Q_t is gross output in country i sector s, $A_t(i,s)$ is productivity, $\widetilde{K}_t(i,s)$ is the service from capital defined as $\widetilde{K}_t(i,s) \equiv u_t(i,s)K_t(i,s)$, $M_t(i,s)$ is intermediate goods used in sector s, γ_Q is the elasticity of substitution between intermediate goods and the value added, α_K is the share parameter for capital service, α_H is the share parameter for hours worked, and $\omega_Q(i,s)$ are the parameters related to the shares of intermediate goods. Similar to consumption and investment, intermediate input is produced in two steps. First, intermediate goods of both domestic and foreign origins are combined to produce sector k's intermediate goods to be used in sector s country i:

$$m_t((i,s),k) = \left[\sum_{j=1}^{I} (\omega_m((i,s),(j,k)))^{\frac{1}{\gamma_m}} (m_t((i,s),(j,k)))^{\frac{\gamma_m-1}{\gamma_m}}\right]^{\frac{\gamma_m}{\gamma_m-1}}, \quad (13)$$

where $m_t((i,s),(j,k))$ is intermediate good bought by country i sector s from country j sector k with the share $\omega_m((i,s),(j,k))$, and $m_t((i,s),k)$ is intermediate goods after aggregating $m_t((i,s),(j,k))$ over sectors of sellers. Second, these sector-specific intermediate goods are aggregated across sectors to be used in each sector s in country i as follows:

$$M_t(i,s) = \left[\sum_{k=1}^{S} (\omega_M((i,s),k))^{\frac{1}{\gamma_M}} (m_t((i,s),k))^{\frac{\gamma_M-1}{\gamma_M}}\right]^{\frac{\gamma_M}{\gamma_M-1}}, \quad (14)$$

where $\omega_M((i,s),k)$ is the share of the sector-specific intermediate goods.

The profit maximization problem for the firm is defined as

$$\Pi_t(i) \equiv \sum_s \left[p_t^Q(i,s)Q_t(i,s) - W_t(i,s)H_t(i,s) - R_t^K(i,s)\widetilde{K}_t(i,s) - p_t^M(i,s)M_t(i,s)\right]$$

where $p_t^Q(i,s)$ is the relative price of $Q_t(i,s)$, and $p_t^M(i,s)$ is the relative price of intermediate goods $M_t(i,s)$.

2.3 Resource Constraints

The market clearing condition for each sector s in each country i is given by:

$$n(i)\,Q(i,s) = \sum_j n(j)\,f_{C,t}(j,(i,s)) + \sum_j \sum_k n(j)\,z_t((j,k),(i,s)) + \sum_j \sum_k n(j)\,m_t((j,k),(i,s)). \tag{15}$$

The model also requires the asset market clearing condition and $\sum_{s=1}^{S} K_t(i,s) = u_t(i)K_t(i)$.

2.4 Shock Processes and Variable Definition

We close the model with the description of the productivity processes. The productivity in each sector in each country follows an AR(1) process with persistence ρ_A given by

$$lnA_t(i,s) = \rho_A lnA_{t-1}(i,s) + e_t^A(i,s), \tag{16}$$

where $e_t^A(i,s)$ has a standard deviation $\sigma(i,s)$ and mean 0.

We define the real value added for country i, $V_t(i)$ to be equal to the sum of all sector s gross output less the intermediate input usage evaluated at steady-state prices as follows.

$$V_t(i) \equiv \sum_s Q_t(i,s) - \sum_s M_t(i,s). \tag{17}$$

The sectoral value added in each sector s of country i, $V_t(i,s)$, is defined to be sector s gross output less the intermediate input usage evaluated at steady-state prices,

$$V_t(i,s) \equiv Q_t(i,s) - M_t(i,s). \tag{18}$$

3 Aggregation Bias in a Simplified Model

To build intuition on how aggregation bias may arise from a one-sector model, we first analyze a simple case of the general framework to derive analytical expressions for the response of value added output in the home country to shocks in the foreign country. We define the aggregation bias as the difference between the response of value added output in the home country in a one-sector model and that in the multi-sector model. Using the analytical solution, we discuss the conditions under which there exists an aggregation bias, i.e. the strength of international shock spillover in a one-sector model is different from that in a multi-sector model.

The simplified version of the full model is as follows. We assume Country 1 is a small open economy (SOE) that trades with the rest of the world (Country 2), so we can approximate the equilibrium variables in Country 2 as those in a closed economy. There are no capital services or intermediate inputs, so labor is the only input in the production function. Households spend all their income in each period and have no savings. This immediately implies that trade is balanced every period. To simplify the exposition, we assume $\gamma_{FC} = \gamma_{fC} \equiv \gamma_f$. In addition, we assume that hours worked in each sector are perfect substitutes, i.e. $\gamma_H = \infty$, and $H_t(i) = \sum_s H_t(i,s)$. Therefore, wages are equalized across sectors. Furthermore, we assume the period-by-period utility function takes the Greenwood, Hercowitz, and Huffman (1988) (GHH) form:

$$U(C_t(i), H_t(i)) = \frac{\left(C_t(i) - \psi H_t(i)^{1+\frac{1}{\nu}}\right)^{1-\sigma}}{1-\sigma},$$

where ν is the labor supply elasticity. When households optimally choose consumption and hours of work, employment has a constant elasticity with respect to real wage: $H_t(i) = \left(\frac{W_t(i)/P_t(i)}{\psi\left(1+\frac{1}{\nu}\right)}\right)^{\nu}$, where $P_t(i)$ is the consumers' price index in country i at period t.

We study the cross-country transmission of productivity shocks by characterizing the equilibrium value added in Country 1 in response to a temporary sectoral productivity shock in Country 2. The productivity shocks are defined as deviations from the steady-state productivities scaled by the steady-state value-added shares. We denote the steady-state value-added shares with $\omega_{Q,ss}(2,s)$, and the productivity shocks as $\widehat{A}_t(2,s) \equiv \omega_{Q,ss}(2,s)\left(\log A_t(2,s) - \log A_{ss}(2,s)\right)$. The productivity shocks are scaled because shocks on $A_t(2,s)$ only affects the value-added component of the production function (see equation (12)).[1]

We first show that, in response to sectoral productivity shocks, one can construct an observationally equivalent one-sector closed economy of a multi-sector closed economy in terms of real value added, real wages and hours worked, up to first-order approximations.

[1]We use $\omega_{Q,ss}(2,s)$ to denote the steady-state value-added share in sector s, Country 2, which in principle can be different from the model primitive $\omega_Q(2,s)$ in the steady state (see equation (12)). However, we can always define units in a way such that the two coincide. To see why the productivity shocks are scaled by $\omega_{Q,ss}(2,s)$, one can log-linearize equation 12, keeping all inputs constant. The change in output in response to the change $\log A_t(2,s) - \log A_{ss}(2,s)$ is $\omega_{Q,ss}(2,s)\left(\log A_t(2,s) - \log A_{ss}(2,s)\right)$.

Lemma 1 *In a closed economy, we can find an observationally equivalent one-sector economy in terms of changes in real value added, real wages, and hours worked up to a first-order approximation. In particular, this requires the hypothetical one-sector economy to have an aggregate productivity shock,* $\widehat{A}_t(2)^{one-sec}$*, that equals the average sectoral productivity shocks in the multi-sector model, weighted by the sectoral output shares:*

$$\widehat{A}_t(2)^{one-sec} = \sum_s \left(\frac{p^Q_{ss}(2,s)Q_{ss}(2,s)}{\sum_k p^Q_{ss}(2,k)Q_{ss}(2,k)} \widehat{A}_t(2,s) \right), \tag{19}$$

where $p^Q_{ss}(2,s)$ *and* $Q_{ss}(2,s)$ *are the steady-state price and gross output in sector* s*, Country* 2*.*

Proof. See Appendix B. ■

Equation (19) is closely related to the results in Hulten (1978). To see the connection, we can rewrite it as

$$\frac{\sum_s p^Q_{ss}(2,s)Q_{ss}(2,s)}{V_{ss}(2)} \times \widehat{A}_t(2)^{one-sec} = \sum_s \frac{p^Q_{ss}(2,s)Q_{ss}(2,s)}{V_{ss}(2)} \widehat{A}_t(2,s), \tag{20}$$

where $V_{ss}(2)$, as in the general model, denotes the value added of Country 2. The right-hand side of equation (20) is the aggregate productivity change due to sectoral shocks according to Hulten (1978) where the ratios $\frac{p^Q_{ss}(2,s)Q_{ss}(2,s)}{V_{ss}(2)}$ are the Domar weights (Domar, 1961). The left-hand side is the corresponding version in a one-sector economy. Setting the hypothetical aggregate productivity shock following equation (19) or (20) ensures that the aggregate productivity changes are the same in the one-sector and multi-sector economies.

We can further show that the aggregate productivity change is a sufficient statistic for changes in the real wage, $\widehat{w}_t(2) - \widehat{p}_t(2)$, employment, $\widehat{H}_t(2)$, and real value added/GDP, $\widehat{V}_t(2)$. In a closed economy without intermediate inputs, production in each sector equals consumption, so the change in the real wage is proportional to aggregate productivity change.[2] Under the GHH preferences, the change in employment is proportional to the change in the real wage, thus proportional to the change in aggregate productivity. Finally, the change in real GDP can be decomposed into the change in aggregate productivity and employment, and therefore is also proportional to the change in aggregate productivity.

Taking Country 2's equilibrium changes in prices and quantities as given, Country 1's equilib-

[2] Using labor as the numeraire, the change in the consumer price index equals the average reduction in sectoral production costs weighted by consumption shares, which, in turn, equals the average improvement in sectoral productivity weighted by consumption (thus production) shares.

rium is solved as in the following proposition:

Proposition 1 *Country 1's real value-added can be expressed as*

$$\widehat{V}_t(1) = \frac{\nu\left(\tilde{a}_{21}^C\right)^2}{\Omega}\left[-\gamma_f \widehat{p}_t^{CM}(1) + \gamma_f \widehat{p}_t(2) + \widehat{C}_t(2)\right], \tag{21}$$

where $\Omega \equiv \tilde{a}_{21}^C\left[1 + \nu\tilde{a}_{21}^C - (1-\gamma_f)\left(2 - \tilde{a}_{21}^C\right)\right]$, $\widehat{p}_t(2)$ *is Country 2's consumer price index,* $\tilde{a}_{21}^c$ *is Country 1's steady-state export-to-GDP ratio, and* $\widehat{p}_t^{CM}(1)$ *is the import price index defined as*

$$\widehat{p}_t^{CM}(1) \equiv \sum_s \frac{p_{ss}^Q(2,s) f_{ss}(1,(2,s))}{\sum_k p_{ss}^Q(2,k) f_{ss}(1,(2,s))} \widehat{p}_t^Q(2,s).$$

Proof. See Appendix B. ■

To better understand Proposition 1, we can rewrite equation (21) as follows:

$$\widehat{V}_t(1) = \frac{\nu\tilde{a}_{21}^C}{\Omega}\left[\underbrace{-\tilde{a}_{21}^C \widehat{p}_t^{CM}(1)}_{\text{direct effect on labor supply}} + \underbrace{(1-\gamma_f)\tilde{a}_{21}^C\left(\widehat{p}_t^{CM}(1) - \widehat{p}_t(2)\right)}_{\text{demand due to consumer substitution}} + \underbrace{\tilde{a}_{21}^C\left(\widehat{p}_t(2) + \widehat{C}_t(2)\right)}_{\text{demand due to foreign total consumption}}\right]. \tag{22}$$

The three terms in the brackets in equation (22) represent different channels through which foreign shocks affect Country 1's excess labor demand. The first term is the direct effect of changing foreign prices on labor supply. The strength of this effect equals the change in the import prices $\widehat{p}_t^{CM}(1)$ multiplied by the import-to-GDP ratio, $\tilde{a}_{21}^c$. The second term captures changes in demand due to foreign consumer substitution between domestic and foreign goods. The magnitude of the substitution effects depends on the consumption elasticity γ_f. Finally, the last term captures changes in labor demand due to foreign income changes. Higher foreign income leads to higher demand for home goods and thus higher home labor demand, scaled by the export-to-GDP ratio.

