Online Appendix for "Uncertainty, Imperfect Information, and Expectation Formation over the Firm's Life Cycle"

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June, 2022

Abstract

In the online appendix, we first provide robustness checks for the stylized facts documented in the paper. Second, we provide a theory appendix that considers alternative setups of the model and their implications on the forecast errors. In particular, we show that the perfect information benchmark and a Jovanovic-type learning model with payoff-relevant noises cannot yield serially correlated forecast errors. Finally, we present additional results from our quantitative analysis.

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1 Additional Empirical Analysis

1.1 Basic Data Description

Our data include firms belonging to Japanese multinational corporations (MNCs) in various countries and industries in 1995-2014. Our baseline regression sample requires a well-defined forecast error (FE) from period t to $t+1$. In this section, we report firm-level statistics in periods when firms make forecasts, therefore up to the year $t = 2013$. In Table OA.1, we report the average number of business groups (groups of firms belonging to the same parent firm) and number of firms in a typical year in four periods, 1995-2000, 2001-2005, 2006-2010 and 2011-2013. These numbers gradually increase over time, while the average/median firm size measured by employment remains stable. On average, we have 6922 firms belonging to 1781 unique parent firms in a typical year during the entire sample period.

Table OA.1: Descriptive Statistics by Time Periods

	Annual Average # of		Employment Statistics			
Year range	Business Groups	Firms	Mean	25th Perc.	50th Perc.	75th Perc.
1995-2000	1059	4701	300.1	20	75	248
2001-2005	1503	6612	335.5	21	79	270
2006-2010	2244	8295	333.6	19	71	244
2011-2013	2919	9592	297.8	16	62	218
1995-2013	1781	6922	319.0	19	71	247

Notes: This table reports the average number of firms/business groups in our baseline regression sample and the corresponding employment statistics in each period.

In Table OA.2, we report number of firms in major countries/regions in 2013. The countries/regions are consistent with our regional analysis in Section 5.3.3 of the paper. A large number of firms in our sample are in major markets of Japanese MNCs such as China, ASEAN countries and the United States. A small number of firms operate in regions such as Africa, Middle East and Eastern Europe. For the list of countries in each region, see Table OA.17.

Table OA.3 reports the number of firms in the top 10 industries in 2013. Our data contains both manufacturing and services firms. Not surprisingly, the industry that contains the largest number of firms is "wholesale and retail trade", followed by "manufacturing of transportation equipment", an industry that is well-known for Japanese firms' overseas footprint. It is clear from the table that

Table OA.2: Number of Firms in Major Countries/Regions, 2013

Major Country/Region	# of Firms
Africa	41
Middle East	70
Eastern Europe	142
Latin America	307
ASEAN	2556
China	3430
Western Europe	920
United States	1287

Notes: This table reports the number of firms in major countries/industries in 2013. See Table OA.17 for the list of countries in each region.

our sample covers a wide range of industries. For all our key facts, we show that they hold in both the whole sample and the manufacturing subsample.

Table OA.3: Number of Firms in Top 10 Industries, 2013

Industry	# of Firms
Wholesale and retail trade	3001
Transportation equipment	1119
Miscellaneous manufacturing industries	622
Other Business Services	611
Chemical and allied products	547
Information and communications equipment	496
Transport	434
Production machinery	385
Electrical machinery, equipment and supplies	347
Information and communications	331

Notes: This table reports the number of firms in the top 10 industries in 2013.

1.2 Alternative Definitions of Forecast Errors and Summary Statistics

We introduce two alternative definitions of forecast errors, which are used for robustness checks later.

First, we define the percentage deviation of the realized sales from the sales forecasts as

$$FE_{t,t+1}^{\text{pct}} = \frac{R_{t+1}}{E_t\left(R_{t+1}\right)} - 1.$$

Second, we construct a measure for the "residual forecast error" measure in an effort to isolate the firm-level idiosyncratic components reflected in the forecast errors. To exclude systemic components, such as business cycles, from the forecast errors, we project the raw forecast error onto country-year and industry-year fixed effects

$$FE_{t,t+1}^{\log} = \delta_{ct} + \delta_{st} + \hat{\epsilon}_{t,t+1}^{FE,\log}, \tag{1}$$

and obtain the residual forecast error $\hat{\epsilon}_{t,t+1}^{FE,\log}$. As it turns out, the fixed effects only account for about 11% of the variation, which indicates that firm-level uncertainty plays a dominant role in generating the firms' forecast errors. We obtain $\hat{\epsilon}_{t,t+1}^{FE,\text{pct}}$ based on the percentage forecast errors for additional robustness checks using the same approach.

The first four rows of Table OA.4 report summary statistics of our main forecast error definition (log deviation, raw) as well as the alternative forecast errors. While the mean of the residual forecast errors, $\hat{\epsilon}_{t,t+1}^{FE,\log}$ and $\hat{\epsilon}_{t,t+1}^{FE,\text{pct}}$, is zero by construction, the mean and median of $FE_{t,t+1}^{\log}$ and $FE_{t,t+1}^{\text{pct}}$ are also close to zero. In the middle four rows, we report the summary statistics of the absolute value of various constructed forecast errors. Since the country-year and industry-year fixed effects account for a small fraction of the variation, the mean, median, and standard deviation of $|\hat{\epsilon}_{t,t+1}^{FE,\log}|$ (and $|\hat{\epsilon}_{t,t+1}^{FE,\text{pct}}|$) are similar to those of $\left|FE_{t,t+1}^{\log}\right|$ (and $\left|FE_{t,t+1}^{\text{pct}}\right|$). The patterns of manufacturing firms' forecast errors are similar to the overall patterns, as shown by the last four rows of the table.

Table OA.4: Summary statistics of the forecast errors

	Obs.	mean	std. dev.	median
$FE^{\text{log}}_{t,t+1}$	131834	-0.024	0.298	-0.005
$FE^{\text{pct}}_{t,t+1}$	132373	0.017	0.332	-0.006
$\hat{\epsilon}^{FE,\text{log}}_{t,t+1}$	131550	-0.000	0.280	0.011
$\hat{\epsilon}^{FE,\text{pct}}_{t,t+1}$	132090	0.000	0.314	-0.022
$\lvert FE^{\text{log}}_{t,t+1}\rvert$	131834	0.200	0.222	0.130
$\lvert FE^{\text{pct}}_{t,t+1}\rvert$	132373	0.203	0.263	0.130
$\lvert\hat{\epsilon}^{FE,\text{log}}_{t,t+1}\rvert$	131550	0.184	0.211	0.115
$\lvert\hat{\epsilon}^{FE,\text{pct}}_{t,t+1}\rvert$	132090	0.189	0.251	0.117
$FE^{\text{log}}_{t,t+1}$ - Manufacturing	80987	-0.022	0.278	-0.004
$FE^{\text{pct}}_{t,t+1}$ - Manufacturing	81244	0.014	0.307	-0.004
$\lvert FE^{\text{log}}_{t,t+1}\rvert$ - Manufacturing	80987	0.186	0.208	0.123
$\lvert FE^{\text{pct}}_{t,t+1}\rvert$ - Manufacturing	81244	0.188	0.242	0.124

Notes: $FE^{\text{log}}_{t,t+1}$ is the log deviation of the realized sales from the sales forecasts, while $FE^{\text{pct}}_{t,t+1}$ is the percentage deviation of the realized sales from the sales forecasts. $\hat{\epsilon}^{FE,\text{log}}_{t,t+1}$ is the residual log forecast error, which we obtain by regressing $FE^{\text{log}}_{t,t+1}$ on a set of industry-year and country-year fixed effects. Similarly, $\hat{\epsilon}^{FE,\text{pct}}_{t,t+1}$ is the residual percentage forecast error, which we obtain by regressing $FE^{\text{pct}}_{t,t+1}$ on a set of industry-year and country-year fixed effects.

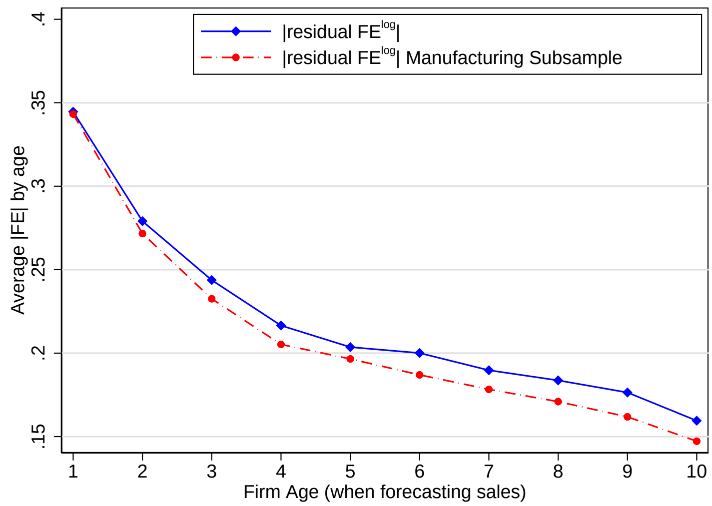
1.3 Robustness Checks for Fact 1: Affiliate Age on Uncertainty

1.3.1 Alternative Measures of FE

We first show that our baseline results in Figure 2 and Table 3 of the paper are robust to alternative measures of forecast errors. Figure OA.1 plots the average absolute value of the residual forecast errors $\hat{\epsilon}^{FE,\text{log}}_{t,t+1}$, for the entire sample and for the manufacturing subsample, respectively. We see a clear pattern that older firms make more precise forecasts.

Tables OA.5 and OA.6 use the absolute value of percentage forecast errors and residual log forecast errors, respectively.

Figure OA.1: $|\hat{\epsilon}_{t,t+1}^{FE,\log}|$ declines with firm age



Note: Average absolute value of residual $FE^{\log}$ by age cohorts.

