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Inconsistency of 2SLS estimators in threshold regression with endogeneity

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HIGHLIGHTS

- We show the inconsistency of three 2SLS estimators in threshold models with endogeneity.
- The CH estimator is inconsistent when q is endogenous.
- The estimator based on the projector of q on z is inconsistent when q is endogenous.
- The estimator based on a misspecified reduced form is inconsistent even if q is exogenous.

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ABSTRACT

This paper shows the inconsistency of three forms of 2SLS estimators to illustrate the specialty of the endogeneity problem in threshold regression.

1. Introduction

Threshold models have received much popularity in current statistical and econometric practice; see Hansen (2011) for a summary of applications, especially in economics. The usual threshold model splits the sample based on an observed threshold variable q:

$$y = x'\beta_1 1(q \le \gamma) + x'\beta_2 1(q > \gamma) + \varepsilon, \tag{1}$$

where $1(\cdot)$ is the indicator function, β_1 and β_2 are threshold parameters in the two regimes defined by whether q exceeds the threshold point γ or not, and all the other variables have the same definitions as in the linear regression framework. To identify γ, usually the conditional moment restriction $E[\varepsilon|x, q] = 0$ is imposed; see Section 3.3 of Yu (2008) for a discussion of this assumption. However, as in the linear regression case, this model may suffer from the endogeneity problem, i.e., $E[\varepsilon|x, q] = 0$ may not hold; see Kourtellos et al. (2009) for some empirical examples where endogeneity is present.

To cure the endogeneity problem, all the existing literature suggests using instruments. Caner and Hansen (2004) use the two-stage least-squares (2SLS) estimator to identify γ in a model in which only x is endogenous while q is exogenous. As shown later, to obtain consistency of the 2SLS estimator, a *stronger* (than usual) assumption on the reduced-form equation between x and the instruments z must be imposed. In the same framework, except that q may also be endogenous, Kourtellos et al. (2009) consider a structural model with *parametric* assumptions on the data distribution. They then apply the technique in sample selection such as Heckman (1979) to estimate γ consistently. In the related structural change literature, Boldea et al. (2012), Hall et al. (2012), and Perron and Yamamoto (forthcoming-a) all use the 2SLS estimator to estimate the structural break points.

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In a semiparametric framework, this paper presents simple examples to show that three forms of the 2SLS estimator are not consistent in estimating γ. First, when q is endogenous, the 2SLS estimator of Caner and Hansen (2004) is inconsistent. Second, when q is endogenous, the naive 2SLS estimator based on the reduced-form equation between q and instruments z is inconsistent. Third, when q is exogenous, and x may be endogenous, the 2SLS estimator of Caner and Hansen (2004) is inconsistent unless a stronger assumption on the reduced-form equation is satisfied. In the structural change framework, the time index t plays the role of q and it is always exogenous, so our arguments in the third case are also relevant to the structural change literature mentioned above.

These negative results clarify the difference between the endogeneity problem in threshold regression and in the usual linear regression. These results also motivate us to search for a general consistent estimator of γ. Yu (2012) provides such an estimator, called the *integrated difference kernel estimator* (IDKE), which can estimate γ consistently even without instruments.

A word on notation: the letter C is used as a generic positive constant, which need not be the same in each occurrence. f and F with subscripts denote the probability distribution function (pdf) and the cumulative distribution function (cdf) of the corresponding random variables, respectively. $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal pdf and cdf, respectively.

2. Inconsistency of 2SLS estimators

In this section, we use two simple setups of (1) to show that three conventional forms of the 2SLS estimator of γ is not consistent. Our discussion on the first two forms of the 2SLS estimator is based on

$$y = \delta_0 1(q \le \gamma_0) + \varepsilon, \tag{2}$$
$$E[\varepsilon|q] = g(q) \neq 0, \qquad E[\varepsilon] = 0.$$