These direct effects will be further moderated by the general equilibrium feedback. For example, higher labor demand tends to increase home wages and leads to higher home prices. This will then lead consumers and producers to substitute away from home goods and reduce home labor demand. Such general equilibrium effects are captured by the coefficient Ω^{-1}. In Appendix B, we show that Ω equals the derivative of Country 1's excess labor demand with respect to the change in home real wage. A larger Ω implies that home labor demand is more sensitive to changes in home wages. With the same direct change in the excess labor demand, we therefore need a smaller increase in home wages to bring the excess labor demand back to zero, dampening the final effect on total employment/real value added.

In the proof of Lemma 1, we express the changes in foreign expenditure and the foreign consumer price index as linear combinations of sectoral productivity shocks $\widehat{A}_t(2,s)$ and both are proportional to $\widehat{A}_t(2)^{one-sec}$. However, this proportionality does not hold for the change in the aggregate import price, $\widehat{p}_t^{CM}(1)$. The lack of a sufficient statistic for $\widehat{p}_t^{CM}(1)$ is the only source of aggregation bias in the simple model without intermediate inputs. More formally, we state this aggregation bias in the following corollary:

Corollary 1 *Denote the actual change in Country 1's real value added as $\widehat{V}_t(1)$ in a multi-sector model and the predicted change in a one-sector model as $\widehat{V}_t(1)^{one-sec}$. The one-sector model is calibrated to match (1) the aggregate variables in Country 2 as in Lemma 1, (2) the steady-state export-to-GDP ratio in Country 1, and (3) the same labor supply elasticity v and elasticity of substitution γ_f as in the multi-sector model. The one-sector model will have a bias in the predicted change in value added as*

$$\widehat{V}_t(1)^{one-sec} - \widehat{V}_t(1) = \frac{\nu \tilde{a}_{21}^C \gamma_f}{\Omega} \left(\widehat{p}_t^{CM}(1) - \widehat{p}_t(2) \right).$$

The term $\widehat{p}_t^{CM}(1) - \widehat{p}_t(2)$ can be expressed as

$$\widehat{p}_t^{CM}(1) - \widehat{p}_t(2) = \sum_s \left[\omega_C(2,(2,s)) - \tilde{\omega}_C(1,(2,s)) \right] \widehat{A}_t(2,s), \tag{23}$$

where $\tilde{\omega}_C(1,(2,s))$ is sectoral import share defined as

$$\tilde{\omega}_C(1,(2,s)) \equiv \frac{p_{ss}^Q(2,s) f_{ss}(1,(2,s))}{\sum_k p_{ss}^Q(2,k) f_{ss}(1,(2,k))}.$$

The bias disappears if any of the following conditions hold:

1. *Shocks are symmetric across sectors: $\widehat{A}_t(2,s) = \widehat{A}_t(2,1), \forall s$*

2. *Sectoral import shares in Country 1 are the same as sectoral consumption shares in Country 2, i.e., $\omega_C(2,(2,s)) = \tilde{\omega}_C(1,(2,s)), \forall s$.*

Proof. See Appendix B. ∎

Intuitively, when all sectors in Country 2 receive the same shock, the multi-sector model behaves like a one-sector model, and abstracting from sectoral heterogeneity does not lead to any bias in aggregate predictions. The bias also disappears when the sectoral import shares are equal to the sectoral consumption shares. This holds, for example, when sectors in Country 2 have the same openness, i.e.,

$$\frac{f_{ss}(1,(2,s))}{f_{ss}(1,(2,s))+f_{ss}(2,(2,s))} = \frac{f_{ss}(1,(2,s))}{Q_{ss}(2,s)} = \frac{f_{ss}(1,(2,1))}{Q_{ss}(2,1)}, \forall s \in \mathcal{S}.$$

To see this, suppose there are two sectors. Sector 1 is tradable while sector 2 is nontradable. We must have $\omega_C(2,(2,s)) < \tilde{\omega}_C(1,(2,s)) = 1$ for $s = 1$. It is immediate that, for non-negative shocks $\widehat{A}_t(2,s)$, the one-sector model under-predicts transmission if the shock is larger in the tradable sector and over-predicts transmission if the shock is larger in the non-tradable sector.

Our simple model provides sharp analytical insights on the sources of aggregation bias, but it also omits other important channels that may affect the magnitude and direction of aggregation bias. In Appendix B, we extend our analysis and incorporate domestic and foreign intermediate inputs in production. We show that the presence of domestic inputs does not change the expressions of the aggregation bias, while foreign intermediate inputs introduce additional terms to the aggregation bias. When foreign intermediate inputs are present, the aggregation bias depends not only on sectoral productivity shocks and sectoral import and consumption shares, but also on the heterogeneity of value added to gross output ratios across sectors, as well as the incorrect calibration of the parameter $\tilde{a}^C_{21}$ in the one-sector model. However, in Appendix C, we show these forces are quantitatively less important than those in Corollary 1.

Do these two conditions in Corollary 1 hold in the data? As plotted in Figure 1, we find that import and consumption shares are different across sectors and countries. In addition, Figure 2 suggests that changes in sectoral productivity are unlikely to be similar across sectors. Therefore, aggregation bias is likely to exist in realistic calibration. The question is how large this bias is, so we turn to the quantitative analysis next.

4 Quantitative Analysis

As the above analysis suggests that aggregation bias in terms of shock transmission across countries can depend on both sectoral linkages and sectoral productivity shocks, this section seeks to quantify aggregation bias based on these channels in our full model. We highlight that using a one-sector model tends to underpredict cross-country shock spillover, and that aggregation bias varies over time.

4.1 Data and Calibration

We first describe the data and parameters to calibrate the full model.

Data Our model is calibrated to two countries, each of the 20 countries in our sample vis-a-vis a composite ROW between 1970 and 2008. The 20 countries are Australia, Austria, Belgium, Canada, Germany, Denmark, Spain, Finland, France, Greece, Ireland, Italy, Japan, Korea, Luxembourg, the Netherlands, Portugal, Sweden, the United Kingdom and the United States. Each country has 31 sectors. The names of the sectors corresponding to the sector numbers are in Appendix Table A-1. The sectoral output data come from EUKLEMS and the world IO tables are from the World Input-Output Database between 1995 and 2011.

Calibration The model is calibrated for annual data. Table 1 summarizes the calibrated values for common parameters across countries in our baseline. The discount factor is set to be 0.96, so the implied steady state annual interest rate is about 4%. We set the labor share parameter, α_H, to be 2/3, which is standard in the literature. The depreciation rate at the steady state is 10% a year. We calibrate three parameters for household preferences, which are sufficient for the linearized model. The inverse of the intertemporal elasticity of substitution parameter, σ, is set to be 2, which is along the line of Backus, Kehoe, and Kydland (1995), and others in the international business cycle literature. The inverse Frisch labor supply parameter, ψ, is set to be 1, following Bilbiie (2009) and others. We set the parameter measuring the wealth elasticity of labor supply, κ, to be 0.1, which implies low wealth effects, similar to Miyamoto and Nguyen (2017).[3] We set a small value of the Uzawa preference, ϕ_β, as standard international business cycle model with incomplete markets (0.0001).

In the literature, the range for the elasticities of substitution across sectors and countries varies widely. The elasticity of substitution across countries in the international business cycle literature is typically small, around 1; for example, Backus, Kehoe, and Kydland (1995) set this parameter to 1.5, and Johnson (2013) imposes 1 in his baseline model and analyzes a lower elasticity case in his robustness check. In contrast, the estimates in the trade literature are much larger, between 6 and 15. A recent paper by Boehm, Levchenko, and Pandalai-Naya (2023) estimates a low short-run trade elasticity of less than one and a long-run elasticity of about 1. In the baseline, as Johnson (2013), we set the elasticities of substitution across both final and intermediate goods in different

[3]The Greenwood, Hercowitz, and Huffman (1988) preferences imply a wealth elasticity parameter of 0, and the separable utility function implies this parameter to be 2.

sectors and countries to 1 and discuss our quantitative results with respect to this parameter later. With this parameterization, the composite intermediate input is Cobb-Douglas in inputs from different countries. We also set the parameter for variable utilization, δ_b, to be 0.05, implying a highly elastic utilization. The steady-state investment adjustment cost parameter is imposed to be 0.25.

As our full model does not assume a small open economy, we calibrate the size of the countries according to the average of the World IO tables between 1995 and 2011. In addition, we set the following steady-state shares in the model based on the average 1995-2011 World IO tables: the shares of intermediate inputs in gross output in each sector in each country ω_q, the shares of foreign goods in sectoral final consumption, investment and intermediate goods for all sectors and countries $\{\omega_{fC}, \omega_Z, \omega_M\}$, the shares of sectoral composite in final consumption, investment and intermediate goods for all countries $\{\omega_{CF}, \omega_z, \omega_m\}$. These steady-state shares are calibrated to completely replicate the 2-country 31-sector world IO table.

Table 1: Calibrated Parameters

Parameter		Value
β	discount factor	0.96
σ	relative risk aversion	2
κ	wealth effect on labor supply	0.1
ψ	inverse of Frisch elasticity	1
s	investment adjustment cost	0.25
α_H	labor share	2/3
δ	depreciation	0.1
δ_b	capacity utilization	0.15
ϕ_β	Uzawa preference	0.0001
γ_F	elasticity across sectors for final good	1
γ_f	elasticity across countries for final good	1
γ_Z	elasticity across countries for investment goods	1
γ_z	elasticity across sectors for investment goods	1
γ_Q	elasticity between value added and intermediate	1
γ_M	elasticity across sectors for intermediate	1
γ_m	elasticity across countries for intermediate	1
ρ_A	persistence of TFP	0.34

4.2 Aggregation bias and sectoral heterogeneity in IO structure

This section quantifies how much sectoral heterogeneity in input-output linkages can generate aggregation bias in shock spillover. As in the simple model analysis, we consider $\widehat{A}_t(2, s)$, which is a productivity shock in sector s in the foreign country, i.e., country 2. We then compute the relative responses of country 1's value added and hours to the responses of country 2's value added and hours in both the multi-sector model and the one-sector model counterparts. In other words, we compute $\frac{\widehat{V}_t(1)}{\widehat{V}_t(2)}$ and $\frac{\widehat{H}_t(1)}{\widehat{H}_t(2)}$ to $\widehat{A}_t\,(2, s)$ for each $s \in \mathbb{S}$. These relative responses are the changes in value added output in country 1 for a 1% change in value added output in country 2. Notice that we do not compute $\widehat{V}_t(1)$ and $\widehat{V}_t(2)$ separately like in the simple model. The reason is that the relative impulse responses normalize the transmission of shocks from country 2 to country 1, and sectoral shock sizes do not matter, as the relative responses measure how much value added output in the home country changes for a given 1% change in the foreign country, which can be driven by different sectoral shocks.