Table OA.5: Age effects on the absolute percentage forecast errors, $|FE_{t,t+1}^{\text{pct}}|$

Sample:	All Firms			Survivors	Manufacturing
Dep.Var: $\lvert FE_{t,t+1}^{\text{pct}}\rvert$	(1)	(2)	(3)	(4)	(5)
$\mathbb{1}(\text{Age}_t = 2)$	-0.068^a	-0.061^a	-0.066^a	-0.065^a	-0.063^a
	(0.008)	(0.008)	(0.009)	(0.012)	(0.012)
$\mathbb{1}(\text{Age}_t = 3)$	-0.104^a	-0.091^a	-0.090^a	-0.086^a	-0.088^a
	(0.008)	(0.008)	(0.009)	(0.012)	(0.011)
$\mathbb{1}(\text{Age}_t = 4)$	-0.131^a	-0.116^a	-0.112^a	-0.101^a	-0.114^a
	(0.009)	(0.009)	(0.009)	(0.013)	(0.012)
$\mathbb{1}(\text{Age}_t = 5)$	-0.142^a	-0.125^a	-0.115^a	-0.110^a	-0.108^a
	(0.008)	(0.008)	(0.009)	(0.015)	(0.012)
$\mathbb{1}(\text{Age}_t = 6)$	-0.145^a	-0.126^a	-0.116^a	-0.114^a	-0.116^a
	(0.008)	(0.008)	(0.009)	(0.015)	(0.012)
$\mathbb{1}(\text{Age}_t = 7)$	-0.157^a	-0.135^a	-0.122^a	-0.131^a	-0.121^a
	(0.008)	(0.008)	(0.010)	(0.016)	(0.012)
$\mathbb{1}(\text{Age}_t = 8)$	-0.159^a	-0.135^a	-0.120^a	-0.117^a	-0.121^a
	(0.008)	(0.008)	(0.010)	(0.018)	(0.012)
$\mathbb{1}(\text{Age}_t = 9)$	-0.161^a	-0.136^a	-0.120^a	-0.118^a	-0.125^a
	(0.009)	(0.009)	(0.010)	(0.020)	(0.012)
$\mathbb{1}(\text{Age}_t \geq 10)$	-0.175^a	-0.139^a	-0.120^a	-0.122^a	-0.119^a
	(0.008)	(0.008)	(0.010)	(0.022)	(0.013)
$\log(\text{Emp})_t$		-0.022^a	-0.033^a	-0.043^a	-0.032^a
		(0.001)	(0.003)	(0.006)	(0.003)
$\log(\text{Parent Emp})_t$		-0.000	0.000	0.005	0.000
		(0.001)	(0.003)	(0.007)	(0.003)
Industry-year FE	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y
Firm FE			Y	Y	Y
N	131757	128931	123609	22090	77062
R^2	0.094	0.110	0.339	0.318	0.338

Notes: Standard errors are clustered at the business group level, c 0.10 b 0.05 a 0.01. The dependent variable is the absolute value of forecast errors in all regressions. Age is the age of the firm when making the forecasts. Regressions in columns 1, 2 and 3 include all firms, while the regression in column 4 only includes firms that have continuously appeared in the sample from age 1 to age 7.

Table OA.6: Age effects on the absolute residual log forecast errors, $|\hat{\epsilon}^{\log}_{FE,t,t+1}|$

Sample:	All Firms			Survivors	Manufacturing
Dep.Var: $|\hat{\epsilon}^{\log}_{FE,t,t+1}|$	(1)	(2)	(3)	(4)	(5)
$\mathbb{1}(\text{Age}_t = 2)$	-0.066[a]	-0.059[a]	-0.065[a]	-0.073[a]	-0.071[a]
	(0.007)	(0.007)	(0.007)	(0.009)	(0.011)
$\mathbb{1}(\text{Age}_t = 3)$	-0.100[a]	-0.087[a]	-0.087[a]	-0.093[a]	-0.098[a]
	(0.007)	(0.007)	(0.007)	(0.010)	(0.010)
$\mathbb{1}(\text{Age}_t = 4)$	-0.126[a]	-0.111[a]	-0.110[a]	-0.110[a]	-0.124[a]
	(0.007)	(0.007)	(0.008)	(0.011)	(0.011)
$\mathbb{1}(\text{Age}_t = 5)$	-0.138[a]	-0.121[a]	-0.115[a]	-0.121[a]	-0.124[a]
	(0.007)	(0.007)	(0.008)	(0.012)	(0.011)
$\mathbb{1}(\text{Age}_t = 6)$	-0.141[a]	-0.123[a]	-0.115[a]	-0.123[a]	-0.127[a]
	(0.007)	(0.007)	(0.008)	(0.012)	(0.011)
$\mathbb{1}(\text{Age}_t = 7)$	-0.151[a]	-0.129[a]	-0.122[a]	-0.135[a]	-0.134[a]
	(0.007)	(0.007)	(0.008)	(0.013)	(0.011)
$\mathbb{1}(\text{Age}_t = 8)$	-0.155[a]	-0.132[a]	-0.122[a]	-0.128[a]	-0.136[a]
	(0.007)	(0.007)	(0.008)	(0.015)	(0.011)
$\mathbb{1}(\text{Age}_t = 9)$	-0.161[a]	-0.136[a]	-0.127[a]	-0.130[a]	-0.141[a]
	(0.007)	(0.007)	(0.008)	(0.017)	(0.011)
$\mathbb{1}(\text{Age}_t \geq 10)$	-0.173[a]	-0.138[a]	-0.125[a]	-0.132[a]	-0.137[a]
	(0.007)	(0.006)	(0.008)	(0.018)	(0.011)
$\log(\text{Emp})_t$		-0.022[a]	-0.027[a]	-0.037[a]	-0.027[a]
		(0.001)	(0.002)	(0.005)	(0.002)
$\log(\text{Parent Emp})_t$		-0.000	0.001	0.011	0.001
		(0.001)	(0.003)	(0.007)	(0.003)
Industry-year FE	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y
Firm FE			Y	Y	Y
N	131230	128429	123111	21982	76823
R^2	0.082	0.104	0.361	0.365	0.352

Notes: Standard errors are clustered at the business group level, c 0.10 b 0.05 a 0.01. The dependent variable is the absolute value of forecast errors in all regressions. Age is the age of the firm when making the forecasts. Regressions in columns 1, 2 and 3 include all firms, while the regression in column 4 only includes firms that have continuously appeared in the sample from age 1 to age 7.

1.3.2 Conditional Variance: a Two-step Approach

Second, we address the concern that the decline in $|FE|$ may reflect a reduction in firms' biases in the level of FEs rather than a reduction in the variance of FEs. We do so by characterizing the conditional variance of FEs using a two step procedure and test whether it depends on the firm's age. To derive this, we first assume that the conditional expectation of forecast errors is linear in the independent variables (including fixed effects)

$$E(FE|X) = \beta X.$$

Therefore, the conditional variance becomes

$$V(FE|X) = E((FE - \beta X)^2|X).$$

To test whether $V(FE|X)$ depends on firm age and other independent variables, we first regress FE on all the regressors and obtain the squared residual term:

$$\hat{\upsilon}_{FE}^2 \equiv (FE - \hat{\beta}X)^2.$$

We then project $\hat{\upsilon}_{FE}^2$ onto X in the second-stage regression.[1] When we include firm age as an independent variable, the coefficient of age in the second-stage regression is informative about whether the variance of firm-level forecast errors is affected by firm age. One can test other potential determinants of the variance in the same way.

In Table OA.7, we perform the two-step procedure, using the log forecast error as the key dependent variable (FE in the derivation above). Though the age coefficients here are not directly comparable to regressions with absolute forecast errors as the dependent variable, this procedure reveals similar patterns as Table 3 of the paper: firm-level uncertainty declines as firms gain more experience. In Column 5, we define forecast errors using percentage deviations, and the effects of age on conditional variance of these errors are similar to that in Column 2 where we use the log

[1] We use $\hat{\upsilon}$ to denote the residual term here to distinguish from the residual forecast errors defined in equation 1. The latter is obtained by purging the country-year and industry-year fixed effects only, while the former purges all regressors that we believe may affect the conditional variance of forecast errors, including the age dummies and other controls.

forecast errors.

Table OA.7: Age effects on the variance forecast errors: conditional variance regressions

Dep. Var.	$\hat{v}^2_{FE,\log}(t,t+1)$				$\hat{v}^2_{FE,\text{pct}}(t,t+1)$
Sample:	All Firms		Survivors	Manufacturing	All Firms
	(1)	(2)	(3)	(4)	(5)
$\mathbb{1}(\text{Age}_t = 2)$	-0.063^a	-0.036^a	-0.047^a	-0.037^a	-0.043^a
	(0.008)	(0.006)	(0.008)	(0.008)	(0.012)
$\mathbb{1}(\text{Age}_t = 3)$	-0.094^a	-0.052^a	-0.057^a	-0.056^a	-0.064^a
	(0.008)	(0.006)	(0.008)	(0.008)	(0.012)
$\mathbb{1}(\text{Age}_t = 4)$	-0.118^a	-0.066^a	-0.068^a	-0.071^a	-0.082^a
	(0.008)	(0.006)	(0.009)	(0.008)	(0.012)
$\mathbb{1}(\text{Age}_t = 5)$	-0.124^a	-0.068^a	-0.073^a	-0.070^a	-0.081^a
	(0.008)	(0.006)	(0.010)	(0.008)	(0.013)
$\mathbb{1}(\text{Age}_t = 6)$	-0.125^a	-0.069^a	-0.074^a	-0.072^a	-0.086^a
	(0.008)	(0.006)	(0.010)	(0.009)	(0.013)
$\mathbb{1}(\text{Age}_t = 7)$	-0.131^a	-0.073^a	-0.081^a	-0.077^a	-0.092^a
	(0.008)	(0.006)	(0.011)	(0.009)	(0.013)
$\mathbb{1}(\text{Age}_t = 8)$	-0.131^a	-0.071^a	-0.072^a	-0.077^a	-0.083^a
	(0.008)	(0.006)	(0.012)	(0.009)	(0.013)
$\mathbb{1}(\text{Age}_t = 9)$	-0.134^a	-0.074^a	-0.073^a	-0.081^a	-0.083^a
	(0.008)	(0.006)	(0.013)	(0.009)	(0.014)
$\mathbb{1}(\text{Age}_t \geq 10)$	-0.135^a	-0.072^a	-0.080^a	-0.077^a	-0.082^a
	(0.007)	(0.006)	(0.014)	(0.009)	(0.013)
$\log(\text{Emp})_t$	-0.017^a	-0.016^a	-0.022^a	-0.015^a	-0.028^a
	(0.001)	(0.002)	(0.004)	(0.002)	(0.004)
$\log(\text{Parent Emp})_t$	0.001	0.002	0.004	0.002	-0.001
	(0.001)	(0.002)	(0.007)	(0.002)	(0.003)
Industry-year FE	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y
Firm FE		Y	Y	Y	Y
N	128429	123111	21982	76823	123609
R^2	0.071	0.317	0.307	0.300	0.261

Notes: Standard errors are clustered at the business group level. Significance levels: c 0.1, b 0.05, a 0.01. Age is the age of the firm when making the forecasts. Regressions in columns 1, 2 and 5 include all firms. Column 3 includes firms that continuously show up in the data from age one to age seven. Column 4 focuses on the manufacturing subsample.

1.3.3 Excluding Naive Forecasts

Third, we show that our results are not driven by firms that use simple forecasting rules. In our data, about 3.4% of the firms use their current sales as their sales forecasts for the next year. Though it is impossible to gauge what fraction of these firms misreport their forecasts, we try to be conservative and drop all of them from our dataset and run the regressions in Table 3 of the paper. The results are almost identical (see Table OA.8).