This model can be interpreted as returns to schooling in the classical literature but with threshold effects. Here, q is a measure of schooling, and y is a measure of earnings. For different schooling levels, the earnings are different, so there is a threshold effect. As usual, q is not exogenous, since ε includes such factors as inborn ability, which is correlated with q. So the threshold variable is endogenous, and the only regressor 1 is exogenous. In Section 2.1, we show that the least-squares estimator (LSE) of γ is inconsistent, which implies that the 2SLS procedure of Caner and Hansen (2004) is not valid when q is endogenous. Section 2.2 shows that the usual 2SLS procedure by projecting q on instruments cannot generate a consistent estimator of γ either. Finally, in Section 2.3, we show that the 2SLS procedure of Caner and Hansen (2004) is invalid even if q is exogenous. To illustrate this point, we assume that

$$y = x1(q \le \gamma_0) + \varepsilon, \tag{3}$$
$$E[\varepsilon|x] \neq 0, \qquad E[\varepsilon|q] = 0.$$

Since q is a valid instrument, the number of instruments is the same as the number of endogenous variables, and no extra instruments are required. When the reduced-form relationship between x and q is misspecified, the 2SLS estimator of γ is not consistent.

2.1. Inconsistency of the LSE

In model (2), $E[y|q] = \delta_0 1(q \le \gamma_0) + g(q)$, which is different from $\delta_0 1(q \le \gamma_0)$, so the LSE based on the objective function

$$\frac{1}{n}\sum_{i=1}^{n}(y_i - \delta 1(q \le \gamma))^2$$

cannot be consistent. To be specific, assume that the joint distribution of (q, ε) is $N\left(\mathbf{0}, \begin{pmatrix} 1 & \rho_0 \\ \rho_0 & 1 \end{pmatrix}\right)$ with $\rho_0 \neq 0$; then $E[e|q] = \rho_0 q$.

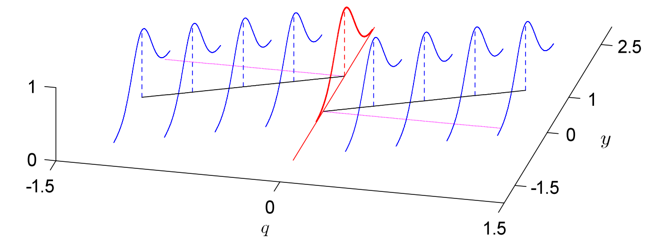

Fig. 1. Joint distribution of (y, q) in threshold regression with endogeneity: the black line is the true $E[y|q]$, and the magenta line is $E[y|q]$ in the original specification. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Now, $E[y|q] = \delta_0 1(q \le \gamma_0) + \rho_0 q$, and

$$y = \begin{cases} \delta_0 + \rho_0 q + \sqrt{1-\rho_0^2}\,u, & \text{if } q \le \gamma_0, \\ \rho_0 q + \sqrt{1-\rho_0^2}\,u, & \text{if } q > \gamma_0, \end{cases}$$

where u is independent of q and follows a standard normal distribution. Compared to the exogenous case, where $\rho_0 = 0$, an extra function $\rho_0 q$ is added to the mean function. Also, the error variance is reduced. The joint distribution of q and y is shown in Fig. 1, where $\rho_0 = 0.9$, $\gamma_0 = 0$, and $\delta_0 = 1$.

The inconsistency of $\widehat{\delta}_{LSE}$ is a standard result in the literature. To separate out the effect of $\widehat{\gamma}_{LSE}$, suppose that γ_0 is known. Then

$$\begin{aligned} &\frac{1}{n}\sum_{i=1}^{n}(y_i - \delta 1(q \le \gamma_0))^2 \\ &= \frac{1}{n}\sum_{i=1}^{n}\left(\delta_0 1(q \le \gamma_0) + \rho_0 q + \sqrt{1-\rho_0^2}\,u - \delta 1(q \le \gamma_0)\right)^2 \\ &\to C + (\delta_0 - \delta)^2 P(q \le \gamma_0) + 2(\delta_0 - \delta)\rho_0 E[q1(q \le \gamma_0)]. \end{aligned}$$

The first-order condition for δ is $2(\delta - \delta_0)P(q \le \gamma_0) = 2\rho_0 E[q1(q \le \gamma_0)]$, i.e., $2(\delta - \delta_0)\Phi(\gamma_0) = -2\rho_0\phi(\gamma_0)$, so $\widehat{\delta}_{LSE}$ converges to $\delta_0 - 2\rho_0\lambda(\gamma_0) \neq \delta_0$, where $\lambda(\gamma_0) = \phi(\gamma_0)/\Phi(\gamma_0)$ is the inverse Mill's ratio. This limit is equal to δ_0 only if $\rho_0 = 0$; the larger $|\rho_0|$ is, the larger the bias is.