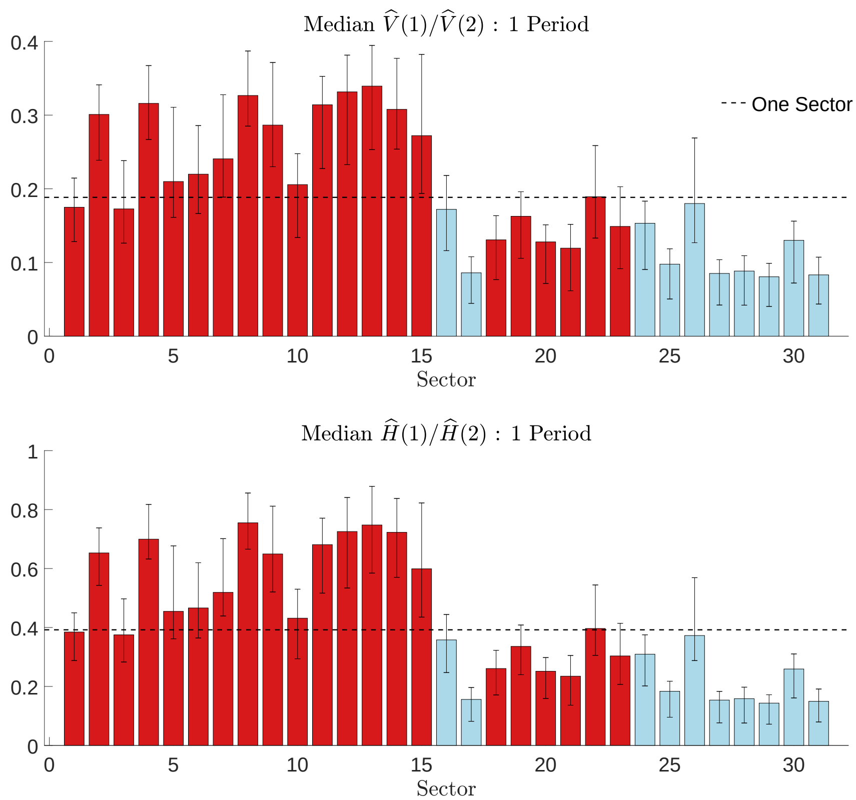
Figure 3 plots the relative changes in country 1 to country 2 in response to each sectoral productivity shock in country 2 in a median country in our sample of 20 pairs of countries (the red and light blue bars) and its 25-75 percentile bands (the thin lines). Given the heterogeneous input-output linkages of each sector, international spillover of sectoral shocks are also different: sectors with higher trade exposures (the red bars) tend to have larger transmission across countries than those with lower trade exposures. Furthermore, the one-sector model can generate smaller spillover across countries if the actual shock originates from sectors such as Manufacturing, but a larger spillover if the shock comes from the Services sector. These results highlight the differences in international spillover of productivity shocks between a multi-sector and a one-sector model as shocks originate from different sectors, which play different roles in the input-output structure. In addition, the one-sector spillover strength is roughly the same as the average of all sectoral shocks' spillover.

4.3 Aggregation bias and sectoral shock heterogeneity

In addition to playing different roles in the input-output structure, sectors can be driven by different productivity shocks, and the sectoral shock composition can be different at each point in time. This implies that aggregation bias can be time varying.

To quantify the aggregation bias in a one-sector model in terms of shock spillover across countries, we first recover all sectoral productivity shocks between 1970 and 2009 using a Kalman smoother. To that end, we match sectoral value added in the model for each country with the data between 1970 and 2009 and back out productivity shocks that replicate the data. Figure 4 plots the median standard deviation of home and foreign sectoral productivity shocks for all 31 sectors. As expected, shock sizes are heterogeneous across sectors. The sector with the largest shock size is sector 7 (Coke, Refined Petroleum and Nuclear Fuel). Services sectors such as sectors 25 (Real estate activities) have lower shock standard deviations, consistent with the observation that these

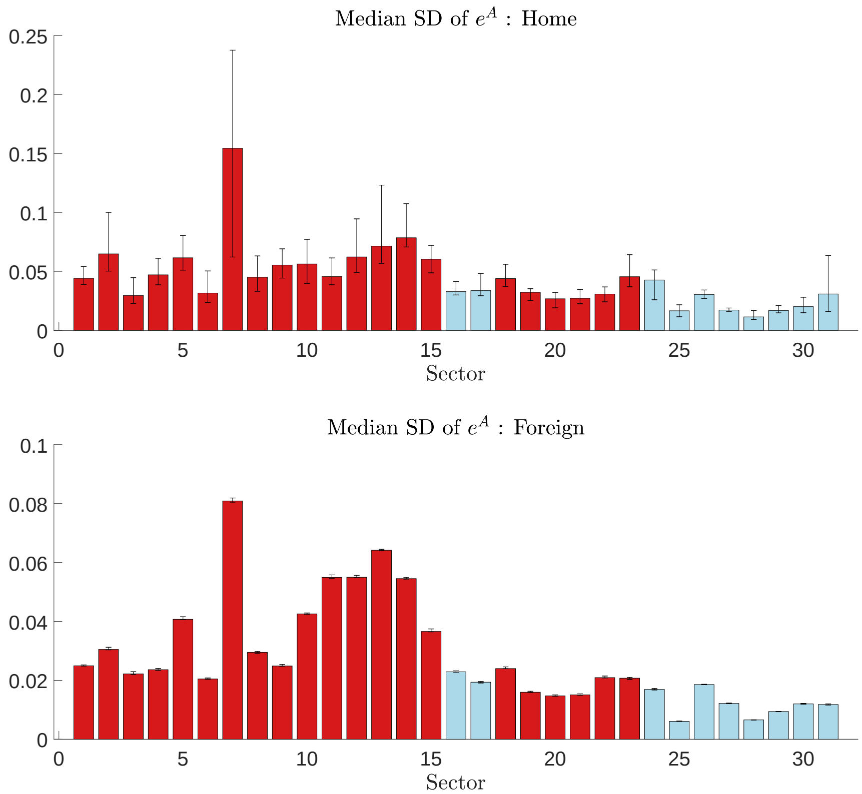
Figure 3: Transmission of sectoral shocks from country 2 to country 1 in a multi-sector model.



Notes: The red (and light blue) bars are the median relative responses across 20 country pairs in sectors with relatively high (and low) trade volume. The lines indicate the 25-75 percentile. The horizontal dashed lines are the shock spillover of the productivity shock in country 2 to country 1.

sectors' value added is less volatile than manufacturing sectors.

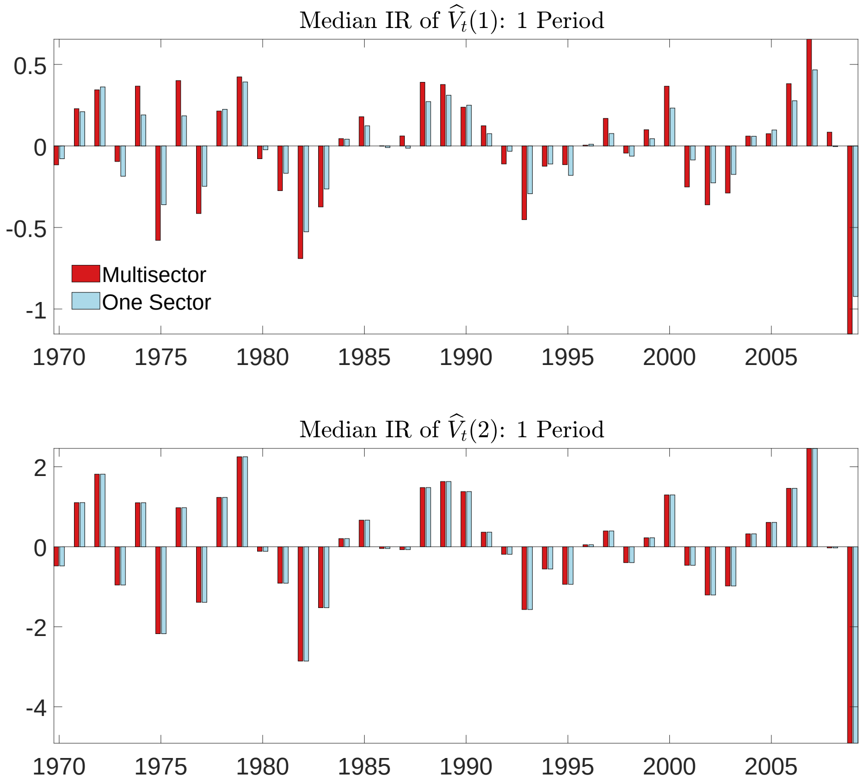
Figure 4: Recovered shocks in a multi-sector model.



Notes: This figure plots the recovered standard deviations of home (upper panel) and foreign (lower panel) sectoral shocks in a median country. The thin bars plot the 25-75 percentile.

What do these heterogeneous shocks imply about the aggregation bias in a one-sector model in terms of international shock spillover? We plot in Figure 5 the impulse responses of both home and foreign value added, $\hat{V}_1$ and $\hat{V}_2$, respectively, to the recovered foreign shocks between 1970 and 2009 in a multi-sector model and a one-sector model counterpart. To facilitate comparison between international transmission implied in a multi-sector model and that in a one-sector model, we normalize foreign value added responses in the multi-sector model to be the same as that in the one-sector model. In almost all years between 1970 and 2009, shock spillover is larger in the multi-sector model than in the one-sector model. For example, in 1973-1974, foreign shock spillovers to the home country are only half as large in the one-sector model as in the multi-sector model. Similarly, the one-sector model predicts a transmission strength of only 75% of that in the multi-sector model in the 2008-2009 Global Financial Crisis. These results suggest that the one-sector model tends to underpredict the transmission of foreign shocks to the domestic economy.

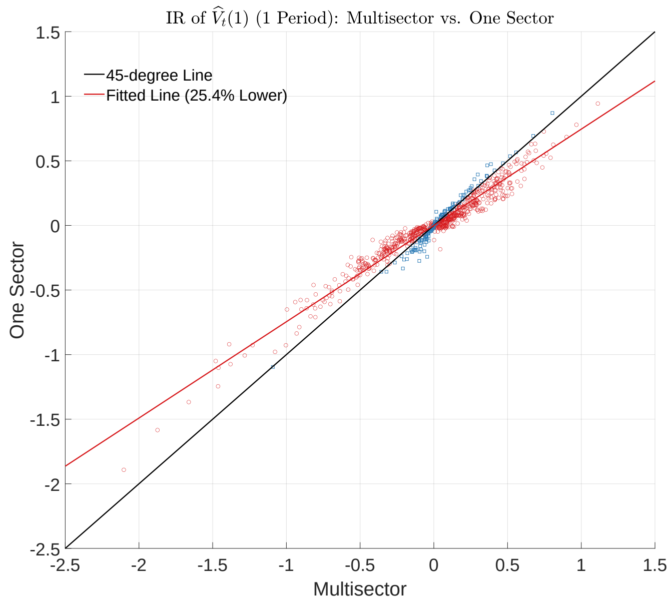
Figure 5: Median aggregation bias across time.



Notes: This figure plots the impulse responses of value added in home (upper panel) and foreign (lower panel) at a one-year horizon in a median country. The red bars are the responses in a multi-sector model, and the light blue bars are those in a one-sector model counterpart.