Table OA.8: Age effects on the absolute value of forecast errors: no naive forecasting rule

Sample:	All Firms			Survivors	Manufacturing
Dep.Var: $\lvert FE^{\log}_{t,t+1}\rvert$	(1)	(2)	(3)	(4)	(5)
$\mathbb{1}(\text{Age}_t = 2)$	-0.068^a	-0.061^a	-0.064^a	-0.071^a	-0.072^a
	(0.007)	(0.007)	(0.008)	(0.010)	(0.011)
$\mathbb{1}(\text{Age}_t = 3)$	-0.102^a	-0.088^a	-0.087^a	-0.095^a	-0.103^a
	(0.007)	(0.007)	(0.008)	(0.010)	(0.011)
$\mathbb{1}(\text{Age}_t = 4)$	-0.126^a	-0.112^a	-0.108^a	-0.108^a	-0.124^a
	(0.007)	(0.007)	(0.008)	(0.011)	(0.011)
$\mathbb{1}(\text{Age}_t = 5)$	-0.142^a	-0.124^a	-0.115^a	-0.122^a	-0.127^a
	(0.007)	(0.007)	(0.008)	(0.012)	(0.011)
$\mathbb{1}(\text{Age}_t = 6)$	-0.143^a	-0.124^a	-0.114^a	-0.123^a	-0.131^a
	(0.007)	(0.007)	(0.008)	(0.013)	(0.011)
$\mathbb{1}(\text{Age}_t = 7)$	-0.152^a	-0.130^a	-0.120^a	-0.138^a	-0.136^a
	(0.007)	(0.007)	(0.008)	(0.014)	(0.011)
$\mathbb{1}(\text{Age}_t = 8)$	-0.156^a	-0.133^a	-0.121^a	-0.130^a	-0.140^a
	(0.007)	(0.007)	(0.009)	(0.015)	(0.012)
$\mathbb{1}(\text{Age}_t = 9)$	-0.160^a	-0.135^a	-0.122^a	-0.133^a	-0.141^a
	(0.007)	(0.007)	(0.009)	(0.017)	(0.012)
$\mathbb{1}(\text{Age}_t \geq 10)$	-0.172^a	-0.137^a	-0.121^a	-0.138^a	-0.135^a
	(0.007)	(0.007)	(0.009)	(0.019)	(0.012)
$\log(\text{Emp})_t$		-0.021^a	-0.023^a	-0.034^a	-0.024^a
		(0.001)	(0.002)	(0.005)	(0.002)
$\log(\text{Parent Emp})_t$		0.001	0.001	0.008	-0.000
		(0.001)	(0.003)	(0.007)	(0.003)
Industry-year FE	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y
Firm FE			Y	Y	Y
N	127278	124872	119615	21481	75179
R^2	0.107	0.124	0.368	0.361	0.365

Notes: Standard errors are clustered at the business group level. Age is the age of the firm when making the forecasts. Regressions in columns 1, 2 and 3 include all firms, while the regression in column 4 only includes firms that have continuously appeared in the sample from age 1 to age 7.

1.3.4 Controlling Market/Product Diversification

As firm ages, it is possible that they enhance their capabilities and diversify their businesses by selling to more markets and selling more products . This diversification argument implies that firm demand becomes less volatile when the firm becomes older and provides an alternative interpretation of the age effects on the absolute forecast errors. To evaluate the relevance of this alternative explanation, we construct various measures of market/product diversification for firms, and show that including them in the regressions does not eliminate the impact of age on the decline in variance of forecast errors. Therefore, we argue that learning about demand provides a good explanation for the patterns documented in the paper.

In Columns 1 and 2 of Table OA.9, we use the number of destination markets as a measure of market diversification and the Herfindahl-Hirschman Index (HHI) as an inverse measure of market diversification, respectively. In our data, we observe the firms' sales up to six markets: the host country (local market), Japan, Asia, North America, Europe and the rest of the world.[2] We therefore define the HHI of firm i as

$$HHI_i^{markets} = \sum_{m=1}^{6} s_{im}^2,$$

where s_{im} is the share of market m sales in firm i's total sales. Consistent with the findings in Garetto et al. (2019), we find that firms grow by diversifying their destination markets (results available upon request). Columns 1 and 2 show that market diversification has a negative impact on the absolute value of forecast errors and reduced the impact of age compared to Column 3 in Table 3 of the paper. However, the age coefficients are still negatively significant and maintain 80% of the magnitude of those in Table 3.

The Japanese foreign activities survey provides limited information on sales by market, and does not break down affiliated firms' sales by product. To construct finer measures of market/product diversification, we merge the subset of firms operating in China with the China customs data (2000 - 2009). This involves translating the firms' names to Chinese (most of them are in English in the foreign activities survey) and matching them with the exporter names in the customs data. We

[2] Affiliates' sales to the four continents exclude the sales in the local market, if they are located in any of these continents.

were able to match 3925 out of the 7317 affiliated firms in China to the customs data between 2000 and 2009. Among the matched firms, the median number of exporting destinations is two (maximum = 149), and the median number of HS 6-digit products is four (maximum = 461).

In Columns 3 and 4, we calculate a refined measure of market diversification by combining the customs data with the six-market diversification measures in Columns 1 and 2. In particular, if the firm can be matched to the customs data, the number of markets it serves equals to the number of export destinations or the number of export destinations plus one, depending on whether it sells locally in China. The HHI of market sales is also calculated by combing the local sales and sales to each export destination. To increase the sample size, we use the six-market diversification measures, if the firm cannot be matched to the customs data. To capture the potential non-linear effects of the number of markets, we use the logarithm of this variable instead of its level. As is shown in the table, these market diversification measures have a negative but insignificant effect on firm-level uncertainty of the affiliated firms in China, while the age effects remain large and significant.

Finally, in Columns 5 and 6, we examine the impact of product diversification. For each given year, we calculate the number of export products at the HS 6-digit level, and also the HHI using product level sales of a firm i in China

$$HHI_i^{products} = \sum_{p=1}^{N_i} s_{ip}^2,$$

where N_i is the total number of products and s_{ip} is the export share of product p in firm i's total exports. One caveat is that we only observe exports by products from the China customs data but do not observe sales by product in the local market, so the product diversification variables inevitably contain measurement errors. However, we believe they still capture the extent to which firms diversify their product portfolio. Similar to Columns 3-4, we see a negative and insignificant impact of product diversification on firm-level uncertainty, while the age effects remain significant and large.

Table OA.9: Age effects on the absolute value of forecast errors: controlling for market/product diversification

Sample:	All Affiliates		All Chinese Affiliates		Matched with China Customs	
Dep.Var: $\lvert FE^{\log}_{t,t+1}\rvert$	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{1}(\text{Age}_t = 2)$	-0.047^a	-0.049^a	-0.059^a	-0.061^a	-0.047	-0.048
	(0.009)	(0.009)	(0.015)	(0.015)	(0.031)	(0.031)
$\mathbb{1}(\text{Age}_t = 3)$	-0.064^a	-0.067^a	-0.079^a	-0.080^a	-0.073^b	-0.073^b
	(0.008)	(0.008)	(0.015)	(0.015)	(0.030)	(0.030)
$\mathbb{1}(\text{Age}_t = 4)$	-0.082^a	-0.085^a	-0.102^a	-0.104^a	-0.077^b	-0.077^a
	(0.008)	(0.009)	(0.016)	(0.016)	(0.030)	(0.030)
$\mathbb{1}(\text{Age}_t = 5)$	-0.091^a	-0.094^a	-0.119^a	-0.122^a	-0.088^a	-0.089^a
	(0.008)	(0.009)	(0.016)	(0.016)	(0.030)	(0.030)
$\mathbb{1}(\text{Age}_t = 6)$	-0.089^a	-0.092^a	-0.116^a	-0.120^a	-0.074^b	-0.074^b
	(0.009)	(0.009)	(0.016)	(0.016)	(0.032)	(0.032)
$\mathbb{1}(\text{Age}_t = 7)$	-0.096^a	-0.100^a	-0.123^a	-0.127^a	-0.074^b	-0.075^b
	(0.009)	(0.009)	(0.017)	(0.017)	(0.032)	(0.032)
$\mathbb{1}(\text{Age}_t = 8)$	-0.097^a	-0.100^a	-0.123^a	-0.126^a	-0.085^b	-0.085^b
	(0.009)	(0.009)	(0.017)	(0.017)	(0.033)	(0.033)
$\mathbb{1}(\text{Age}_t = 9)$	-0.100^a	-0.104^a	-0.128^a	-0.132^a	-0.085^b	-0.086^b
	(0.009)	(0.009)	(0.017)	(0.017)	(0.034)	(0.034)
$\mathbb{1}(\text{Age}_t \geq 10)$	-0.100^a	-0.103^a	-0.127^a	-0.131^a	-0.078^b	-0.079^b
	(0.009)	(0.009)	(0.018)	(0.018)	(0.037)	(0.037)
# of Markets at t	-0.003^a					
	(0.001)					
HHI Market Sales at t		0.015^a		0.010		
		(0.005)		(0.009)		
log # of Markets at t			-0.002			
			(0.003)			
log # of HS6 Products at t					-0.002	
					(0.004)	
HHI HS6 Product Exports at t						0.006
						(0.014)
$\log(\text{Emp})_t$	-0.022^a	-0.023^a	-0.026^a	-0.026^a	-0.030^a	-0.030^a
	(0.002)	(0.002)	(0.004)	(0.004)	(0.011)	(0.011)
$\log(\text{Parent Emp})_t$	-0.001	-0.001	0.000	-0.001	0.001	0.001
	(0.003)	(0.003)	(0.006)	(0.006)	(0.010)	(0.010)
Industry-year FE	Y	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y	Y
Firm FE	Y	Y	Y	Y	Y	Y
N	109102	104598	27103	26514	8066	8177
R^2	0.372	0.376	0.376	0.378	0.396	0.393

Notes: Standard errors are clustered at the business group level, c 0.10 b 0.05 a 0.01.. Age is the age of the firm when making the forecasts. Columns 1-2 include all firms, columns 3-4 include all firms operating in China, while columns 5-6 include firms that can be matched to the China Customs data. In columns 1-2, we calculate # of markets and HHI of market sales using information on firms' sales in six markets: the host country, Japan, Asia, North America, Europe and Latin America, where the sales to the four continents exclude those in the host country if the firm locates in one of the continents. In columns 3-4, one market refers to one country if the firm can be found in the customs data, while the market is defined in the same way as columns 1-2 if the match is unsuccessful. In columns 5-6, we only focus on the firms that can be found in the customs data. The number of products and the HHI index are calculated at the HS 6-digit product level that the firm exports.

1.3.5 Partial Year Effects

In Table OA.10, we show that the age effects, especially the difference between age one and age two firms, are not driven by the "partial year effects". The partial year effects are potentially relevant here since some age one firms entered relatively late in its founding year. As a result, they may not have enough information to make a precise forecast at the time of the survey. To investigate this issue, we use the information on the firms' founding months and split the age one firms into two groups: those that entered in the first half of the founding year and those that entered in the second half of the founding year.