$g(q)$ affects the estimation of γ only indirectly. For simplicity, suppose that $\delta_0 = 1$ is known. It is easy to show that

$$\begin{aligned} &\frac{1}{n}\sum_{i=1}^{n}(y_i - \delta_0 1(q \le \gamma))^2 \\ &= \frac{1}{n}\sum_{i=1}^{n}\left(\delta_0 1(q \le \gamma_0) + \rho_0 q + \sqrt{1-\rho_0^2}\,u - \delta_0 1(q \le \gamma)\right)^2 \\ &\to C + E\left[(\delta_0 1(q \le \gamma_0) + \rho_0 q - \delta_0 1(q \le \gamma))^2\right] \\ &= C + E\left[(E[y|q] - \delta_0 1(q \le \gamma))^2\right]. \end{aligned}$$

So $\widehat{\gamma}_{LSE}$ converges to the γ value such that the L^2 distance between the true $E[y|q]$ and its original specification $\delta_0 1(q \le \gamma)$ reaches its minimum. In Fig. 1, we choose a location of the jump in the magenta line to minimize its L^2 distance to the black line. In the specification above,

$$\begin{aligned} &E\left[(E[y|q] - \delta_0 1(q \le \gamma))^2\right] \\ &= \begin{cases} \rho_0^2 + 2\rho_0(\phi(\gamma_0) - \phi(\gamma)) + (\Phi(\gamma_0) - \Phi(\gamma)) \\ \quad \text{if } \gamma < \gamma_0; \\ \rho_0^2 + 2\rho_0(\phi(\gamma) - \phi(\gamma_0)) + (\Phi(\gamma) - \Phi(\gamma_0)) \\ \quad \text{if } \gamma \ge \gamma_0. \end{cases} \end{aligned}$$

The corresponding minimizer γ of $E\left[(E[y|q] - \delta_0 1(q \le \gamma))^2\right]$ as a function of γ_0 and ρ_0 is shown in Fig. 2. Note that the minimizer is not a smooth function of γ_0, since $E\left[(E[y|q] - \delta_0 1(q \le \gamma))^2\right]$ is not smooth in γ and γ_0. From Fig. 2, $\widehat{\gamma}_{LSE}$ is obviously inconsistent as long as $\rho_0 \neq 0$. The larger ρ_0 is, the more significant the

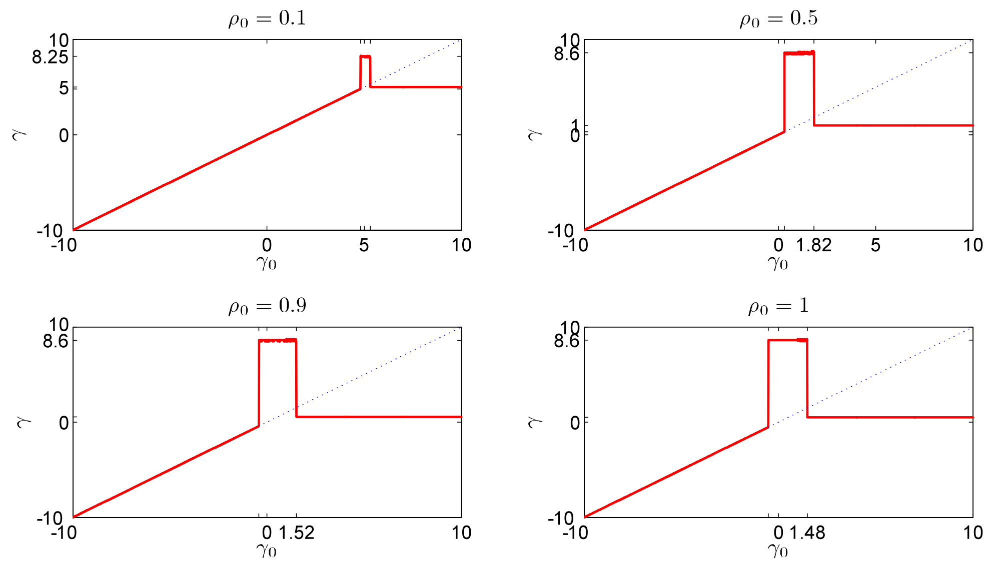

Fig. 2. Limit of $\widehat{\gamma}_{LSE}$ as a function of γ_0 and ρ_0.

inconsistency is: the case with $\rho_0 = 1$ is the limit case illustrating the most possible inconsistency; when $\rho_0 = 0$, the inconsistency disappears.