Finally, we plot in Figure 6 the responses of the home country to foreign shocks in all countries between 1970 and 2009 in the multi-sector model against that in the one-sector model. The responses are computed in the one-year horizon. The blue dots show the cases when the one-sector model overpredicts transmission, and the red dots are when the one-sector model underpredicts transmission. Overall, consistent with the median analysis, we find that the one-sector model predicts 25.4% less transmission of foreign shocks to the domestic economy over time. This result implies that the multi-sector model should, on average, generate larger comovement across countries than the one-sector model, and the one-sector model may underpredict the importance of foreign shocks in domestic business cycles.

Figure 6: Aggregation bias across countries over time.



Notes: Each point presents the domestic response to a foreign shock in the one-sector against the multi-sector models at each point in time for each pair of countries.

5 Conclusion

We investigate the differences in terms of foreign shock transmission to domestic economies in a multi-sector model and an aggregated one-sector model counterpart. Our analytical solution shows that the condition in which international shock spillover predicted in a one-sector model is the same as that in a multi-sector model. Quantitatively, we find that the one-sector model tends to underpredict shock spillover from foreign countries, suggesting that the multi-sector model may help generate larger comovement across countries conditional on productivity shocks compared to the one-sector model. While we consider a model that has been a basis of many international business cycle models, future work can incorporate other frictions to understand how these frictions may amplify or reduce the aggregation bias in a one-sector model.

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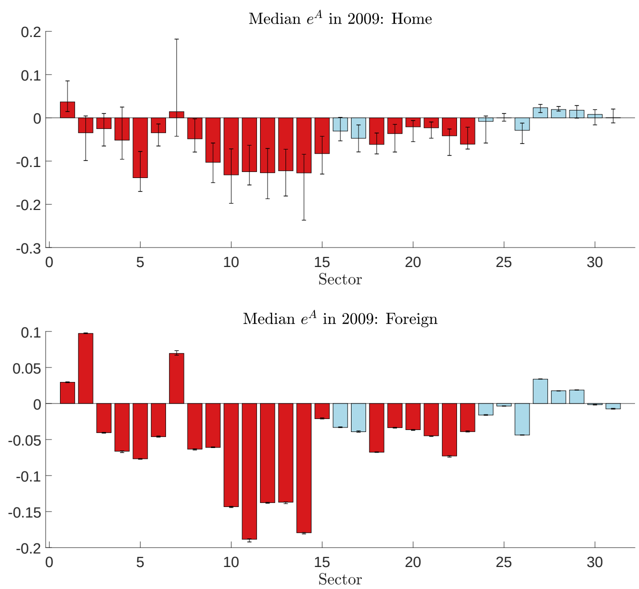
Online Appendix

A Tables and Figures

Table A-1: Available Sectors

Sector code	Number	Name
AtB	Sector 1	Agriculture, Hunting, Forestry and Fishing
C	Sector 2	Mining and Quarrying
D15t16	Sector 3	Food, Beverages and Tobacco
D17t19	Sector 4	Textiles, Textile, Leather and Footwear
D20	Sector 5	Wood and of wood and cork
D21t22	Sector 6	Pulp, Paper, Paper, Printing and Publishing
D23	Sector 7	Coke, Refined Petroleum and Nuclear Fuel
D24	Sector 8	Chemicals and Chemical Products
D25	Sector 9	Rubber and Plastics
D26	Sector 10	Other Non-Metallic Mineral
D27t28	Sector 11	Basic Metals and Fabricated Metal
D29	Sector 12	Machinery, Nec
D30t33	Sector 13	Electrical and Optical Equipment
D34t35	Sector 14	Transport Equipment
D36t37	Sector 15	Manufacturing, Nec; Recycling
E	Sector 16	Electricity, Gas and Water Supply
F	Sector 17	Construction
50	Sector 18	Sale, maintenance and repair of motor vehicle and retail sale of fuel
51	Sector 19	Wholesale trade and commission trade, except motor vehicles
52	Sector 20	Retail trade, except of motor vehicles
H	Sector 21	Hotels and Restaurants
60t63	Sector 22	Transport and Storage
64	Sector 23	Post and Telecommunications
J	Sector 24	Financial Intermediation
70	Sector 25	Real estate activities
71t74	Sector 26	Renting of M&EQ and other business activities
L	Sector 27	Public admin and defence
M	Sector 28	Education
N	Sector 29	Health and Social work
O	Sector 30	Other Community, social and personal services
P	Sector 31	Private households with employed persons

Figure A-1: Sectoral shocks during the Global Financial Crisis 2009.



Notes: The recovered shocks in each sector in a median country between 2008-2009.

B More general results and proofs for Section 3

In this section, we present more general results and proofs for Section 3 in the main text. The main extension is that we allow for intermediate inputs in production.

Before we present the results, we define some additional notation in Table B-2 to facilitate the discussion.

Table B-2: Define other vectors and matrices

Notation	Description	Dimensions	Value
$\mathbf{p}_{Q,i}$	output prices	$S \times 1$	$\left[p^Q_{ss}(i,s)\right]_s$
$\mathbf{q}_i$	output	$S \times 1$	$\left[Q_{ss}(i,s)\right]_s$
$\boldsymbol{\lambda_i}$	output share	$S \times 1$	$P^Q_{ss}(i,s)Q_{ss}(i,s)/\left(\mathbf{p}_{Q,i}\right)^T \mathbf{q}_i$
$\mathbf{f}_i$	final consumption	$S \times 1$	$[f_{ss}(i,s)]_s$
$\mathbf{a}_i$	productivity	$S \times 1$	$[A_{ss}(i,s)]_s$
$\boldsymbol{\omega}_{Q,i} \circ \widehat{\mathbf{a}}_i \equiv \widehat{\mathbf{z}}_i$	change in effective productivity	$S \times 1$	$\left[\omega_Q(i,s)\widehat{A}_t(i,s)\right]_s$
$\boldsymbol{\omega}_{Q,i}$	value added share in production	$S \times 1$	$\left[\omega_Q(i,s)\right]_s$
$\text{diag}(\mathbf{1}-\boldsymbol{\omega}_{Q,i})\boldsymbol{\Theta}^m_{ij} \equiv \mathbf{A}_{ij}$	input share in gross output	$S \times S$	$(1-\omega_Q(i,s))\theta_m((i,s),(j,k))$
$\boldsymbol{\Psi}^m_{ij}$	share of output used by downstream	$S \times S$	
$\boldsymbol{\Psi}^f_{ij}$	share of output consumed	$S \times 1$	
$\tilde{\boldsymbol{\Theta}}^m_{ii}$	Leontief inverse	$S \times S$	$\left(\mathbf{I}-\text{diag}(\mathbf{1}-\boldsymbol{\omega}_{Q,i})\boldsymbol{\Theta}^m_{ii}\right)^{-1}$
$\tilde{\boldsymbol{\Psi}}^m_{ii}$	Downstream usage inverse	$S \times S$	$(\mathbf{I}-\boldsymbol{\Psi}^m_{ii})^{-1}$
$\boldsymbol{\omega}_{C,ij}$	consumption shares	$S \times 1$	$\omega_C(j,(i,s)) \equiv \frac{\theta_f(j,(i,s))}{\sum_s \theta_f(j,(i,s))}$
$\boldsymbol{\Gamma}^w_i$	sector's share of employment	$1 \times S$	$\left[H_{ss}(i,s)/H_{ss}(i)\right]_s$

The results in Lemma 1 can be extended to a closed economy with intermediate inputs without any modifications. We therefore provide a proof for the more general case with intermediate inputs.

Proof of Lemma 1 with intermediate inputs. To solve the equilibrium in Country 2 as a closed economy, we first apply Walras's Law and normalize the change in wages, $\widehat{w}_t(2)$, to zero. As illustrated in Table B-2, we denote the effective productivity shocks in Country 2 as $\widehat{\mathbf{z}}_2 \equiv \boldsymbol{\omega}_{Q,2} \circ \widehat{\mathbf{a}}_2$, where $\circ$ indicates element-wise product.

Perfect competition and the production function of each sector imply that the changes in sectoral prices must follow

$$\widehat{\mathbf{p}}_{Q,2} = -\widehat{\mathbf{z}}_2 + \text{diag}(\mathbf{e}_S - \boldsymbol{\omega}_{Q,2})\boldsymbol{\Theta}^m_{22}\widehat{\mathbf{p}}_{Q,2},$$

where $\mathbf{e}_S$ is a $S \times 1$ vector of ones and the operator diag transforms the vector into a diagonal matrix. The first term on the right-hand side is the direct effect of the productivity shocks, while the second term captures the direct effect of the input prices.

Based on this equation, we can solve the changes in prices as a linear transformation of changes in productivities, $\widehat{\mathbf{p}}_{Q,2} = -(\mathbf{I} - \text{diag}(\mathbf{1} - \boldsymbol{\omega}_{Q,2})\boldsymbol{\Theta}^m_{22})^{-1}\widehat{\mathbf{z}}_2 = -\tilde{\boldsymbol{\Theta}}^m_{22}\widehat{\mathbf{z}}_2$. The change in the aggregate price index is the average of sectoral price changes, weighted by consumption shares $\boldsymbol{\omega}_{C,22}$:

$$\widehat{p}_t(2) = \boldsymbol{\omega}_{C,22}^T \widehat{\mathbf{p}}_{Q,2} = -\boldsymbol{\omega}_{C,22}^T \tilde{\boldsymbol{\Theta}}_{22}^m \widehat{\mathbf{z}}_2.$$

Due to GHH preferences, the change in employment must satisfy

$$\widehat{H}_t(2) = \nu(\widehat{w}_t(2) - \widehat{p}_t(2)) = \nu \boldsymbol{\omega}_{C,22}^T \tilde{\boldsymbol{\Theta}}_{22}^m \widehat{\mathbf{z}}_2.$$

Since Hulten (1978), we know that the change in real value added can be written as

$$\widehat{V}_t(2) = \sum_s \underbrace{\frac{p_{ss}^Q(2,s) Q_{ss}(2,s)}{V_{ss}(2)}}_{\text{Domar weights}} \omega_Q(2,s) \widehat{A}_t(2,s) + \widehat{H}_t(2).$$

Note that the Domar weights equals $\boldsymbol{\omega}_{C,22}^T \tilde{\boldsymbol{\Theta}}_{22}^m$. To see this, consider the goods market clearing conditions

$$\begin{aligned}
&(\mathbf{p}_{Q,2}\mathbf{q}_2)^T = (\mathbf{p}_{Q,2}\mathbf{f}_2)^T + (\mathbf{p}_{Q,2}\mathbf{q}_2)^T \operatorname{diag}(\mathbf{1} - \boldsymbol{\omega}_{Q,2}) \boldsymbol{\Theta}_{22}^m \\
\Leftrightarrow\, &(\mathbf{p}_{Q,2}\mathbf{q}_2)^T = (\mathbf{p}_{Q,2}\mathbf{f}_2)^T \times \tilde{\boldsymbol{\Theta}}_{22}^m \\
\Leftrightarrow\, &\frac{1}{V_{ss}(2)} (\mathbf{p}_{Q,2}\mathbf{q}_2)^T = \boldsymbol{\omega}_{C,22}^T \tilde{\boldsymbol{\Theta}}_{22}^m
\end{aligned}$$

Therefore, both changes in employment and value added are proportional to the Domar weights weighted productivity change $\boldsymbol{\omega}_{C,22}^T \tilde{\boldsymbol{\Theta}}_{22}^m \widehat{\mathbf{z}}_2$. We need a one-sector economy in which

$$\boldsymbol{\omega}_{C,22}^T \tilde{\boldsymbol{\Theta}}_{22}^m \widehat{\mathbf{z}}_2 = \frac{\sum_s p_{ss}^Q(2,s) Q_{ss}(2,s)}{V_{ss}(2)} \widehat{z}_2^{one-sec},$$

where $\frac{\sum_s p_{ss}^Q(2,s) Q_{ss}(2,s)}{V_{ss}(2)}$ is the output-to-value-added ratio (also the Domar weight in a one-sector economy). When labor is the only input, we have $\boldsymbol{\omega}_{Q,2} = \mathbf{e}_s$ thus $\widehat{\mathbf{z}}_2 = \widehat{\mathbf{a}}_2$. Therefore, we have obtained equation (20). ■

In the main text, we have discussed that the aggregate productivity change (20) is a sufficient statistic for changes in the real wage, $\widehat{w}_t(2) - \widehat{p}_t(2)$, employment, $\widehat{H}_t(2)$, and real value added/GDP, $\widehat{V}_t(2)$. With intermediate inputs, the impacts of $\widehat{A}_t(2,s)$ are amplified because $\widehat{A}_t(2,s)$ has both a direct effect (changes in productivities) and an indirectly effect (intermediate input prices). The overall effects are exactly captured by the Domar weights (see the proof). Under the GHH preferences, the change in the real wage, employment and real value added/GDP are still proportional to the change in aggregate productivity.