In Columns 1 and 2, we treat the age one firms that entered in the second half of the year as the base group. These firms have less than six months of experience at the time of survey (age $\in (0, 0.5)$), and should arguably have the highest forecast error. We then include the other age dummies, including one dummy indicating age one firms that entered in the first half of the year (age $\in (0.5, 1)$). We find some suggestive evidence that an additional six month of experience reduces the absolute forecast errors, though the effect is not significant when we include firm fixed effects. On the other hand, age two firms have significantly smaller forecast errors than both groups of age one firms.

In Columns 3-4, we provide additional robustness checks by excluding age one firms that entered in the second half of the founding year. In Column 5, we exclude age one firms and show that the decline in forecast errors is still significant after age two, though at a smaller scale. All these results are consistent with learning and cannot be totally driven by the partial year effect of age one firms.

Table OA.10: Age effects on the absolute residual forecast errors: robustness to partial year effects.

Sample:	All Affiliates		Excluding Age 0-0.5		Excluding Age 0-1
Dep.Var: $\lvert FE^{\log}_{t,t+1}\rvert$	(1)	(2)	(3)	(4)	(5)
$\mathbb{1}(\text{Age}_t \in (0.5, 1))$	-0.022^c	-0.011			
	(0.013)	(0.015)			
$\mathbb{1}(\text{Age}_t = 2)$	-0.069^a	-0.068^a	-0.048^a	-0.058^a	
	(0.010)	(0.010)	(0.009)	(0.011)	
$\mathbb{1}(\text{Age}_t = 3)$	-0.100^a	-0.093^a	-0.079^a	-0.084^a	-0.027^a
	(0.009)	(0.010)	(0.010)	(0.011)	(0.005)
$\mathbb{1}(\text{Age}_t = 4)$	-0.124^a	-0.115^a	-0.103^a	-0.106^a	-0.049^a
	(0.009)	(0.010)	(0.010)	(0.011)	(0.006)
$\mathbb{1}(\text{Age}_t = 5)$	-0.136^a	-0.122^a	-0.115^a	-0.113^a	-0.056^a
	(0.009)	(0.010)	(0.009)	(0.011)	(0.006)
$\mathbb{1}(\text{Age}_t = 6)$	-0.135^a	-0.119^a	-0.114^a	-0.110^a	-0.053^a
	(0.009)	(0.010)	(0.009)	(0.011)	(0.006)
$\mathbb{1}(\text{Age}_t = 7)$	-0.142^a	-0.126^a	-0.121^a	-0.116^a	-0.060^a
	(0.009)	(0.010)	(0.009)	(0.011)	(0.006)
$\mathbb{1}(\text{Age}_t = 8)$	-0.144^a	-0.126^a	-0.123^a	-0.116^a	-0.060^a
	(0.010)	(0.011)	(0.010)	(0.012)	(0.006)
$\mathbb{1}(\text{Age}_t = 9)$	-0.146^a	-0.128^a	-0.125^a	-0.118^a	-0.062^a
	(0.009)	(0.011)	(0.009)	(0.012)	(0.006)
$\mathbb{1}(\text{Age}_t \geq 10)$	-0.148^a	-0.126^a	-0.127^a	-0.118^a	-0.062^a
	(0.009)	(0.011)	(0.009)	(0.012)	(0.007)
$\log(\text{Emp})_t$	-0.021^a	-0.024^a	-0.020^a	-0.023^a	-0.020^a
	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)
$\log(\text{Parent Emp})_t$	0.001	0.001	0.001	0.001	-0.000
	(0.001)	(0.003)	(0.001)	(0.003)	(0.003)
Industry-year FE	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y
Firm FE		Y		Y	Y
N	128429	123111	126914	121671	120217
R^2	0.122	0.366	0.118	0.362	0.361

Notes: Standard errors are clustered at the business group level, c 0.10 b 0.05 a 0.01. In columns 1-2, we use age one firms that entered in the second half of the founding year as the base group (age $\in (0, 0.5)$) and include an additional dummy variable indicating whether the age one firms entered in the first half of the founding year (age $\in (0.5, 1)$). In column 3-4, we exclude age one affiliated firms that entered in the second half of the founding year. Column 5 excludes all age one firms.

1.3.6 Idiosyncratic Shocks or Heterogeneous Exposure to Aggregate Shocks?

Though we have shown that our results are robust to using the "residual forecast errors", which arguably tease out the systematic forecast errors due to aggregate shocks, they may still be affected by the aggregate economy since firms may have heterogeneous exposure to aggregate shocks (David et al., 2019). In this subsection, we construct alternative measures of residual forecast errors to tease out such heterogeneous exposure.

There are multiple mechanisms through which firms have heterogeneous exposure to aggregate shocks. David et al. (2019) show that, all else equal, (1) labor intensive firms are more exposed to cyclical movements in wages (2) firms facing a high demand elasticity (setting a lower markup) respond more strongly to aggregate shocks, and (3) high-quality products are more cyclical since households tend to consume higher quality goods in booms due to non-homothetic preferences. To account for such heterogeneous exposure, we construct an alternative residual forecast error by running the following regression

$$FE^{\log}_{it,t+1} = \delta^{\text{labor}}_b \times \delta_{ct} + \delta^{\text{markup}}_b \times \delta_{ct} + \delta^{\text{quality}}_b \times \delta_{ct} + \delta_{st} + \hat{\epsilon}^{FE,\log}_{it,t+1}$$

where $\delta^{\text{labor}}_b \times \delta_{ct}$ indicates a set of labor-share-bin-country-year fixed effects. The labor share bins are obtained by dividing our sample into ten equally-sized bins based on the firms' labor share (wage bill divided by total sales). We define $\delta^{\text{markup}}_b \times \delta_{ct}$ and $\delta^{\text{quality}}_b \times \delta_{ct}$ in similar ways. We use the ratio of total sales to material costs as a measure of the markup and workers' average wage as a measure of output quality. The markup measure is proportional to price over marginal cost as long as (1) the output elasticity with respect to materials is constant and (2) materials are a flexible input, i.e., not subject to adjustment frictions (de Loecker and Warzynski, 2012). We use workers' wage to approximate firm output quality, as previous studies show that firms producing high-quality output tend to be more skill intensive. (see, for example, Verhoogen (2008); Fieler et al. (2018)) Finally, δ_{st} is a set of industry-year fixed effects, which we also include when calculating the baseline residual forecast errors.

This specification captures heterogeneous responses to aggregate shocks (country-year fixed effects) based on firm characteristics such as labor share, markup and output quality. It includes substantially more fixed effects compared to the regression we use to obtain the baseline residual forecast errors (only country-year and industry-year fixed effects). The expanded set of fixed effects explains 23% of the variation in the raw forecast errors. The residuals, capturing forecast errors due to idiosyncratic shocks, still maintain 77% of the variation in the raw forecast errors.

In Table OA.11, we replicate regressions in Table 3 of the paper. Though the age coefficients are smaller, they are still significantly negative and are about 85% of those estimated with raw forecast errors. Note that the number of observations are smaller than in the paper, as much more singletons are dropped when we estimate the residual forecast errors due to the added fixed effects.

Table OA.11: Age effects on the absolute value of alternative residual forecast errors, where we have purged an expanded set of fixed effects.

Sample:	All Firms			Survivors	Manufacturing
Dep.Var: $\left\|\hat{\epsilon}^{\log}_{FE,t,t+1}\right\|$	(1)	(2)	(3)	(4)	(5)
$\mathbb{1}(\text{Age}_t = 2)$	-0.034^a	-0.035^a	-0.037^a	-0.048^a	-0.019
	(0.008)	(0.008)	(0.009)	(0.012)	(0.012)
$\mathbb{1}(\text{Age}_t = 3)$	-0.051^a	-0.047^a	-0.043^a	-0.053^a	-0.028^b
	(0.008)	(0.008)	(0.009)	(0.012)	(0.012)
$\mathbb{1}(\text{Age}_t = 4)$	-0.081^a	-0.075^a	-0.065^a	-0.067^a	-0.052^a
	(0.008)	(0.008)	(0.009)	(0.012)	(0.012)
$\mathbb{1}(\text{Age}_t = 5)$	-0.090^a	-0.082^a	-0.069^a	-0.081^a	-0.052^a
	(0.008)	(0.008)	(0.009)	(0.013)	(0.012)
$\mathbb{1}(\text{Age}_t = 6)$	-0.096^a	-0.087^a	-0.071^a	-0.078^a	-0.056^a
	(0.008)	(0.008)	(0.009)	(0.014)	(0.012)
$\mathbb{1}(\text{Age}_t = 7)$	-0.105^a	-0.093^a	-0.078^a	-0.098^a	-0.065^a
	(0.008)	(0.008)	(0.009)	(0.015)	(0.012)
$\mathbb{1}(\text{Age}_t = 8)$	-0.107^a	-0.095^a	-0.077^a	-0.090^a	-0.064^a
	(0.008)	(0.008)	(0.009)	(0.016)	(0.012)
$\mathbb{1}(\text{Age}_t = 9)$	-0.114^a	-0.099^a	-0.081^a	-0.093^a	-0.071^a
	(0.008)	(0.008)	(0.009)	(0.017)	(0.013)
$\mathbb{1}(\text{Age}_t \geq 10)$	-0.123^a	-0.099^a	-0.078^a	-0.090^a	-0.065^a
	(0.008)	(0.008)	(0.009)	(0.019)	(0.013)
$\log(\text{Emp})_t$		-0.019^a	-0.022^a	-0.028^a	-0.023^a
		(0.001)	(0.002)	(0.005)	(0.003)
$\log(\text{Parent Emp})_t$		-0.001	0.006^b	0.009	0.007^a
		(0.001)	(0.003)	(0.008)	(0.002)
Industry-year FE	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y
Firm FE			Y	Y	Y
N	98102	97968	93145	16494	61000
R^2	0.075	0.094	0.352	0.369	0.340

Notes: Standard errors are clustered at the business group level, c 0.10 b 0.05 a 0.01.