An immediate corollary of the discussion above is that the 2SLS estimator of Caner and Hansen (2004) is inconsistent. Suppose that we have an instrument z in hand; then Caner and Hansen (2004) project the only regressor $x = 1$ on the instruments $(1, z)$ and the resulting projector is 1, so the 2SLS estimator is the same as the LSE and is inconsistent. In the general case (1) with $E[e|q] \neq 0$, we regress x on the instruments z to obtain the predictor $\widehat{x}$, and then minimize $\sum_{i=1}^{n} \left(y_i - \widehat{x}_i'\beta_1 1(q_i \leq \gamma) - \widehat{x}_i'\beta_2 1(q_i > \gamma)\right)^2$ to estimate γ. The insight in the simple example above can be applied here: this 2SLS estimator is not consistent, since the endogeneity of q is not considered in the estimating procedure.

2.2. Inconsistency of the 2SLS estimator when q is endogenous

Suppose that there exists a valid instrument z such that $E[\varepsilon|z] = 0$ and $\mathrm{Cov}(z, q) \neq 0$. To simplify our discussion, let $\delta_0 = 1$ be known, and normalize $E[\varepsilon] = E[q] = E[z] = 0$ and $E[\varepsilon^2] = E[q^2] = E[z^2] = 1$. We can estimate γ by minimizing $\frac{1}{n}\sum_{i=1}^{n}(y_i - 1(z_i\widehat{\pi} \leq \gamma))^2$, where $\widehat{\pi}$ is from the first-step regression $q_i = z_i\pi + v_i$. Note that $\pi_0 = \mathrm{Cov}(z, q) \neq 0$. By some preliminary calculations,[1] we can show that

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^{n}(y_i - 1(z_i\widehat{\pi} \leq \gamma))^2 &\to E\left[(y - 1(z\pi_0 \leq \gamma))^2\right]\\ &= E\left[(1(q \leq \gamma_0) + \varepsilon - 1(z\pi_0 \leq \gamma))^2\right]\\ &= C + E[(1(q \leq \gamma_0) - 1(z\pi_0 \leq \gamma))^2]\\ &= C + P(z \leq \gamma/\pi_0) - 2\int_{-\infty}^{\gamma/\pi_0} F_{q|z}(\gamma_0|z)f(z)dz. \qquad (4)\end{aligned}$$

If we estimate γ using least-squares, then we can show that

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^{n}(y_i - 1(q_i \leq \gamma))^2 &\to C + E[(1(q \leq \gamma_0) - 1(q \leq \gamma))^2] - 2E[\varepsilon 1(q \leq \gamma)]\\ &= C + P(q \leq \gamma) - 2F_q(\gamma_0 \wedge \gamma) - 2E[g(q)1(q \leq \gamma)]. \qquad (5)\end{aligned}$$

If there is no endogeneity, then $E[g(q)1(q \leq \gamma)] = 0$, and the minimizer of $P(q \leq \gamma) - 2F_q(\gamma_0 \wedge \gamma)$ is γ_0. Comparing (4) and (5), we can see that the instrument z indeed controls the bias induced by the correlation between q and ε (the corresponding term of $E[1(q \leq \gamma)\varepsilon]$ in (4) is $E[1(z\pi_0 \leq \gamma)\varepsilon] = 0$), but meanwhile it introduces extra bias by changing $1(q \leq \gamma)$ to $1(z\pi_0 \leq \gamma)$. As a result, the minimizer of $P(z \leq \gamma/\pi_0) - 2\int_{-\infty}^{\gamma/\pi_0} F_{q|z}(\gamma_0|z)f(z)dz$ may not be γ_0. Actually, from (4), the probability limit of the 2SLS estimator is the same as replacing q by $z\pi_0$ in the least-squares estimation of the simplest threshold model $y = 1(q \leq \gamma)$. From the first-order condition of (4), $\frac{f_z(\gamma/\pi_0)}{\pi_0} - 2F_{q|z}(\gamma_0|\gamma/\pi_0)\frac{f_z(\gamma/\pi_0)}{\pi_0} = 0$, so the 2SLS estimator converges to γ such that $F_{q|z}(\gamma_0|\gamma/\pi_0) = 1/2$, which is the γ value of $z\pi_0$ such that γ_0 is the median of the conditional distribution $F_{q|z}$.[2] Of course, such a γ value need not be γ_0.