Since Country 1 is a small open economy, we can take Country 2's equilibrium changes in prices and quantities as given and solve for Country 1's equilibrium. To proceed, we express Country 1's prices and quantities (in log changes) as functions of the shocks (Country 2's prices and quantities) and Country 1's wage. We then calculate the excess labor demand in Country 1. Setting the excess demand to zero will allow us to solve for the wage and thus the other equilibrium variables.

We first derive Country 1's prices as functions of the shocks and Country 1's wage. As summarized in Table B-2, we denote the $S \times 1$ vector of output prices in country i as $\mathbf{p}_{Q,i} \equiv \left[p_{ss}^{Q}(i,s)\right]_s$, the $S \times 1$ vector of value added share in sectors of country i as $\boldsymbol{\omega}_{Q,i} \equiv [\omega_Q(i,s)]_s$, the $S \times S$ matrix of Country j's input share of Country i's sectoral gross output as $\boldsymbol{\Theta}_{ij}^{m} \equiv [(1-\omega_Q(i,s))\omega_M((i,s),k)\omega_m((i,s),(j,k))]_{s,k}$, where the row index s refers to the output sector and the column index k refers to the input sector. Perfect competition and log-linearization of the cost function associated with (12) imply that the changes in sectoral output prices must satisfy

$$\widehat{\mathbf{p}}_{Q,1} = \boldsymbol{\omega}_{Q,1}\widehat{w}_t(1) + \boldsymbol{\Theta}_{12}^{m}\widehat{\mathbf{p}}_{Q,2} + \boldsymbol{\Theta}_{11}^{m}\widehat{\mathbf{p}}_{Q,1}.$$

From this we can solve the change in prices as $\widehat{\mathbf{p}}_{Q,1} = \tilde{\boldsymbol{\Theta}}_{11}^{m}(\boldsymbol{\omega}_{Q,1}\widehat{w}_t(1) + \boldsymbol{\Theta}_{12}^{m}\widehat{\mathbf{p}}_{Q,2})$, where $\tilde{\boldsymbol{\Theta}}_{11}^{m} \equiv (\mathbf{I} - \boldsymbol{\Theta}_{11}^{m})^{-1}$. Intuitively, price changes in Country 1 depend on changes in its wage and foreign input prices. The direct propagation of the foreign price shocks depends on the input coefficients $\boldsymbol{\Theta}_{12}^{m}$, but will be amplified by the domestic input coefficients as captured by the Leontief inverse $\tilde{\boldsymbol{\Theta}}_{11}^{m}$.

The consumer price index in Country 1 is an average of sectoral prices weighted by consumption shares

$$\begin{aligned}\widehat{p}_t(1) &= \boldsymbol{\omega}_{C,11}^{T}\widehat{\mathbf{p}}_{Q,1} + \boldsymbol{\omega}_{C,21}^{T}\widehat{\mathbf{p}}_{Q,2} \\ &= \underbrace{\boldsymbol{\omega}_{C,11}^{T}\tilde{\boldsymbol{\Theta}}_{11}^{m}\boldsymbol{\omega}_{Q,1}}_{\tilde{a}_{1L}^{c}}\widehat{w}_t(1) + \underbrace{(\boldsymbol{\omega}_{C,11}^{T}\tilde{\boldsymbol{\Theta}}_{11}^{m}\boldsymbol{\Theta}_{12}^{m} + \boldsymbol{\omega}_{C,21}^{T})}_{\text{sum}\equiv\tilde{a}_{21}^{c}}\widehat{\mathbf{p}}_{Q,2},\end{aligned}$$

where the $S \times 1$ vector, $\boldsymbol{\omega}_{C,ji} \equiv [\omega_{fC}(i,(j,s))\omega_{FC}(i,s)]_s$, denotes the share of goods from each Country j's sector in Country i's final consumption. In the second line of the above expression, the coefficient before wage, $\tilde{a}_{1L}^{c} \equiv \boldsymbol{\omega}_{C,11}^{T}\tilde{\boldsymbol{\Theta}}_{11}^{m}\boldsymbol{\omega}_{Q,1}$, summarizes the impact of wage on the consumer price index. The $1 \times S$ vector, $\boldsymbol{\omega}_{C,11}^{T}\tilde{\boldsymbol{\Theta}}_{11}^{m}\boldsymbol{\Theta}_{12}^{m} + \boldsymbol{\omega}_{C,21}^{T}$, summarizes the impact of foreign sectoral price changes on Country'1 consumer price index. The first term captures the indirect impact through foreign inputs used in domestic production and the second term captures the direct impact through consumption of foreign final goods. We further denote the sum of these S coefficients $\tilde{a}^c_{21}$. We verify in the proof of Proposition B-1 that $\tilde{a}^c_{1L} + \tilde{a}^c_{21} = 1$. We define the consumption-relevant import price index as

$$\widehat{p}_t^{CM}(1) \equiv \frac{\boldsymbol{\omega}^T_{C,11}\tilde{\boldsymbol{\Theta}}^m_{11}\boldsymbol{\Theta}^m_{12} + \boldsymbol{\omega}^T_{C,21}}{\tilde{a}^c_{21}}\widehat{\mathbf{p}}_{Q,2},$$

a weighted average of import prices where the weights are proportional to the overall impact of each foreign sector's price on the home consumer price index. The two price indices are closely related: $\widehat{p}_t(1) = \tilde{a}^c_{1L}\widehat{w}_t(1) + \tilde{a}^c_{21}\widehat{p}_t^{CM}(1)$.

Instead of solving the nominal wage change $\widehat{w}_t(1)$, we solve the wage change relative to the consumption-relevant import price index, $\widehat{w}_t(1) - \widehat{p}_t^{CM}(1)$. There are two reasons for this choice. First, due to the GHH preferences, the change in Country 1's labor supply is proportional to this difference, i.e., $\widehat{H}_t(1) = \nu(\widehat{w}_t(1) - \widehat{p}_t(1)) = \nu\tilde{a}^c_{21}(\widehat{w}_t(1) - \widehat{p}_t^{CM}(1))$. In addition, the change in employment must equal the change in real GDP because Country 1's aggregate TFP is not affected by foreign price and demand shocks, a result first shown in Kehoe and Ruhl (2008). Therefore, the change in wage relative to the import price index is a sufficient statistic for the change in aggregate employment and value added. Second, when foreign goods are not used as intermediate inputs, i.e., $\boldsymbol{\Theta}^m_{12} = \mathbf{0}$, all home sectors will have the same price change, $\widehat{w}_t(1)$. Therefore, $\widehat{w}_t(1)$ is also the change in Country 1's export price index.[4] In addition, since foreign goods are only used as final goods by home households, $\widehat{p}_t^{CM}(1)$ coincides with the overall import price index. In this special case (no foreign intermediate inputs), $\widehat{w}_t(1) - \widehat{p}_t^{CM}(1)$ is also the change in the terms of trade.

We define the excess labor demand (in log changes) as the difference between the log change in labor demand and the log change in labor supply and denote it as $D_1(\widehat{w}_t(1) - \widehat{p}_t^{CM}(1)|\widehat{\mathbf{p}}_{Q,2}, \widehat{\mathbf{q}}_2, \widehat{C}_t(2))$, where $\widehat{\mathbf{q}}_i$ denotes an $S \times 1$ vector of changes in Country i's sectoral gross output and $\widehat{C}_t(2)$ is the change in foreign real consumption. All three shocks, $\widehat{\mathbf{p}}_{Q,2}, \widehat{\mathbf{q}}_2, \widehat{C}_t(2)$, are functions of the foreign productivity changes and base-period shares. We use them directly to better explain the transmission. We can write the excess labor demand as

$$D_1(\widehat{w}_t(1) - \widehat{p}_t^{CM}(1)|\widehat{\mathbf{p}}_{Q,2}, \widehat{\mathbf{q}}_2, \widehat{C}_t(2)) = (1-\gamma_Q)\boldsymbol{\Gamma}^w_1(\widehat{w}_t(1)\mathbf{e}_S - \widehat{\mathbf{p}}_{Q,1}) + \boldsymbol{\Gamma}^w_1\widehat{\mathbf{p}_{Q,1}\mathbf{q}_1} - \widehat{w}_t(1) - \nu\left(\widehat{w}_t(1) - \widehat{p}_t(1)\right)$$

[4]A sector that uses less labor will see a smaller direct impact of wage, but a larger indirect impact through its intermediate inputs. Eventually, all sectors' price changes are the same as the wage. Mathematically, we know that total input shares in each sector equals $1-\omega_Q(1,s)$. Denoting an $S \times 1$ vector of ones as $\mathbf{e}_S$, we have $\boldsymbol{\Theta}^m_{11}\mathbf{e}_S = \mathbf{e}_S - \boldsymbol{\omega}_{Q,1}$. Therefore, we have $\widehat{\mathbf{p}}_{Q,1} = \tilde{\boldsymbol{\Theta}}^m_{11}\boldsymbol{\omega}_{Q,1}\widehat{w}_t(1) = \widehat{w}_t(1)$.

where $\mathbf{e}_S$ denotes an $S \times 1$ vector of ones and $\boldsymbol{\Gamma}_1^w$ denotes a $1 \times S$ vector of employment shares of Country 1's sectors. The first term on the right-hand side captures the change in labor share in each sector, the second term captures the change in labor demand due to changes in output and the last term is the change in labor supply. The last term, $\nu\left(\widehat{w}_t(1) - \widehat{p}_t(1)\right)$, captures the change in labor supply. An equilibrium is solved by setting $D_1(\widehat{w}_t(1) - \widehat{p}_t^{CM}(1) | \widehat{\mathbf{p}}_{Q,2}, \widehat{\mathbf{q}}_2, \widehat{C}_t(2)) = 0$. In the expression above, $\widehat{\mathbf{p}}_{Q,1}$ and $\widehat{\mathbf{q}}_1$ are functions of the shocks $\widehat{\mathbf{p}}_{Q,2}, \widehat{\mathbf{q}}_2, \widehat{C}_t(2)$ and the equilibrium wage change $\widehat{w}_t(1)$.[5]