1.4 Robustness Checks for Fact 2

1.4.1 Auto-correlations using alternative forecast errors

Table OA.12: Correlation of $FE_{t,t+1}$ and $FE_{t-1,t}$, overall and by age group

Sample	All ages	Age 2-4	Age 5-7	Age $\geq$ 8
All industries				
$corr(FE^{\text{pct}}_{t,t+1}, FE^{\text{pct}}_{t-1,t})$	0.105	0.137	0.120	0.093
	[96967]	[10578]	[13875]	[72514]
$corr(\hat{\epsilon}^{FE,\log}_{t,t+1}, \hat{\epsilon}^{FE,\log}_{t-1,t})$	0.113	0.154	0.141	0.092
	[96194]	[10373]	[13764]	[72057]
$corr(\hat{\epsilon}^{FE,\text{pct}}_{t,t+1}, \hat{\epsilon}^{FE,\text{pct}}_{t-1,t})$	0.087	0.122	0.111	0.070
	[96707]	[10541]	[13838]	[72328]
Manufacturing				
$corr(FE^{\text{pct}}_{t,t+1}, FE^{\text{pct}}_{t-1,t})$	0.108	0.172	0.116	0.089
	[60364]	[5906]	[8623]	[45835]
$corr(\hat{\epsilon}^{FE,\log}_{t,t+1}, \hat{\epsilon}^{FE,\log}_{t-1,t})$	0.118	0.177	0.139	0.092
	[60049]	[5817]	[8580]	[45652]
$corr(\hat{\epsilon}^{FE,\text{pct}}_{t,t+1}, \hat{\epsilon}^{FE,\text{pct}}_{t-1,t})$	0.092	0.160	0.103	0.070
	[60289]	[5895]	[8612]	[45782]

Notes: $FE^{\text{pct}}_{t,t+1}$ is the percentage deviation of realized sales from expected sales. The other two measures, $\hat{\epsilon}^{FE,\log}_{t,t+1}$ and $\hat{\epsilon}^{FE,\text{pct}}_{t,t+1}$, are the residual forecast errors, which we obtain by regressing $FE^{\log}_{t,t+1}$ and $FE^{\text{pct}}_{t,t+1}$ on a set of industry-year and country-year fixed effects.. Age is measured at the end of year t. The manufacturing subsample is constructed in the same way as the previous section. Number of observations used for each correlation is shown in the brackets below.

1.4.2 AR(1) Models with Age Interactions

In this section, we perform several robustness checks of the regressions in Table 5 of the paper. We first replace the log forecast errors with alternative definitions of forecast errors. Table OA.13 uses percentage forecast errors, while Table OA.14 uses residual log forecast errors. The results in Table OA.14 are almost identical to those obtained using log forecast errors, while the magnitudes of the estimates in Table OA.13 are slightly smaller. Next, we exclude firms that use current sales as their sales forecasts for the next year and re-run the regressions in Table 5. The results are very similar (see Table OA.15).

Table OA.13: AR(1) regressions with Age Interactions, Percentage Forecast Errors

Sample:	All Affiliates				Manufacturing			
Dep.Var: $FE^{\text{pct}}_{t+1,t+2}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$FE^{\text{pct}}_{t,t+1}$	0.071[a]	0.068[a]	0.106[a]	0.098[a]	0.085[a]	0.080[a]	0.112[a]	0.103[a]
	(0.014)	(0.013)	(0.018)	(0.017)	(0.020)	(0.020)	(0.024)	(0.024)
$\times \max\{\text{Age}_t, 10\}$	-0.005[a]		-0.008[a]		-0.007[a]		-0.009[a]	
	(0.002)		(0.002)		(0.003)		(0.003)	
$\times \log(\text{Age}_t)$		-0.015[a]		-0.023[a]		-0.024[a]		-0.029[a]
		(0.006)		(0.007)		(0.009)		(0.010)
$\log(\text{Emp})_t$	-0.003[a]	-0.003[a]	-0.004[a]	-0.004[a]	-0.004[a]	-0.004[a]	-0.005[a]	-0.005[a]
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\log(\text{Parent Emp})_t$	-0.010[b]	-0.010[b]	-0.009[b]	-0.009[b]	-0.009	-0.009	-0.012[c]	-0.012[c]
	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.006)	(0.006)
Industry-year FE	Y	Y	Y	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y	Y	Y	Y
Business Group FE	Y	Y			Y	Y		
Busi.Group-Age FE			Y	Y			Y	Y
N	93971	93971	85278	85278	58862	58862	52720	52720
R^2	0.181	0.180	0.250	0.250	0.198	0.198	0.266	0.266

Notes: Standard errors are clustered at the business group level. c 0.10 b 0.05 a 0.01. The "manufacturing" subsample refers to affiliated firms that are in manufacturing sectors.

Table OA.14: AR(1) regressions with Age Interactions, Residual Log Forecast Errors

Sample:	All Affiliates				Manufacturing			
Dep.Var: $\hat{\epsilon}^{FE,\log}_{t+1,t+2}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\hat{\epsilon}^{FE,\log}_{t,t+1}$	0.106[a]	0.101[a]	0.138[a]	0.128[a]	0.118[a]	0.116[a]	0.147[a]	0.144[a]
	(0.014)	(0.014)	(0.019)	(0.018)	(0.020)	(0.020)	(0.026)	(0.026)
$\times \max\{\text{Age}_t, 10\}$	-0.006[a]		-0.009[a]		-0.009[a]		-0.011[a]	
	(0.002)		(0.002)		(0.003)		(0.003)	
$\times \log(\text{Age}_t)$		-0.019[a]		-0.025[a]		-0.030[a]		-0.035[a]
		(0.006)		(0.007)		(0.009)		(0.011)
$\log(\text{Emp})_t$	0.003[a]	0.003[a]	0.002[c]	0.002[c]	0.002	0.002	0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\log(\text{Parent Emp})_t$	-0.010[b]	-0.010[b]	-0.010[b]	-0.010[b]	-0.011[c]	-0.011[c]	-0.014[b]	-0.014[b]
	(0.004)	(0.004)	(0.004)	(0.005)	(0.006)	(0.006)	(0.007)	(0.007)
Industry-year FE	Y	Y	Y	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y	Y	Y	Y
Business Group FE	Y	Y			Y	Y		
Busi.Group-Age FE			Y	Y			Y	Y
N	93478	93478	84839	84839	58630	58630	52510	52510
R^2	0.097	0.097	0.168	0.168	0.111	0.111	0.182	0.182

Notes: Standard errors are clustered at the business group level. c 0.10 b 0.05 a 0.01. The "manufacturing" subsample refers to affiliated firms that are in manufacturing sectors.

Table OA.15: AR(1) regressions with Age Interactions, excluding firms with zero expected growth rates

Sample:	All Affiliates				Manufacturing			
Dep.Var: $FE^{\log}_{t+1,t+2}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$FE^{\log}_{t,t+1}$	0.101^a	0.096^a	0.098^a	0.093^a	0.111^a	0.109^a	0.105^a	0.104^a
	(0.014)	(0.014)	(0.014)	(0.014)	(0.019)	(0.019)	(0.019)	(0.020)
$\times$ max{Age$_t$, 10}	-0.005^a		-0.004^a		-0.007^a		-0.007^a	
	(0.002)		(0.002)		(0.002)		(0.002)	
$\times$ log(Age$_t$)		-0.014^b		-0.013^b		-0.025^a		-0.023^b
		(0.006)		(0.006)		(0.009)		(0.009)
log(Emp)$_t$			0.002^b	0.002^b			0.001	0.001
			(0.001)	(0.001)			(0.001)	(0.001)
log(Parent Emp)$_t$			-0.011^a	-0.011^a			-0.011^c	-0.011^c
			(0.004)	(0.004)			(0.006)	(0.006)
Industry-year FE	Y	Y	Y	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y	Y	Y	Y
Business Group FE	Y	Y	Y	Y	Y	Y	Y	Y
Age FE	Y	Y	Y	Y	Y	Y	Y	Y
N	92871	92871	91378	91378	58214	58214	57646	57646
R^2	0.206	0.206	0.208	0.208	0.231	0.231	0.233	0.233

Notes: Standard errors are clustered at the business group level. c 0.10 b 0.05 a 0.01. The "manufacturing" subsample refers to affiliated firms that are in manufacturing sectors.

1.5 Robustness Checks for Fact 3

Table OA.16: AR(1) coef and horse race between country characteristics

	Dep.Var: $FE_{t+1,t+2}$							
$FE^{\log}_{t,t+1}$	0.1488^a	0.1338^a	0.1175^a	0.1160^a	0.1632^a	0.1410^a	0.1272^a	0.1237^a
	(0.0222)	(0.0174)	(0.0176)	(0.0177)	(0.0221)	(0.0180)	(0.0182)	(0.0181)
× Management Score (WMS 2015)	-0.0130				-0.0127			
	(0.0084)				(0.0082)			
× Time Diff from Japan		0.0163^b		0.0301^a		0.0151^b		0.0291^a
		(0.0073)		(0.0087)		(0.0072)		(0.0087)
× log GDP p.c. 1995			-0.0110^c	-0.0272^a			-0.0110^c	-0.0269^a
			(0.0067)	(0.0080)			(0.0066)	(0.0080)
× $\log(\text{Age})_t$	-0.0291^a	-0.0282^a	-0.0220^a	-0.0220^a				
	(0.0087)	(0.0070)	(0.0070)	(0.0070)				
× $\min\{Age, 10\}$					-0.0103^a	-0.0091^a	-0.0076^a	-0.0073^a
					(0.0025)	(0.0021)	(0.0021)	(0.0021)
Industry-year FE	Y	Y	Y	Y	Y	Y	Y	Y
Country-year FE	Y	Y	Y	Y	Y	Y	Y	Y
Busi.Group-Age FE	Y	Y	Y	Y	Y	Y	Y	Y
N	53433	86271	86271	86271	53433	86271	86271	86271
R^2	0.284	0.270	0.270	0.271	0.284	0.270	0.270	0.271

Notes: Standard errors are clustered at the business group level. Significance levels: c 0.1, b 0.05, a 0.01. Management score is from the World Management Survey up to 2015 . Management score, time zone differences and log GDP per capita are all standardized to faciliate interpretation of the coefficients.

2 Theory Appendix

2.1 Full Information Rational Expectation Models

In this subsection, we derive the expression of the forecast error in the full information rational expectation (FIRE) model. We calculate the logarithm of *realized* sales in period t as

$$\log(R_n(\theta,\varphi_{n-1})) = (\sigma-1)\left[\log(\sigma-1)-\log(\sigma)\right] + \log(Y) + (\sigma-1)\log(P)$$
$$+\theta + (\sigma-1)\left[b(\varphi_{n-1},n)-\log(w)\right] + \frac{\sigma-1}{\sigma}\log(\varphi_n),$$

where

$$b(\varphi_{n-1},n) \equiv E\left(\varphi_n^{\frac{\sigma-1}{\sigma}} | \varphi_{n-1}, n\right).$$

Since the firm knows θ in the FIRE model, the logarithm of forecasted sales is

$$\log(R_n(\theta,\varphi_{n-1})) = (\sigma-1)\left[\log(\sigma-1)-\log(\sigma)\right] + \log(Y) + (\sigma-1)\log(P)$$
$$+\theta + (\sigma-1)\left[b(\varphi_{n-1},n)-\log(w)\right] + b(\varphi_{n-1},n),$$

which leads to

$$FE^{\log}_{n-1,n} = \frac{(\sigma-1)\nu_n}{\sigma} - \frac{(\sigma-1)^2\sigma^2_{\nu_n}}{2\sigma^2}. \tag{2}$$

Thus, we have

$$Cov\left(FE^{\log}_{n-1,n}, FE^{\log}_{n,n+1}\right) = Cov\left(\frac{(\sigma-1)\nu_n}{\sigma}, \frac{(\sigma-1)\nu_{n+1}}{\sigma}\right) = 0.$$

Therefore, forecast errors are serially uncorrelated in FIRE models.