To be more specific, consider the following two examples of f_{qz}. First, suppose that z and q are jointly normal; then the conditional distribution of q given z is $N(\pi_0 z, 1 - \pi_0^2)$. Now, setting $F_{q|z}(\gamma_0|\gamma/\pi_0) = \Phi\left(\frac{\gamma_0-\gamma}{\sqrt{1-\pi_0^2}}\right) = \frac{1}{2}$, we have $\gamma = \gamma_0$ for any $|\pi_0| \in (0, 1)$. Second, suppose that $f_{qz}(q, z) = \frac{1}{12}\left(1 + \frac{zq}{3}\right), z, q \in [-\sqrt{3}, \sqrt{3}]$; then both q and z have marginal uniform distributions on $[-\sqrt{3}, \sqrt{3}]$, $\pi_0 = \frac{1}{3}$ and $F_{q|z}(q|z) = \frac{zq^2}{12\sqrt{3}} - \frac{z}{4\sqrt{3}} + \frac{q}{2\sqrt{3}} + \frac{1}{2}$. Solving $F_{q|z}(\gamma_0|\gamma/\pi_0) = \frac{\gamma_0^2\gamma/\pi_0}{12\sqrt{3}} - \frac{\gamma/\pi_0}{4\sqrt{3}} + \frac{\gamma_0}{2\sqrt{3}} + \frac{1}{2} = \frac{1}{2}$, we have $\gamma = \frac{2\gamma_0}{3-\gamma_0^2}$.[3] $\gamma = \gamma_0$ if and only if $\gamma_0 = 0$ and ± 1. Fig. 3 illustrates the calculation above intuitively. Although the asymptotic bias of the 2SLS estimator seems smaller than that in Section 2.1 (given that part of the endogeneity in q is controlled by z), it is not completely eliminated.

When δ_0 is unknown, there is no explicit-form solution for the limit of $\widehat{\gamma}_{2SLS}$. In this case, $\widehat{\gamma}_{2SLS}$ is not consistent even when q and z are jointly normal. Also, the inconsistency of $\widehat{\gamma}_{2SLS}$ will contaminate the consistency of regular parameters; that is, $\widehat{\delta}_{2SLS}$ will not be

[1] The estimation of π_0 may affect the asymptotic distribution of $\widehat{\gamma}_{2SLS}$ but will not affect its probability limit. This is similar to the case in the usual 2SLS estimation.

[2] In the more general case $y = x'\delta_0 1(q \leq \gamma_0) + \varepsilon$, this solution is more complicated. For example, suppose that δ_0 is known and only q is endogenous; then $\widehat{\gamma}_{2SLS}$ converges to the solution of $\frac{\delta_0' E[xx' 1(q\leq\gamma_0)|z'\pi_0=\gamma]\delta_0}{\delta_0' E[xx'|z'\pi_0=\gamma]\delta_0} = \frac{1}{2}$, where z includes x and some extra instruments, and π_0 is the true value in the regression $q = z'\pi + v$.

[3] γ_0 cannot be $\sqrt{3}$, which is the usual assumption that the threshold point cannot stay on the boundary of the threshold variable.