We have the following proposition

Proposition B-1 *The change in equilibrium wage relative to the consumption-relevant import price index can be solved as*

$$
\begin{aligned}
\widehat{w}_t(1) - \widehat{p}_t^{CM}(1) = \Omega^{-1} \Bigg\{ & \underbrace{-\tilde{a}_{21}^c \widehat{p}_t^{CM}(1)}_{\text{direct effect on labor supply}} + \underbrace{\boldsymbol{\Gamma}_1^w \tilde{\boldsymbol{\Psi}}_{11}^m \left(\boldsymbol{\Psi}_{12}^f \widehat{p_t(2) C_t}(2) + \boldsymbol{\Psi}_{12}^m \widehat{\mathbf{p}_{Q,2} \mathbf{q}_2} \right)}_{\text{demand due to foreign consumption and output changes}} \\
& + \underbrace{(1-\gamma_f) \boldsymbol{\Gamma}_1^w \tilde{\boldsymbol{\Psi}}_{11}^m \left[(\boldsymbol{\Psi}_{11}^f + \boldsymbol{\Psi}_{12}^f) \circ (\tilde{\boldsymbol{\Theta}}_{11}^m \boldsymbol{\Theta}_{12} \Delta \widehat{\mathbf{p}}_{Q,2}) + \boldsymbol{\Psi}_{12}^f (\widehat{p}_t^{CM}(1) - \widehat{p}_t(2)) \right]}_{\text{demand due to consumer substitution}} \\
& + \underbrace{(1-\gamma_Q) \boldsymbol{\Gamma}_1^w \tilde{\boldsymbol{\Psi}}_{11}^m \left[(\tilde{\boldsymbol{\Theta}}_{11}^m \boldsymbol{\Theta}_{12} \Delta \widehat{\mathbf{p}}_{Q,2}) \circ ((\boldsymbol{\Psi}_{11}^m + \boldsymbol{\Psi}_{12}^m) \mathbf{e}_S) - \left(\tilde{\boldsymbol{\Theta}}_{11}^m \boldsymbol{\Theta}_{12} + \boldsymbol{\Psi}_{12}^m \right) \Delta \widehat{\mathbf{p}}_{Q,2} \right]}_{\text{demand due to producer substitution}} \Bigg\},
\end{aligned}
\tag{B-1}
$$

where $\boldsymbol{\Psi}_{ij}^f$ and $\boldsymbol{\Psi}_{ij}^m$ are matrices of final consumption and intermediate shares, which capture propagation from downstream sectors' demand to upstream sectors. Similar to the definition of $\tilde{\boldsymbol{\Theta}}_{11}^m$, $\tilde{\boldsymbol{\Psi}}_{11}^m$ equals $(\mathbf{I} - \boldsymbol{\Psi}_{11}^m)^{-1}$ and captures the total requirement of inputs from downstream sectors' expansion. $\Delta \widehat{\mathbf{p}_{Q,2}}$ is defined as the difference between $\mathbf{p}_{Q,2}$ and $\widehat{p}_t^{CM}(1)$.[6] *The denominator, Ω, is the coefficient of $\widehat{w}_t(1)$ in the excess labor demand, or equivalently, $\partial D_1 / \partial \widehat{w}_t(1)$.*

The change in employment and real value added is $\widehat{V}_t(1) = \widehat{H}_t(1) = \nu \tilde{a}_{21}^c \left(\widehat{w}_t(1) - \widehat{p}_t^{CM}(1) \right)$.

Proof of Proposition B-1. To solve the equilibrium in Country 1, we express all other prices and quantities as functions of the endogenous variable $\widehat{w}_t(1)$, and finally use the labor market clearing condition to solve $\widehat{w}_t(1)$.

Perfect competition and the production functions imply that the changes in sectoral output

[5] Note that $\widehat{\mathbf{p}}_{Q,2}, \widehat{\mathbf{q}}_2, \widehat{C}_t(2)$, in turn, depend on the productivity shocks $\widehat{A}_t(2, s)$. We work with the former for better interpretations.

[6] The detailed definitions of the matrices are collected in Table B-2.

prices must satisfy

$$\widehat{\mathbf{p}}_{Q,1} = \boldsymbol{\omega}_{Q,1}\widehat{w}_t(1) + \text{diag}(\mathbf{1} - \boldsymbol{\omega}_{Q,1})\boldsymbol{\Theta}^m_{12}\widehat{\mathbf{p}}_{Q,2} + \text{diag}(\mathbf{1} - \boldsymbol{\omega}_{Q,1})\boldsymbol{\Theta}^m_{11}\widehat{\mathbf{p}}_{Q,1}.$$

From this we can solve the change in prices as $\widehat{\mathbf{p}}_{Q,1} = \tilde{\boldsymbol{\Theta}}^m_{11}(\boldsymbol{\omega}_{Q,1}\widehat{w}_t(1) + \mathbf{A}_{12}\widehat{\mathbf{p}}_{Q,2})$.

The goods market clearing conditions are

$$\widehat{\mathbf{p}_{Q,1}\mathbf{q}_1} = \mathbf{F}_1 + \boldsymbol{\Psi}^m_{11}\widehat{\mathbf{p}_{Q,1}\mathbf{q}_1},$$

where the first term collects demand from consumers in both countries and intermediate input usage by sectors in Country 2. The second term reflects intermediate input usage by sectors in Country 1, taking the input shares as fixed. From the above equation, we can solve $\widehat{\mathbf{p}_{Q,1}\mathbf{q}_1} = \tilde{\boldsymbol{\Psi}}^m_{11}\mathbf{F}_1$. The s-th element of the vector $\mathbf{F}_1$ is

$$\begin{aligned}
F_{1s} = {} & \psi^f_{1s,1}\Big((1-\gamma_f)(\widehat{p}^Q_t(1,s) - \widehat{p}_t(1)) + \widehat{p_t(1)C_t}(1)\Big) \\
& + \sum_{k\in\mathcal{S}} \psi^m_{1s,1k}\Big[(1-\gamma_Q)(\widehat{p}^M_t(1,k) - \widehat{p}^Q_t(1,k)) + (1-\gamma_m)(\widehat{p}^Q_t(1,s) - \widehat{p}^M_t(1,k))\Big] \\
& + \psi^f_{1s,2}\Big((1-\gamma_f)(\widehat{p}^Q_t(1,s) - \widehat{p}_t(2)) + \widehat{p_t(2)C_t}(2)\Big) \\
& + \sum_{k\in\mathcal{S}} \psi^m_{1s,2k}\Big[(1-\gamma_Q)(\widehat{p}^M_t(2,k) - \widehat{p}^Q_t(2,k)) + (1-\gamma_m)(\widehat{p}^Q_t(1,s) - \widehat{p}^M_t(2,k)) + \widehat{p}^Q_t(2,k) + \widehat{Q}_t(2,k)\Big].
\end{aligned}$$

We can then write $\mathbf{F}_1$ in matrix format

$$\begin{aligned}
\mathbf{F}_1 = {} & (1-\gamma_f)\boldsymbol{\Psi}^f_{11} \circ (\widehat{\mathbf{p}}_{Q,1} - \widehat{p}_t(1)\mathbf{e}_S) + \boldsymbol{\Psi}^f_{11}\widehat{p_t(1)C_t}(1) + (1-\gamma_m)[\widehat{\mathbf{p}}_{Q,1} \circ (\boldsymbol{\Psi}^m_{11}\mathbf{e}_S) - \boldsymbol{\Psi}^m_{11}\widehat{\mathbf{p}}_{Q,1}] \\
& + (1-\gamma_f)\boldsymbol{\Psi}^f_{12} \circ (\widehat{\mathbf{p}}_{Q,1} - \widehat{p}_t(2)\mathbf{e}_S) + \boldsymbol{\Psi}^f_{12}\widehat{p_t(2)C_t}(2) + (1-\gamma_m)[\widehat{\mathbf{p}}_{Q,1} \circ (\boldsymbol{\Psi}^m_{12}\mathbf{e}_S) - \boldsymbol{\Psi}^m_{12}\widehat{\mathbf{p}}_{Q,2}] + \boldsymbol{\Psi}^m_{12}\widehat{\mathbf{p}_{Q,2}\mathbf{q}_2},
\end{aligned}$$

where the first and third terms are the changes in demand due to domestic and foreign consumers' substitution between goods from different sectors, and the second and fourth terms reflect changes in the share of the goods as intermediate inputs in each sector. The fifth term is caused by a change in foreign demand for intermediate inputs, taking the input shares as fixed.

According to the GHH preference, labor supply measured in wage bills can be expressed as

$$\widehat{w}_t(1) + \widehat{H}_t(1) = \widehat{w}_t(1) + \nu(\widehat{w}_t(1) - \widehat{p}_t(1)),$$

where $\widehat{p}_t(1) = \boldsymbol{\omega}^T_{C,11}\widehat{\mathbf{p}}_{Q,1} + \boldsymbol{\omega}^T_{C,21}\widehat{\mathbf{p}}_{Q,2}$ is the aggregate price index in Country 1. Substituting in the solution of $\widehat{\mathbf{p}}_{Q,1}$ obtained earlier, we have

$$\widehat{p}_t(1) = \underbrace{\boldsymbol{\omega}_{C,11}^T \tilde{\boldsymbol{\Theta}}_{11}^m \boldsymbol{\omega}_{Q,1}}_{\tilde{a}_{1L}^c} \widehat{w}_t(1) + \underbrace{(\boldsymbol{\omega}_{C,11}^T \tilde{\boldsymbol{\Theta}}_{11}^m \mathbf{A}_{12} + \boldsymbol{\omega}_{C,21}^T)}_{\text{sum} \equiv \tilde{a}_{21}^c} \widehat{\mathbf{p}}_{Q,2} = \tilde{a}_{1L}^c \widehat{w}_t(1) + \tilde{a}_{21}^c \widehat{p}_t^{CM}(1),$$

where we have used the definitions of $\tilde{a}_{1L}^c$, $\tilde{a}_{21}^c$ and $\widehat{p}_t^{CM}(1)$ and the fact that $\boldsymbol{\omega}_{C,11}^T \tilde{\boldsymbol{\Theta}}_{11}^m \boldsymbol{\omega}_{Q,1} + \left(\boldsymbol{\omega}_{C,11}^T \tilde{\boldsymbol{\Theta}}_{11}^m \mathbf{A}_{12} + \boldsymbol{\omega}_{C,21}^T\right) \mathbf{e}_S = 1$.

Meanwhile, labor demand measured in wage bills can be written as

$$\sum_s \frac{H_{ss}(1,s)}{H_{ss}(1)} \left(\frac{\widehat{w_t(1) H_t(1,s)}}{p_t^Q(1,s) Q_t(1,s)} + \widehat{p}_t^Q(1,s) + \widehat{Q}_t(1,s) \right) = (1-\gamma_Q) \boldsymbol{\Gamma}_1^w (\widehat{w}_t(1) \mathbf{e}_S - \widehat{\mathbf{p}}_{Q,1}) + \boldsymbol{\Gamma}_1^w \widehat{\mathbf{p}_{Q,1} \mathbf{q}_1},$$

where the first term in the parentheses in the left-hand side is the change in the value-added share in sector s.

Equating labor demand and supply and collecting terms, we obtain the solution to $\widehat{w}_t(1) - \widehat{p}_t^{CM}(1)$ stated in the theorem. ■

Due to log-linearization, the excess labor demand is linear in $\widehat{w}_t(1) - \widehat{p}_t^{CM}(1)$ and foreign shocks, and can be written as

$$D_1 = \nabla_{\widehat{\mathbf{p}}_{Q,2}} D_1 \cdot \widehat{\mathbf{p}}_{Q,2} + \nabla_{\widehat{\mathbf{q}}_2} D_1 \cdot \widehat{\mathbf{q}}_2 + \frac{\partial D_1}{\partial \widehat{C}_t(2)} \widehat{C}_t(2) - \frac{\partial D_1}{\partial \widehat{w}_t(1)} \left(\widehat{w}_t(1) - \widehat{p}_t^{CM}(1)\right).$$

The first three terms on the right-hand side are the direct effect of foreign shocks on D_1. Our proof verifies that they are equal to the value of terms in the curly brackets of equation (B-1). Setting the right-hand side to zero and we prove Proposition B-1. Therefore, the equilibrium changes in home wages, employment and real value added are all linear combinations of foreign shocks. The coefficients only depend on the elasticities and steady-state shares.