2.2 Full Information Rational Expectation Models with Endogenous Exits

In this subsection, we consider the case in which incumbent firms can choose to exit after observing the its productivity shock and the demand draw. There are two sub-cases to discuss. First, following the same timing assumption adopted in the paper, we assume that the firm observes its productivity shock at age $n-1$ when choosing to stay at age n. In this case, there is an exit cutoff on the productivity shock $\bar{\varphi}_{n-1}(\theta)$ (depending on θ) below which incumbent firms exit. Thus, incumbents that have survived at both ages n and $n+1$ must satisfy

$$\log\varphi_{n-1} = \mu_\varphi + \rho\log\varphi_{n-2} + \nu_{n-1} \geq \log\left(\bar{\varphi}_{n-1}(\theta)\right), \quad \log\varphi_n = \mu_\varphi + \rho\log\varphi_{n-1} + \nu_n \geq \log\left(\bar{\varphi}_n(\theta)\right), \tag{3}$$

Conditioning on $\log\varphi_{n-2}$ and survival at both ages n and $n+1$, there is a negative correlation between ν_{n-1} and ν_n implied by equation (3) as $\log\varphi_{n-1} = \mu_\varphi + \rho\log\varphi_{n-2} + \nu_{n-1}$. The intuition is that a better contemporaneous productivity innovation at age $n-1$ (that pushes up the productivity realization at age $n-1$) makes survival at age $n+1$ (that depends on the productivity realization at age n) easier, which implies worse productivity innovations at age n on average. This leads to a negative correlation between the contemporaneous productivity innovations at ages $n-1$ and n, conditioning on survival. However, the autocovariance of the forecast errors at ages n and $n+1$ is still zero, as the productivity innovation at age $n+1$ that enter into the forecast error at age $n+1$

is still random conditioning on the survival at ages $n-1$ and n:

$$
\begin{aligned}
& Cov(FE^{\log}_{n-1,n}, FE^{\log}_{n,n+1} | \text{surviving at both ages } n \text{ and } n+1) \\
= \; & Cov\Big(\frac{(\sigma-1)\nu_n}{\sigma}, \frac{(\sigma-1)\nu_{n+1}}{\sigma} \Big| \log\varphi_{n-1} \geq \log\left(\bar{\varphi}_{n-1}(\theta)\right), \mu_\varphi + \rho\log\varphi_{n-1} + \nu_n \geq \log\left(\bar{\varphi}_n(\theta)\right)\Big) \\
= \; & 0.
\end{aligned}
$$

Second, we consider the sub-case that the firm observes its productivity shock at age n when choosing to stay at age n which is different from the assumption used in the paper but common in most firm dynamics models (e.g., Hopenhayn (1992)). In this case, then exit cutoff at age n is related to the productivity shock at age n or $\bar{\varphi}_n(\theta)$ below which incumbent firms exit. Again, a better contemporaneous productivity innovation at age n makes survival at age $n+1$ easier, which implies worse productivity innovations at age $n+1$ on average. This leads to a negative correlation between the contemporaneous productivity innovations at ages $n-1$ and n, conditioning on survival. Thus, survivors at ages n and $n+1$ must satisfy

$$
\mu_\varphi + \rho\log\varphi_{n-1} + \nu_n \geq \log\left(\bar{\varphi}_n(\theta)\right), \quad \mu_\varphi + \rho\log\varphi_n + \nu_{n+1} \geq \log\left(\bar{\varphi}_{n+1}(\theta)\right), \tag{4}
$$

Conditioning on $\log\varphi_{n-1}$ and survival at both ages n and $n+1$, there is a negative correlation between ν_n and ν_{n+1} implied by equation (4) as $\log\varphi_n = \mu_\varphi + \rho\log\varphi_{n-1} + \nu_n$. Therefore, the correlation of forecast errors becomes

$$
\begin{aligned}
& Cov(FE^{\log}_{n-1,n}, FE^{\log}_{n,n+1} | \text{surviving at both ages } n \text{ and } n+1) \\
= \; & Cov\Big(\frac{(\sigma-1)\nu_n}{\sigma}, \frac{(\sigma-1)\nu_{n+1}}{\sigma} \Big| \mu_\varphi + \rho\log\varphi_{n-1} + \nu_n \geq \log\left(\bar{\varphi}_n(\theta)\right), \mu_\varphi + \rho\log\varphi_n + \nu_{n+1} \geq \log\left(\bar{\varphi}_{n+1}(\theta)\right)\Big) \\
< \; & 0.
\end{aligned}
$$

Finally, the proof would be the same (with changes in notations), if we assume that the demand shifter θ follows an AR(1) process and the productivity shock is time-invariant. In total, the FIRE model cannot be used to rationalize forecast errors made in two consecutive periods are positively correlated.

2.3 ε is a Real Shock as in Jovanovic (1982)

In this section, we show that forecast errors made by firms in Jovanovic (1982) are serially uncorrelated—a property that rational expectations models with *full information* also inherit. In order to show this property, we modify our model presented in the paper in the following way. We assume that the firm-specific demand shifter, $a_t(\omega)$, is the sum of a time-invariant permanent demand draw $\theta(\omega)$ and a transitory demand shock $\varepsilon_t(\omega)$ as in Arkolakis et al. (2018):

$$a_t(\omega) = \theta(\omega) + \varepsilon_t(\omega). \tag{5}$$

Firms understand that $\theta(\omega)$ is drawn from a normal distribution $N\left(\bar{\theta}, \sigma_\theta^2\right)$, and the independently and identically distributed (i.i.d.) transitory demand shock, $\varepsilon_t(\omega)$, is drawn from another normal distribution $N\left(0, \sigma_\varepsilon^2\right)$. We assume that the firm observes the sum of the two demand components, $a_t(\omega)$, at the end of each period, not each of them separately. Thus, the firm needs to learn about its permanent demand every period by forming an posterior belief about the distribution of θ. In summary, we drop the "pure" informational noise from the model and assume that the firm cannot differentiate the permanent demand draw from the transitory demand shock. As a result, the realized overall demand shifters, $a_1, a_2, \ldots, a_t$, become the noisy signals for the permanent demand draw $\theta(\omega)$. The crucial difference here is that the transitory demand shock now acts as both an informational noise *and* as a "real" shock that directly affects the firm's overall demand.

We modify the firm's belief updating process as follows. Since both the prior and the realized demand shifters are normally distributed, the posterior belief is also normally distributed. A firm that is $n+1$ years old has observed the realized demand shifters in the past n periods: $a_1, a_2, \ldots, a_n$, the Bayes' rule implies that the posterior belief about θ is normally distributed with mean μ_n and variance σ_n^2 where

$$\mu_n = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + n\sigma_\theta^2}\bar{\theta} + \frac{n\sigma_\theta^2}{\sigma_\varepsilon^2 + n\sigma_\theta^2}\bar{a}_n, \ \sigma_n^2 = \frac{\sigma_\varepsilon^2 \sigma_\theta^2}{\sigma_\varepsilon^2 + n\sigma_\theta^2}. \tag{6}$$

The history of signals $(a_1, a_{2,}, \ldots, a_n)$ is summarized by age n and the average demand shifter:

$$\bar{a}_n \equiv \frac{1}{t}\sum_{i=1}^{n} a_i \text{ for } n \geq 1; \quad \bar{a}_0 \equiv \bar{\theta}.$$

Therefore, the firm believes that the overall demand shifter in period $t+1$, $a_{n+1} = \theta + \varepsilon_{n+1}$, has a normal distribution with mean μ_n and variance $\sigma_n^2 + \sigma_\varepsilon^2$. The difference from the paper is that it is the average demand shifter $\bar{a}_n$ (not $\bar{s}_n$) that is the firm's state variable.

We study the firm's static optimization problem under the modified assumptions now. Given the belief about a_n, an age-n firm chooses employment level l_n to maximize its expected per-period profit at age n, $E_{a_n,\varphi_n|\bar{a}_{n-1},\varphi_{n-1},n}(\pi_n)$. The realized per-period profit at age n is

$$\pi_n = p_n(a_n)\varphi_n l_n - w \times l_n - wf.$$

Firms set the price after observing the realized demand a_n and the productivity shock φ_n to sell all the output. Maximizing $E_{a_n,\varphi_n|\bar{a}_{n-1},\varphi_{n-1},n}(\pi_n)$, the optimal employment of an age-n in period t is[3]

$$l_t = \left(\frac{\sigma-1}{\sigma}\right)^\sigma \left(\frac{b\left(\varphi_{n-1},\bar{a}_{n-1},n-1\right)}{w}\right)^\sigma YP^{\sigma-1}, \tag{7}$$

where

$$\begin{aligned} b\left(\varphi_{n-1},\bar{a}_{n-1},n-1\right) &\equiv E\left(e^{\frac{a_t}{\sigma}}\varphi_n^{\frac{\sigma-1}{\sigma}}|\varphi_{n-1},\bar{a}_{n-1},n\right) \\ &= \exp\left\{\frac{\mu_{n-1}}{\sigma} + \frac{\sigma_{n-1}^2+\sigma_\varepsilon^2}{2\sigma^2} + \frac{\sigma-1}{\sigma}\left((1-\rho)\mu_\varphi + \rho\log\varphi_{n-1}\right) + \frac{(\sigma-1)^2\sigma_{\nu_n}^2}{2\sigma^2}\right\}, \end{aligned} \tag{8}$$

and n is the firm's age. As a result, the logarithm of realized sales and the logarithm of forecasted sales of an age-n firm are

$$\log(R_n(\theta)) = \log\left(\frac{Y}{P^{1-\sigma}}\right) + \frac{a_t}{\sigma} + \frac{\sigma-1}{\sigma}\log\left(\varphi_n\right) + (\sigma-1)\log b\left(\varphi_{n-1},\bar{a}_{n-1},n-1\right) + (\sigma-1)\Big[\log\left(\frac{\sigma-1}{\sigma w}\right)\Big], \tag{9}$$

and

$$\log\Big(E_{n-1}(R_n)\Big) = \log\left(\frac{Y}{P^{1-\sigma}}\right) + \sigma\log b\left(\varphi_{n-1},\bar{a}_{n-1},n-1\right) + (\sigma-1)\Big[\log\left(\frac{\sigma-1}{\sigma w}\right)\Big].$$

The resulting log forecast error of sales is

$$FE_{n-1,n}^{\log} = \frac{(\sigma-1)\nu_{n+1}}{\sigma} - \frac{(\sigma-1)^2\sigma_{\nu_{n+1}}^2}{2\sigma^2} + \frac{\varepsilon_t + (\theta-\mu_{n-1})}{\sigma} - \frac{\sigma_{n-1}^2+\sigma_\varepsilon^2}{2\sigma^2},$$

[3]Since we always consider the steady state, time script t does not play a role in the optimization problem.

which can be rewritten as

$$
\begin{aligned}
FE^{\log}_{n-1,n} &= \frac{(\sigma-1)\nu_{n+1}}{\sigma} - \frac{(\sigma-1)^2\sigma^2_{\nu_{n+1}}}{2\sigma^2} \\
&+ \frac{(1-\zeta(n-1,\lambda))(\theta-\bar{\theta}) + \varepsilon_t - \zeta(n-1,\lambda)\frac{\sum_{i=t-n+1}^{t-1}\varepsilon_i}{n-1}}{\sigma} - \frac{\sigma^2_{n-1}+\sigma^2_{\varepsilon}}{2\sigma^2},
\end{aligned} \tag{10}
$$

where

$$
\lambda \equiv \frac{\sigma^2_\theta}{\sigma^2_\varepsilon}; \quad \zeta(n-1,\lambda) \equiv \frac{(n-1)\lambda}{1+(n-1)\lambda}.
$$

The autocovariance of (log) sales forecast errors is simply

$$
cov(FE^{\log}_{n-1,n}, FE^{\log}_{n,n+1}) = \frac{1}{\sigma^2}\left[\frac{\lambda\sigma^2_\varepsilon}{(1+\lambda n)(1+\lambda(n-1))} - \frac{\lambda n\sigma^2_\varepsilon}{n(1+\lambda n)} + \frac{\lambda n\lambda(n-1)\sigma^2_\varepsilon}{n(1+\lambda n)(1+\lambda(n-1))}\right] = 0.
$$

Therefore, the "real" demand shock that also acts as an informational noise cannot generate non-zero autocorrelation of forecast errors.