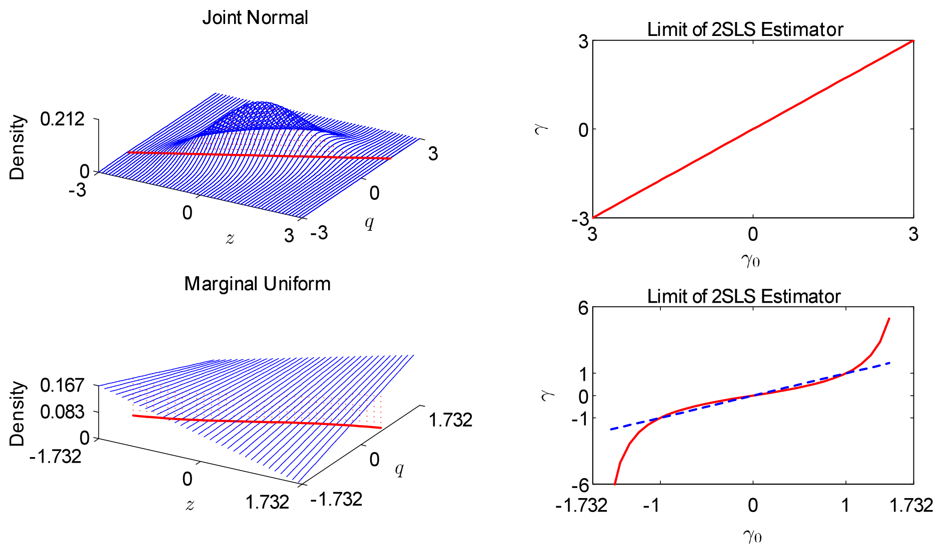

Fig. 3. Limits of $\widehat{\gamma}_{2SLS}$ associated with different $f_{z,q}$: $\pi_0 = 0.5$ in the joint normal case; the red solid lines in the left two graphs are the locations of the conditional median of $f_{q|z}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

consistent either. In the general case (1) with $E[e|q] \neq 0$, we regress $(x', q)'$ on the instruments z to obtain the predictor $(\widehat{x}', \widehat{q})'$, and then minimize $\sum_{i=1}^n \left(y_i - \widehat{x}_i'\beta_1 1(\widehat{q}_i \le \gamma) - \widehat{x}_i'\beta_2 1(\widehat{q}_i > \gamma)\right)^2$ to estimate γ. The insights in the simple examples above can be applied here: this 2SLS estimator is not consistent, since asymptotically the threshold variable is changed from q to the probability limit of $\widehat{q}$, say, $z'\pi_0$.

2.3. Inconsistency of the 2SLS estimator when q is exogenous

Consider (3). Denoting the linear projector of x on q as $q\widehat{\pi}$, then

$$\widehat{\gamma}_{CH} = \arg\min_{\gamma} (y_i - q_i\widehat{\pi} 1(q_i \le \gamma))^2 .$$

If $E[x|q]$ is a linear function of q, say $q\pi_0$, then the objective function of $\widehat{\gamma}_{CH}$ converges to

$$C + E\left[(q\pi_0 1(q \le \gamma) - q\pi_0 1(q \le \gamma_0))^2\right],$$

which is minimized at $\gamma = \gamma_0$. If $E[x|q]$ is a nonlinear function of q, but we mistakenly project x on the linear space spanned by q, then the 2SLS estimator may be inconsistent. To be concrete, suppose that $E[x|q] = q^3$ and $q \sim N(0, 1)$; then the linear projection of x on q is $3q$. Now $\widehat{\gamma}_{CH}$ converges to the minimizer of

$$\begin{aligned} &E\left[\left(q^3 1(q \le \gamma_0) - 3q1(q \le \gamma)\right)^2\right] \\ &\quad = \left[15\Phi(\gamma_0) - \left(\gamma_0^5 + 5\gamma_0^3 + 15\gamma_0\right)\phi(\gamma_0)\right] \\ &\qquad - 6\left[3\Phi(\gamma \wedge \gamma_0) - \left((\gamma \wedge \gamma_0)^3 + 3(\gamma \wedge \gamma_0)\right)\phi(\gamma \wedge \gamma_0)\right] \\ &\qquad + 9\left[\Phi(\gamma) - \gamma\phi(\gamma)\right]. \end{aligned}$$

The left panel of Fig. 4 shows the limit of $\widehat{\gamma}_{CH}$ as a function of γ_0. Obviously, the 2SLS estimator is inconsistent. To understand why the limit of $\widehat{\gamma}_{CH}$ is not always γ_0, we provide further intuition in the right panel of Fig. 4. Let $\gamma_0 = 0$; then obviously the area between $q^3 1(q \le 0)$ and $3q1(q \le -1.22)$ is smaller than that between $q^3 1(q \le 0)$ and $3q1(q \le 0)$. Here, note that -1.22 is the point where $|q^3 - 3q| = |q^3|$, and, when $-1.22 < q < 0$, $|q^3 - 3q| > |q^3|$.