All four terms in the curly brackets of equation (B-1) have intuitive explanations. The first term is the direct effect of changing foreign prices on labor supply. The strength of this effect equals the change of the consumption-relevant import price index $\widehat{p}_t^{CM}(1)$ scaled by its weight in the consumer price index, $\tilde{a}_{21}^c$. The second term captures changes in demand due to foreign consumption and output changes. When foreign prices change, they affect home prices directly through input-output linkages. Changes in foreign and domestic prices will induce home and foreign consumers to switch between goods produced in the two countries. The third term captures such substitution effects, and their magnitude depends on the consumption elasticity γ_f. Finally, the last term captures changes in labor demand due to producer substitution between home and foreign inputs, and its strength depends on the elasticity of substitution between value added and intermediate inputs γ_Q.

These direct effects will be further moderated by the general equilibrium feedback. For example, higher labor demand tends to increase home wages and leads to higher home prices. This will then lead to consumers and producers to substitute away from home goods and reduce home labor demand. Such general equilibrium effects are captured by the coefficient Ω^{-1}.

Note that Proposition B-1 also applies to a one-sector model. All matrices in equation (B-1), including those implicit in Ω, will turn into scalars. The discrepancies in the coefficients between the one- and multi-sector models can cause "aggregation biases", despite the fact that the one-sector model does not have aggregation biases in the foreign country (see Lemma 1). In addition, the one-sector model has only one foreign price, so $\widehat{p}_t^{CM}(1) = \widehat{p}_t(2)$ and $\Delta\widehat{\mathbf{p}}_{Q,2} = 0$. As we illustrate below, such relationships do not necessarily hold in a multi-sector economy. This is another cause of aggregation biases.

We now prove a relationship that sheds light on the economic meanings of the coefficients $\tilde{a}_{1L}^c$ and $\tilde{a}_{21}^c$. It will also be used in later proofs.

Lemma B-2 *The coefficient $\tilde{a}_{1L}^c$ can be written as*

$$\tilde{a}_{1L}^c = \boldsymbol{\omega}_{C,11}^T \tilde{\boldsymbol{\Theta}}_{11}^m \boldsymbol{\omega}_{Q,1} = \boldsymbol{\Gamma}_1^w \tilde{\boldsymbol{\Psi}}_{11}^m \boldsymbol{\Psi}_{11}^f.$$

When imports from Country 2 are only used for final consumption, i.e., $\boldsymbol{\Theta}_{12}^m = 0$, the coefficient $\tilde{a}_{21}^c$ is the export-to-GDP ratio.

Proof. We need to prove the following relationship

$$\frac{\left(\boldsymbol{\lambda}_1 \circ \boldsymbol{\Psi}_{11}^f\right)^T}{\boldsymbol{\lambda}_1^T \boldsymbol{\omega}_{Q,1}} \left(\mathbf{I} - \operatorname{diag}(\mathbf{1} - \boldsymbol{\omega}_{Q,1}) \boldsymbol{\Theta}_{11}^m\right)^{-1} \boldsymbol{\omega}_{Q,1} = \frac{\left(\boldsymbol{\lambda}_1 \circ \boldsymbol{\omega}_{Q,1}\right)^T}{\boldsymbol{\lambda}_1^T \boldsymbol{\omega}_{Q,1}} \left(\mathbf{I} - \boldsymbol{\Psi}_{11}^m\right)^{-1} \boldsymbol{\Psi}_{11}^f,$$

where $\boldsymbol{\lambda}_1$ is a vector of sectoral output shares in Country 1.

Note that the denominators are GDP-to-output ratios. We can actually prove a stronger result

$$\left(\boldsymbol{\lambda}_1 \circ \boldsymbol{\Psi}_{11}^f\right)^T \left(\operatorname{diag}(\mathbf{1} - \boldsymbol{\omega}_{Q,1}) \boldsymbol{\Theta}_{11}^m\right)^n \boldsymbol{\omega}_{Q,1} = \left(\boldsymbol{\lambda}_1 \circ \boldsymbol{\omega}_{Q,1}\right)^T \left(\boldsymbol{\Psi}_{11}^m\right)^n \boldsymbol{\Psi}_{11}^f, \; n = 0, 1, 2, \ldots$$

This expression holds trivially when $n = 0$. When $n > 0$, notice that there is a tight relationship between $\text{diag}(\mathbf{1}-\boldsymbol{\omega}_{Q,1})\boldsymbol{\Theta}_{11}^m$ and $\boldsymbol{\Psi}_{11}^m$ as follows

$$\boldsymbol{\Psi}_{11}^m = \text{diag}(\boldsymbol{\lambda}_1)^{-1}\left(\text{diag}(\mathbf{1}-\boldsymbol{\omega}_{Q,1})\boldsymbol{\Theta}_{11}^m\right)^T \text{diag}(\boldsymbol{\lambda}_1).$$

Substituting this into the right-hand side and apply the fact that $\boldsymbol{\lambda}\circ\mathbf{x} = \text{diag}(\boldsymbol{\lambda}_1)\mathbf{x}$, as well as the fact that the two sides are scalars and to show $LHS = RHS$ is the same as to show $LHS^T = RHS$, we prove the equality.

When $\boldsymbol{\Theta}_{12}^m = \mathbf{A}_{12} = 0$, it is straightforward that $\tilde{a}_{21}^c = \boldsymbol{\omega}_{C,21}^T\mathbf{e}_S$, which equals the import-to-GDP (and export-to-GDP) share. ■

B.1 Example: no (foreign) inputs

In this section, we consider a special case of Proposition B-1 by shutting down trade in intermediate inputs. Mathematically, it requires $\boldsymbol{\Psi}_{12}^m = 0$ and $\boldsymbol{\Theta}_{12}^m = 0$. This also nests the case without any intermediate inputs, which is the case we consider in Proposition 1.[7] We can show that Proposition 1 holds exactly even when we allow for domestic inputs.

Proof of Proposition 1, allowing for domestic intermediate inputs.

When labor is the only input and no domestic or foreign intermediate inputs are used, we have $\boldsymbol{\Psi}^m = 0, \boldsymbol{\Theta}^m = 0, \tilde{\boldsymbol{\Theta}}_{ii}^m = \mathbf{I}, \tilde{\boldsymbol{\Psi}}_{ii}^m = \mathbf{I}, \boldsymbol{\omega}_{Q,1} = \mathbf{e}_S$. Applying the formula in Proposition B-1, we have

$$\begin{aligned}\widehat{w}_t(1) - \widehat{p}_t^{CM}(1) = &\left\{(1-\boldsymbol{\Gamma}_1^w\boldsymbol{\Psi}_{11}^f)(1+v\tilde{a}_{21}^c) - (1-\gamma_f)\boldsymbol{\Gamma}_1^w[\boldsymbol{\Psi}_{11}^f\circ(\boldsymbol{\omega}_{Q,1}-\tilde{a}_{1L}^c\mathbf{e}_S) + \boldsymbol{\Psi}_{12}^f\circ\boldsymbol{\omega}_{Q,1}]\right\}^{-1} \\ &\times\left\{-(1-\boldsymbol{\Gamma}_1^w\boldsymbol{\Psi}_{11}^f)\widehat{p}_t^{CM}(1) + (1-\gamma_f)\boldsymbol{\Gamma}_1^w\left[\boldsymbol{\Psi}_{12}^f(\widehat{p}_t^{CM}(1)-\widehat{p}_t(2))\right] + \boldsymbol{\Gamma}_1^w\boldsymbol{\Psi}_{12}^f p_t\widehat{(2)C_t}(2)\right\}\end{aligned}$$

In addition, we know that $\boldsymbol{\Theta}_{12}^m = \mathbf{A}_{12} = 0$. From Lemma B-2, we know that $\tilde{a}_{1L}^c = \boldsymbol{\Gamma}_1^w\boldsymbol{\Psi}_{11}^f, \tilde{a}_{21}^c = \boldsymbol{\Gamma}_1^w\boldsymbol{\Psi}_{12}^f$. $\tilde{a}_{21}^c$ is also the export-to-GDP ratio. We can then simplify the above formula and obtain the expression in Proposition 1. ■

Because there is no trade in intermediate inputs, all imports are used for final consumption and their prices are "consumption-relevant". Recall that $\tilde{a}_{21}^C$ captures how the import price index affects the consumer price index. In a model without imported intermediate inputs, the coefficient equals the ratio of total imports to consumption, which is also the export-to-GDP ratio because of trade balance.

A perhaps surprising result is that the input-output structure and the elasticity between labor and inputs, γ_Q, are *irrelevant* to Country 1's aggregate response to the shocks. One may think

[7] When labor is the only input and no domestic or foreign intermediate inputs are used, we have $\boldsymbol{\Psi}^m = 0, \boldsymbol{\Theta}^m = 0, \tilde{\boldsymbol{\Theta}}_{ii}^m = \mathbf{I}, \tilde{\boldsymbol{\Psi}}_{ii}^m = \mathbf{I}, \boldsymbol{\omega}_{Q,1} = \mathbf{e}_S$.

that the existence of locally produced intermediate inputs allows firms to substitute between labor and intermediate inputs, which may further affect labor demand. However, unlike the case with imported intermediate inputs, in which domestic goods prices are directly affected, domestic goods prices here are only affected by domestic wages. Any change in home wages will affect both the intermediate input prices and the labor costs. Given that all inputs are sourced locally and the total requirement of labor is one in all sectors, domestic goods prices move one-for-one with the wage. Therefore, there is no substitution between labor and domestic inputs, and the final consumption shares of different sectors remain unchanged.

The formula (21) in Proposition 1 applies to both the one- and multi-sector models. We know from Lemma 1 that calibrating a one-sector model to Country 2 does not generate any bias in aggregate variables such as $\widehat{p}_t(2)$ and $\widehat{C}_t(2)$. In addition, $\widetilde{a}^C_{21}$ is calibrated to the same export-to-GDP ratio in the data, whether we assume one or multiple sectors. Given the same elasticities ν and γ_f, the only term that may lead to a discrepancy in the prediction of real GDP is the import price index, $\widehat{p}^{CM}_t(1)$. We can therefore prove Corollary 1 as follows:

Proof of Corollary 1.

Price changes are symmetric across sectors: $\widehat{p}_t(2,s) = \widehat{p}_t(2,1), \forall s$. When Country 2 only uses labor in production, this condition is equivalent to same productivity shocks across sectors, i.e., $\widehat{A}_t(2,s) = \widehat{A}_t(2,1), \forall s$.

The corollary is a direct result of Proposition 1. To obtain the predicted change in real value added in the one-sector model $\widehat{V}_t(1)^{one-sec}$, we simply set the change in import price $\widehat{p}^{CM}_t(1)^{one-sec}$ to the change in Country 2's price index, $\widehat{p}_t(2)$, because there is only one tradable sector. The term $\widehat{p}^{CM}_t(1)^{one-sec} - \widehat{p}_t(2)$ can be expressed as

$$\begin{aligned}\widehat{p}^{CM}_t(1) - \widehat{p}_t(2) &= -\sum_s \left[\omega_C(2,(2,s)) - \tilde{\omega}_C(1,(2,s))\right] \widehat{p}_t(2,s)\\ &= \sum_s \left[\omega_C(2,(2,s)) - \tilde{\omega}_C(1,(2,s))\right] \widehat{A}_t(2,s).\end{aligned}$$

The first equality holds even when we allow for domestic intermediate inputs. The second equality holds only when there are no intermediate inputs, because we have applied the relationship $\widehat{p}_t(2,s) = -\widehat{A}_t(2,s)$.