For forecast errors made at ages n and $n+1$, they share two common components in equation (10): $\theta - \bar{\theta}$ and $\sum_{i=t-n+1}^{t-1}\varepsilon_i$. Thus, if the prior mean of θ is below (or above) the actual permanent demand shifter, the firm would make positive (or negative) forecast errors at ages n and $n+1$. Similarly, if the sum of the past transitory shocks (up to age $n-1$) is negative (or positive), the firm would make positive (or negative) forecast errors at ages n and $n+1$. In any case, the forecast errors are positively autocorrelated. This is exactly the reason why forecast errors are positively autocorrelated in the paper, as the transitory (information) shocks do not enter into the realization of overall demand shifter. However, as the transitory demand shock, ε_t also enter into the realized demand shifter, there is the third term ε_t which enters into $FE^{\log}_{n-1,n}$ positively but into $FE^{\log}_{n,n+1}$ negatively. The existence of the payoff-relevant noise in Jovanovic (1982), ε_t, causes the negative autocorrelation of forecast errors. And, this additional force perfectly offsets the two forces that cause the positive autocorrelation of forecast errors discussed above.

2.3.1 Alternative intuition

Another way to gain some intuition about the uncorrelated forecast errors in Jovanovic (1982) is that the Bayesian updating with an unbiased prior yields the best linear unbiased estimator

(BLUE) for the overall demand shifter at age n, $a_n = \theta + \varepsilon_n$. To see this, recall that

$$E(\theta | a_{n-1}, a_{n-2}, \dots, a_1) = \mu_{n-1}.$$

According to Hayashi (2000) Proposition 2.7, the conditional expectation is the "best predictor" (i.e., minimizes mean squared error). Since μ_{n-1} is a weighted average of the prior $\bar{\theta}$ and previous signals $a_{n-1}, \dots, a_1$, it must be the "best linear predictor". Note that this property also holds if the goal is to predict $a_n = \theta + \varepsilon_n$, since ε_n is independent of past shocks.

In Jovanovic (1982), the (log) forecast error of sales will be proportional to $a_n - \mu_{n-1} = \theta + \varepsilon_n - \mu_{n-1}$. The previous forecast error is proportional $a_{n-1} - \mu_{n-2}$, a linear combination of $a_{n-1}, \dots, a_1$. Since $E(a_n - \mu_{n-1} | a_{n-1}, \dots, a_1) = 0$, we must have $E(a_n - \mu_{n-1} | a_{n-1} - \mu_{n-2}) = 0$.

When ε_n is *payoff-irrelevant* as in our model, the forecast errors are defined as $\theta - \mu_{n-1}$ instead of $a_n - \mu_{n-1}$. Therefore, we do not have $E(\theta - \mu_{n-1} | \theta - \mu_{n-2}) = 0$, though from the previous discussion we know that $E(\theta - \mu_{n-1} | a_{n-1} - \mu_{n-2}) = 0$. Consider regressing $\theta - \mu_{n-1}$ on $\theta - \mu_{n-2}$. If we use $a_{n-1} - \mu_{n-2} = \theta + \varepsilon_{n-1} - \mu_{n-2}$, then we will obtain a zero coefficient. Regressing the current forecast error on the previous forecast error defined in our model, $\theta - \mu_{n-2}$, creates a "non-classic measurement error" in the regressor. The direction of the "bias" can be seen from the covariance below:

$$Cov(\theta - \mu_{n-1}, \theta - \mu_{n-2}) = Cov(\theta - \mu_{n-1}, a_{n-1} - \mu_{n-2} - \varepsilon_{n-1}) = Cov(\mu_{n-1}, \varepsilon_{n-1}).$$

Since ε_{n-1} enters μ_{n-1} positively, the covariance is positive. Therefore the auto-covariance and the AR(1) coefficient of the forecast errors will be positive.

2.4 ε is a Real Shock and θ is time-varying

In this subsection, we show that the sales forecast errors are still uncorrelated over time, even when we assume that the permanent demand draw, θ, is time-varying. In particular, we assume that θ_t follows an AR(1) structure:

$$\theta_t = \rho \theta_{t-1} + \zeta_t$$

and

$$a_t = \theta_t + \varepsilon_t.$$

In addition, we make the assumption an age-n firm only observes $a_{t-n+1}, ..., a_{t-1}$ up to the beginning of period t (i.e., $n-1$ signals).

The forecast error of firm sales still consists of two parts: the demand-side error and the supply-error:

$$\begin{aligned}
FE^{log}_{t,t+1} &\equiv \log R_{t+1} - \log E_t R_{t+1} = \frac{a_{t+1}}{\sigma} + \frac{\sigma-1}{\sigma}\log\varphi_{t+1} - \log E_t(e^{a_{t+1}/\sigma}\varphi_{t+1}^{\frac{\sigma-1}{\sigma}}) \\
&= \underbrace{\frac{a_{t+1}}{\sigma} - \log E_t(e^{a_{t+1}/\sigma})}_{FE^d_{t,t+1}} + \underbrace{\frac{\sigma-1}{\sigma}\log\varphi_{t+1} - \log E_t(\varphi_{t+1}^{\frac{\sigma-1}{\sigma}})}_{FE^s_{t,t+1}} \\
&= \frac{\theta_{t+1} - \mu_{t+1} + \varepsilon_{t+1}}{\sigma} - \frac{\sigma_t^2}{2\sigma^2} + \frac{(\sigma-1)\nu_{t+1}}{\sigma} - \frac{(\sigma-1)^2\sigma^2_{\nu_{t+1}}}{2\sigma^2} \qquad (11) \\
&= \frac{e_{t+1} + \varepsilon_{t+1}}{\sigma} - \frac{\sigma_t^2}{2\sigma^2} + \frac{(\sigma-1)\nu_{t+1}}{\sigma} - \frac{(\sigma-1)^2\sigma^2_{\nu_{t+1}}}{2\sigma^2}, \qquad (12)
\end{aligned}$$

where $\mu_{t+1} \equiv E_t\theta_{t+1}$ is the forecast of θ_{t+1} made in period t and e_{t+1} is the forecast error of θ_{t+1}. The term of σ_t^2 is the variance of forecast errors in period $t+1$. Variable ν_{t+1} and the term of $\sigma^2_{\nu_{t+1}}$ are the productivity innovation and its variance in period $t+1$. Note that both σ_t^2 and $\sigma^2_{\nu_{t+1}}$ are non-stochastic terms and thus uncorrelated over time. Moreover, the productivity innovation is i.i.d. both over time and across firms (and independent of demand innovations), thus we have

$$Cov\left(\frac{(\sigma-1)\nu_{t+1}}{\sigma}, \frac{(\sigma-1)\nu_t}{\sigma}\right) = 0,$$

and

$$Cov\left(FE^{log}_{t-1,t}, FE^{log}_{t,t+1}\right) = Cov\left(\frac{e_t + \varepsilon_t}{\sigma}, \frac{e_{t+1} + \varepsilon_{t+1}}{\sigma}\right) = Cov\left(\frac{a_t - \mu_t}{\sigma}, \frac{e_{t+1} + \varepsilon_{t+1}}{\sigma}\right).$$

Now, we calculate the correlation between the forecast error of θ_{t+1} (i.e., e_{t+1}) and the realized demand shock a_i where $i \in t-n+2, t-n+3, ..., t$ of age-n firms. The case discussed in the subsection is a variant of Muth (1960)'s model. The optimal forecasting rule concerning θ_t follows:

$$\mu_t = (\rho - K_{t-1})\mu_{t-1} + K_{t-1}a_{t-1},$$

where K_{t-1} is is the Kalman gain. Note that $forecast_t$ is the belief formed at the beginning of period t without observing a_t. To be consistent with the notation in earlier sections, we denote forecast for θ_t using μ_t.

Forecast error (FE) for the hidden state variable θ_{t+1} is

$$\begin{aligned} e_{t+1} &= \theta_{t+1} - \mu_{t+1} \\ &= \theta_{t+1} - (\rho - K_t)\mu_t - K_t a_t \\ &= \rho\theta_t + \zeta_{t+1} - (\rho - K_t)\mu_t - K_t(\theta_t + \varepsilon_t) \\ &= (\rho - K_t)e_t + \zeta_{t+1} - K_t\varepsilon_t. \end{aligned}$$

Now, we calculate variance of both sides and denote $\Sigma_t \equiv Var(e_t)$ to obtain

$$\Sigma_{t+1} = (\rho - K_t)^2\Sigma_t + \sigma_\zeta^2 + K_t^2\sigma_\varepsilon^2.$$

Given Σ_t, we can use the first order condition to derive the optimal Kalman gain as

$$K_t = \frac{\rho\Sigma_t}{\Sigma_t + \sigma_\varepsilon^2}.$$

We discuss the correlation of FEs in the steady state (i.e., $t \to \infty$). The two equations that pin down the steady-state Kalman gain and variance of FEs are

$$\begin{aligned} K &= \rho\Sigma/(\Sigma + \sigma_\varepsilon^2) \\ \Sigma &= (\rho - K)^2\Sigma + \sigma_\zeta^2 + K^2\sigma_\varepsilon^2. \end{aligned}$$

We can solve these equations analytically:

$$K = \frac{\sqrt{(1 + \lambda - \rho^2)^2 + 4\rho^2\lambda} - (1 + \lambda - \rho^2)}{2\rho},$$

where $\lambda = \sigma_\zeta^2/\sigma_\varepsilon^2$ is the noise-to-signal ratio.

Now, we prove the key result of this subsection: $cov(e_{t+1}, a_s) = 0$ for any $s \leq t$ in the steady state. Since it is the steady state, we write $K_t = K$. Iterating backwards, one can express μ_{t+1}

(forecast of θ_{t+1} with information prior to $t+1$) as

$$\begin{aligned}\mu_{t+1} &= (\rho - K)\mu_t + K a_t \\ &= K \sum_{j=0}^{\infty} (\rho - K)^j a_{t-j}.\end{aligned}$$

Thus, we have

$$\begin{aligned}e_{t+1} &= \theta_{t+1} - \mu_{t+1} \\ &= \rho\theta_t + \zeta_{t+1} - K \sum_{j=0}^{\infty} (\rho - K)^j a_{t-j}.\end{aligned}$$

Covariance between a_s and e_{t+1} is (for $s \leq t$)

$$Cov(a_s, e_{t+1}) = \rho Cov(a_s, \theta_t) - K \sum_{j=0}^{\infty} (\rho - K)^j Cov(a_{t-j}, a_s).$$

Note that a_s and θ_t can be rewritten as

$$\begin{aligned}\theta_t &= \sum_{j=0}^{\infty} \rho^j \zeta_{t-j} \\ a_s &= \theta_s + \varepsilon_s = \sum_{j=0}^{\infty} \rho^j \zeta_{s-j} + \varepsilon_s.\end{aligned}$$

Therefore, covariance between θ_t and a_s is

$$Cov(\theta_t, a_s) = \rho^{t-s}\sigma_\theta^2,$$

where $\sigma_\theta^2 = \sigma_\zeta^2/(1-\rho^2)$ is the steady-state variance of θ.

For the covariance between a_{t-j} and a_s, there are three cases:

$$\begin{aligned}
Cov(a_{t-j}, a_s) &= Cov(\sum_{m=0}^{\infty} \rho^m \zeta_{s-m} + \varepsilon_s, \sum_{m=0}^{\infty} \rho^m \zeta_{t-j-m} + \varepsilon_{t-j}) \\
&= \begin{cases} \rho^{t-j-s}\sigma_\theta^2 & \text{if } t-j > s \\ \sigma_\theta^2 + \sigma_\varepsilon^2 & \text{if } t-j = s, \\ \rho^{s-(t-j)}\sigma_\theta^2 & \text{if } t-j < s \end{cases}
\end{aligned}$$

where $\sigma_\theta^2 =\equiv \frac{\sigma_\zeta^2}{1-\rho^2}$ is the variance of the demand shocks in the steady state. Adding up each part, we have

$$\begin{aligned}
Cov(a_s, e_{t+1}) &= \rho^{t-s+1}\sigma_\theta^2 - K\sum_{j=0}^{t-s}(\rho-K)^j \rho^{t-j-s}\sigma_\theta^2 \\
&\quad -K(\rho-K)^{t-s}\sigma_\varepsilon^2 - K \sum_{j=t-s+1}^{\infty} (\rho-K)^j \rho^{s-(t-j)}\sigma_\theta^2 \\
&= \rho^{t-s+1}\sigma_\theta^2 - \rho^{t-s+1}\left(1 - \left(\frac{\rho-K}{\rho}\right)^{t-s+1}\right)\sigma_\theta^2 \\
&\quad -K(\rho-K)^{t-s}\sigma_\varepsilon^2 - \frac{\rho K(\rho-K)^{t-s+1}}{1-\rho(\rho-K)}\sigma_\theta^2 \\
&= \frac{(\rho-K)^{t-s+1}}{1-\rho(\rho-K)}\sigma_\zeta^2 - K(\rho-K)^{t-s}\sigma_\varepsilon^2 \\
&= (\rho-K)^{t-s}\sigma_\varepsilon^2\left(\frac{\lambda(\rho-K)}{1-\rho(\rho-K)} - K\right) \\
&= 0.
\end{aligned}$$

Therefore, FE at period $t+1$ is uncorrelated to any variable that has been relied up to period t. In particular, we have

$$\begin{aligned}
&\sigma^2 Cov\left(FE_{t-1,t}^{log}, FE_{t,t+1}^{log}\right) \\
= \;& Cov(a_{t+1} - \mu_{t+1}, a_t - \mu_t) \\
= \;& Cov(e_{t+1} + \varepsilon_{t+1}, a_t - \mu_t) \\
= \;& Cov(e_{t+1}, a_t - K\sum_{j=0}^{\infty}(\rho-K)^j a_{t-1-j}) = 0,
\end{aligned}$$

as $cov(e_{t+1}, a_s) = 0$ for any $s \leq t$ and the transitory shock ε_{t+1} is independent of any shock that has been relied up to period t. Therefore, the (log) forecast errors of sales are serially uncorrelated, even if the demand shock follows an AR(1) process.

3 Additional Quantitative Results

3.1 Details of Calibration by Region

Table OA.17 provides the list of countries in each region analyzed in Section 5.3.3 of the paper. Note that China and United States are not listed here since they are single countries.

Table OA.17: List of countries by region

Region	Countries
Africa	Cote d'Ivoire; Egypt, Arab Rep.; Kenya; Nigeria; South Africa; Swaziland; Tanzania; Tunisia; Zimbabwe;
Middle East	Iran, Islamic Rep.; Israel; Kuwait; Saudi Arabia; United Arab Emirates;
Eastern Europe	Czech Republic; Hungary; Poland; Romania; Russian Federation; Slovak Republic; Slovenia; Ukraine;
Latin America	Argentina; Bolivia; Brazil; Chile; Colombia; Ecuador; El Salvador; Guatemala; Honduras; Mexico; Nicaragua; Peru; Puerto Rico; Trinidad and Tobago; Uruguay; Venezuela, RB;
ASEAN	Brunei Darussalam; Cambodia; Indonesia; Lao PDR; Malaysia; Myanmar; Philippines; Thailand; Vietnam;
Western Europe	Austria; Belgium; Croatia; Denmark; Finland; France; Germany; Greece; Italy; Netherlands; Norway; Portugal; Spain; Sweden; United Kingdom;

Table OA.18 is a longer version of Table 12 in the paper. It presents the model moments together with the data moments that are targeted. It also shows the change in the price indices when we consider perfect information in each region.

Table OA.18: Calibration by region, data and model moments

	Parameters						Data Moments					Model Moments					Gains from Perfect Info.	
Region	σ_θ	σ_ε	$\sigma_\theta^2/\sigma_\varepsilon^2$	σ_{ν_1}	$\sigma_{\nu_{10}}$	f	Cov_1	Cov_{10}	Var_1	Var_{10}	exit rate	Cov_1	Cov_{10}	Var_1	Var_{10}	exit rate	% Δ P	% Δ Q/L
Panel A: change five parameters																		
Africa	0.86	2.57	0.11	0.51	0.37	0.0152	0.040	0.019	0.186	0.100	0.106	0.040	0.020	0.186	0.100	0.105	-7.82	12.56
Middle East	0.83	2.64	0.10	0.58	0.45	0.0142	0.038	0.019	0.226	0.134	0.104	0.038	0.019	0.226	0.134	0.102	-7.81	12.16
Eastern Europe	1.41	1.80	0.62	0.58	0.32	0.0079	0.065	0.014	0.283	0.072	0.103	0.068	0.014	0.283	0.072	0.101	-6.32	9.44
Latin America	1.62	1.66	0.95	0.39	0.39	0.0070	0.069	0.013	0.218	0.097	0.106	0.073	0.013	0.218	0.097	0.103	-3.94	7.13
ASEAN	0.44	1.50	0.09	0.70	0.34	0.0075	0.011	0.006	0.264	0.073	0.078	0.008	0.006	0.264	0.072	0.078	-2.69	3.54
China	1.12	1.48	0.57	0.64	0.31	0.0074	0.044	0.010	0.275	0.065	0.089	0.044	0.010	0.276	0.065	0.089	-4.82	7.16
Western Europe	0.91	1.47	0.39	0.50	0.31	0.0131	0.033	0.009	0.178	0.065	0.107	0.034	0.009	0.179	0.065	0.106	-4.25	6.95
United States	0.78	1.49	0.27	0.52	0.31	0.0147	0.028	0.009	0.180	0.063	0.112	0.028	0.009	0.180	0.063	0.110	-4.49	7.11
Panel B: change four parameters, fix f																		
Africa	0.86	2.57	0.11	0.51	0.37	0.0093	0.040	0.019	0.186	0.100	0.106	0.039	0.020	0.184	0.098	0.073	-5.88	9.77
Middle East	0.83	2.64	0.10	0.58	0.45	0.0093	0.038	0.019	0.226	0.134	0.104	0.037	0.018	0.225	0.130	0.074	-6.11	9.83
Eastern Europe	1.41	1.80	0.62	0.58	0.32	0.0093	0.065	0.014	0.283	0.072	0.103	0.068	0.015	0.281	0.072	0.112	-6.15	9.86
Latin America	1.62	1.66	0.95	0.39	0.39	0.0093	0.069	0.013	0.218	0.097	0.106	0.071	0.014	0.219	0.098	0.125	-4.63	7.84
ASEAN	0.44	1.50	0.09	0.70	0.34	0.0093	0.011	0.006	0.264	0.073	0.078	0.011	0.006	0.268	0.072	0.102	-3.42	3.87
China	1.12	1.48	0.57	0.64	0.31	0.0093	0.044	0.010	0.275	0.065	0.089	0.042	0.010	0.280	0.065	0.104	-4.98	7.70
Western Europe	0.91	1.47	0.39	0.50	0.31	0.0093	0.033	0.009	0.178	0.065	0.107	0.034	0.010	0.177	0.066	0.081	-3.58	5.93
United States	0.78	1.49	0.27	0.52	0.31	0.0093	0.028	0.009	0.180	0.063	0.112	0.029	0.008	0.177	0.063	0.077	-3.27	5.55

Notes: Panel (A) shows the results when we re-calibrate five parameters for each region $(\sigma_\theta, \sigma_\varepsilon, \kappa_1, \kappa_0, f)$. We present age-dependent volatility $\sigma_{\nu_1}, \sigma_{\nu_{10}}$ instead of κ_1, κ_0 to facilitate interpretation. We target five moments in this calibration, $Cov(FE_{n-1,n}, FE_{n,n+1})$ for $n = 1$ and $n \geq 10$, $Var(FE_{n,n+1})$ for $n = 1$ and $n \geq 10$ and incumbent exit rates, respectively. $\%\Delta P$ and $\%\Delta Q/L$ are the percentage changes in the price index and the labor productivity when we change the model from the calibrated imperfect information case to perfect information. Panel (B) reports the results when we re-calibrate the learning and uncertainty related parameters but keep the fixed costs at the baseline value $f = 0.0093$. We target the first four moments but do not attempt to match the exit rates in the data. The model matches the data moments well (other than the untargeted exit rates in Panel B). A full list of countries in each region can be found in Online Appendix Table OA.17.

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