It is clear from the above equation that $\widehat{p}^{CM}_t(1) - \widehat{p}_t(2)$ is zero when either condition (1) or (2) in the corollary holds. Under either of these two conditions, the one-sector model predicts the same change in real value added as the multi-sector model. ■

B.2 Example: all intermediate inputs are imported

In this section, we consider a special case of Proposition B-1 by allowing Country 1 to import foreign intermediate inputs but assuming away domestic inputs. We also assume home goods are not used as intermediate inputs in the foreign country to make the expressions simpler. Mathematically, we have the following conditions: $\mathbf{\Theta}_{11}^{m}=0, \mathbf{\Psi}_{11}^{m}=0, \mathbf{\Psi}_{12}^{m}=0$.

Applying these to Proposition B-1, we have

Proposition B-2 *Country 1's real value added can be expressed as*

$$\widehat{V}_t(1)=\frac{\nu\widetilde{a}_{21}^{C}}{\Omega}\left(-\widetilde{a}_{21}^{C}\widehat{p}_t^{CM}(1)+(1-\gamma_f)\,\widetilde{a}_{21}^{C}\left(\widehat{p}_t^{CM}(1)-\widehat{p}_t(2)\right)-(\gamma_f-\gamma_Q)\,\mathbf{\Gamma}_1^{w}\mathbf{\Theta}_{12}\Delta\hat{\mathbf{p}}_{Q,2}+\widetilde{a}_{21}^{C}\left(\widehat{p}_t(2)+\widehat{C}_t(2)\right)\right), \tag{B-1}$$

where Ω *can be written as*

$$\Omega=\widetilde{a}_{21}^{C}\left(1+v\widetilde{a}_{21}^{C}\right)-(1-\gamma_f)\left[\bar{\omega}_{Q,1}-\left(\widetilde{a}_{1L}^{C}\right)^2\right]-(1-\gamma_Q)\left[1-\bar{\omega}_{Q,1}\right]+(\gamma_f-\gamma_Q)\sum_j\frac{\lambda(1,s)\left(\Delta\omega_Q(1,s)\right)^2}{\bar{\omega}_{Q,1}},$$

and $\bar{\omega}_{Q,1}$ *is defined to be the aggregate value added share in gross output,* $\Delta\omega_Q(1,s)$ *is the sector-specific valued added share relative to the aggregate share and* $\lambda(1,s)$ *is the gross output share of sector* s *in Country 1.*

Compared with the case without foreign intermediate inputs, there is an additional term in the direct impact of foreign shocks, $-(\gamma_f-\gamma_Q)\,\mathbf{\Gamma}_1^{w}\mathbf{\Theta}_{12}\Delta\hat{\mathbf{p}}_{Q,2}$. This term reflects the heterogeneous impact of foreign price shocks on domestic sectoral output, and the resulted heterogeneous changes in domestic prices lead to reallocation of labor demand across sectors due to consumer and producer substitutions. It is the combined effects of the third and fourth terms in equation (B-1). These two effects are exactly canceled out if the two elasticities are the same, $\gamma_f=\gamma_Q$.

Similar to the first example, a one-sector model can predict biased changes in real value added because the actual consumption-relevant import price index differs from the overall price change in Country 2. Moreover, the one-sector model misses the term $-(\gamma_f-\gamma_Q)\,\mathbf{\Gamma}_1^{w}\mathbf{\Theta}_{12}\Delta\hat{\mathbf{p}}_{Q,2}$ because there is one output price in Country 2.

Nevertheless, even when all foreign prices have the same change, the one-sector model may still predict a biased change because it uses incorrect coefficients before the shocks in equation (B-1). First, the last term in Ω, $(\gamma_f-\gamma_Q)\sum_j\frac{\lambda(1,s)\left(\Delta\omega_Q(1,s)\right)^2}{\bar{\omega}_{Q,1}}$, relates to the variance of value added shares across sectors and is positive unless all sectors have the same share. To understand why the variance matters, recall that Ω is the derivative of the excess labor demand with respect to $\widehat{w}_t(1)$. When there is more variation in $\omega_Q(1,s)$, a rise in wage induces a larger dispersion in the changes of the output prices and thus stronger substitution by consumers toward low $\omega_Q(1,s)$ sectors. However, such sectors are also more likely to hire more labor than other industries. This means that the general equilibrium effects that dampen wage changes will be stronger. Similarly, for an industry with a larger $\omega_Q(1,s)$, a rise in wage induces a larger change in output prices but a smaller reduction in value added shares, the magnitude of which is governed by the parameter $\gamma_Q(1,s)$. Given that such an industry is also more likely to hire more labor, a larger dispersion in $\omega_Q(1,s)$ will lead to a smaller dampening effect of the substitution between labor and inputs. This variance term, however, disappears in the one-sector model and becomes a source of aggregation bias.

The second and last source of incorrect coefficients is the parameter $\widetilde{a}_{21}^C$ (or equivalently, $\widetilde{a}_{1L}$). To see this, consider the expression of $\widetilde{a}_{1L}^C$

$$\widetilde{a}_{1L}^C = \boldsymbol{\omega}_{C,11}^T \boldsymbol{\omega}_{Q,1} = \sum_s \omega_C(1,(1,s))\omega_Q(1,s).$$

In the corresponding one-sector model, we calibrate the sectoral value added share in gross output to the aggregate share in the multi-sector model, $\bar{\omega}_{Q,1}$, so in the one-sector model, this coefficient equals $\bar{a}_{1L}^C \equiv \bar{\omega}_{Q,1}\sum_s \omega_C(1,(1,s))$, the ratio of consumption of domestic goods to gross output. In general, $\bar{a}_{1L}^C \neq \widetilde{a}_{1L}^C$. The two coincide when there is no heterogeneity in the value-added shares across sectors.

C Model-based decomposition of the bias with input-output linkages

In this section, we offer a decomposition of the aggregation bias based on the equation (B-1). We denote the predicted change in real GDP in the multi- and one-sector model as $\widehat{V}_t^{ms}(1)$ and $\widehat{V}_t^{os}(1)$, respectively. Given the shocks $\widehat{A}_t(2,s)$, we can calculate $\widehat{\mathbf{p}}_{Q,2}, \widehat{\mathbf{q}}_2, \widehat{C}_t(2)$ using the multi-sector model formula. Substituting these into equation (B-1) and the associated change in real GDP, we obtain $\widehat{V}_t^{ms}(1)$. We start from $\widehat{V}_t^{ms}(1)$ and gradually move to $\widehat{V}_t^{os}(1)$.

0. start from the multi-sector prediction

$$\widehat{V}_t^0(1) \equiv \widehat{V}_t^{ms}(1).$$

1. replace $\widehat{p}_t^{CM}(1)$ with $\widehat{p}_t(2)$ in the direct effect on labor supply

2. replace $\widehat{p}_t^{CM}(1)$ with $\widehat{p}_t(2)$ in the demand due to (foreign) consumer substitution

3. drop the term with $\Delta\widehat{\mathbf{p}}_{Q,2}$ in the demand due to consumer substitution

4. drop the term due to producer substitution because $\Delta\widehat{\mathbf{p}}_{Q,2} = 0$

5. The second term in the multi-sector model, demand due to foreign consumption and output changes, can be rewritten as

$$\mathbf{\Gamma}_1^w \tilde{\mathbf{\Psi}}_{11}^m \left(\mathbf{\Psi}_{12}^f \widehat{p_t(2)C_t}(2) + \mathbf{\Psi}_{12}^m \widehat{\mathbf{p}_{Q,2}\mathbf{q}_2} \right) = \mathbf{\Gamma}_1^w \tilde{\mathbf{\Psi}}_{11}^m \left(\mathbf{\Psi}_{12}^f + \mathbf{\Psi}_{12}^m (\mathbf{I} - \mathbf{\Psi}_{22}^m)^{-1} \mathbf{\Psi}_{22}^f \right) \widehat{p_t(2)C_t}(2).$$

   The one sector model can replicate $\widehat{p_t(2)C_t}(2)$, but has different coefficients, which we write in terms of scalar instead of matrix operations: $\Gamma_1^w \tilde{\Psi}_{11}^m \left(\Psi_{12}^f + \Psi_{12}^m (1 - \Psi_{22}^m)^{-1} \Psi_{22}^f \right)$. Replacing the scalar in the multi-sector model with the one in the one-sector model, we complete this step.

6. We change $\widetilde{a}_{21}^{C,ms}$ to $\widetilde{a}_{21}^{C,os}$.

7. We change Ω^{ms} to Ω^{os}, and finally obtain $\widehat{V}_t^{os}(1)$.

Note that when $\gamma_f = \gamma_Q = 1$, steps 2, 3 and 4 do not change the predicted real GDP.

We use the estimated sectoral shocks from Section 4 to carry out this decomposition. Similar to the methods used in Figure 6, we use the estimated sectoral shocks to first compute the predicted change in real GDP in the multi-sector model in each country and year, and then carry out the above steps to gradually move to the one-sector model. For each step, we run a regression of the predicted change in real GDP on that from the multi-sector baseline to obtain the regression coefficient b. The bias at each step is then calculated as $100 \times (b - 1)$.

In Table C-3, we report the results of this decomposition for different values of γ_f and γ_Q. The first row shows that the aggregation bias, calculated in this way, is -19.9% (i.e., the one-sector model underpredicts the change in real GDP by 19.9% on average) when $\gamma_f = 1.0$ and $\gamma_Q = 1.0$. This is quite close to the bias (-25.4%) reported in Figure 6, suggesting that the simplified model in Section B does not miss important mechanisms driving the aggregation bias in the full model with capital and other ingredients. Notice that the first step, simply replacing the consumption-relevant import price index with the overall foreign price index, already results in an aggregation bias of -28.6%, which accounts for the majority of the total bias. Though numbers vary with different elasticities, this pattern is quite general and holds for all combinations of γ_f and γ_Q reported in the table. We therefore conclude that the most important source of aggregation bias is the mis-measurement of the consumption-relevant import price index, as emphasized throughout the paper.

Table C-3: Decomposition of the biases (% deviation using regression coefficients $100 \times (b-1)$) of alternative models (from multi-sector to one-sector) from the multi-sector model

		Steps							
γ_f	γ_Q	0	1	2	3	4	5	6	7
1.0	1.0	0	-28.6	-28.6	-28.6	-28.6	-22.4	-15.1	-19.9
2.0	1.0	0	-24.5	-32.6	-30.3	-30.3	-33.5	-27.0	-26.9
0.5	1.0	0	-31.3	-26.1	-27.6	-27.6	-15.6	-8.0	-16.7
1.0	0.5	0	-23.3	-23.3	-23.3	-29.7	-22.6	-16.3	-27.6
0.5	0.5	0	-26.0	-21.7	-22.9	-30.0	-17.9	-12.0	-30.4

Notes. Step 0 is the multi-sector model, Step 7 is the one-sector model. Steps 2 and 3 are consumer substitution and do not change the biases if $\gamma_f = 1$. Step 4 is producer substitution, and does not change the biases if $\gamma_Q = 1$.