The consistency of the 2SLS estimator in Caner and Hansen (2004) critically relies on the assumption that $E[x|q]$ is known up to finite parameters. However, such an assumption does not necessarily hold in the usual 2SLS procedure to get a consistent estimator. Recall that, for $y = x'\beta_0 + \varepsilon$ with $E[x\varepsilon] \neq 0$, the 2SLS procedure just linearly projects x on instruments z without assuming $E[x|z]$ to be linear in z. To achieve consistency of the 2SLS estimator in threshold regression, we need to consistently estimate $E[x|q]$ even in this special case that q is exogenous. Of course, consistent estimation of $E[x|q]$ must involve some nonparametric techniques, which will complicate the 2SLS procedure of Caner and Hansen (2004).

To examine why we must estimate $E[x|q]$ to achieve consistency of the 2SLS estimator, consider the general model (1). Note that γ_0 is the location where the *conditional mean* of y given x, rather than the *projection* of y on x, changes. This point is clearly illustrated in the counter-example of Section 3.3 of Yu (2008). When there is endogeneity, the conditional mean of y given x still changes at γ_0 as long as $E[\varepsilon|x]$ is continuous, which is the key insight in the IDKE. In other words,

$$\gamma_0 = \arg\min_{\gamma} E[(y - E[y|x, q]1(q \le \gamma) - E[y|x, q]1(q > \gamma))^2].$$

However, γ_0 need not equal $\arg\min_\gamma E[(y - x'\beta_1 1(q \le \gamma) - x'\beta_2 1(q > \gamma))^2]$, i.e., the LSE of γ is not consistent.[4] In the 2SLS estimation, γ_0 is the location where the *conditional mean* of y given z, rather than the *projection* of y on z, changes. To understand this point, taking the conditional mean given z (which includes q as a component) on both sides of (1), we have

$$E[y|z] = E[x|z]'\beta_1 1(q \le \gamma) + E[x|z]'\beta_2 1(q > \gamma),$$

where $E[\varepsilon|z] = 0$ is required. In other words,

$$\gamma_0 = \arg\min_{\gamma} E[(y - E[x|z]'\beta_1 1(q \le \gamma) + E[x|z]'\beta_2 1(q > \gamma))^2],$$

so minimizing $n^{-1}\sum_{i=1}^n (y_i - \widehat{E}[x_i|z_i]'\beta_1 1(q \le \gamma) - \widehat{E}[x_i|z_i]'\beta_2 1(q_i > \gamma))^2$ will generate a consistent estimator of γ_0, where $\widehat{E}[x_i|z_i]$ is a nonparametric estimator of $E[x_i|z_i]$.[5] However, γ_0 need not equal $\arg\min_\gamma E[(y - \mathbb{P}(x|z)'\beta_1 1(q \le \gamma) - \mathbb{P}(x|z)'\beta_2 1(q > \gamma))^2]$ if $E[x|z] \neq \mathbb{P}(x|z)$, where $\mathbb{P}(x|z)$ is the linear projection of x on z, i.e., $\widehat{\gamma}_{CH}$ is not consistent. Assume that $\mathbb{P}(x|z) = \Pi z$. Then there may even be an identification problem in the 2SLS estimation if $\Pi'\beta_1 = \Pi'\beta_2$, i.e., there is no structural change at all in the projection problem.

[4] The LSE of Perron and Yamamoto (forthcoming-b) is generally inconsistent. This result is also observed by Hall et al. (2012).

[5] This estimator is based on the global fitting of the conditional mean of y. However, as argued in Yu (2012), the asymptotic distribution of $\widehat{\gamma}_{CH}$ based on such an objective function is hard to derive. As an alternative, the IDKE proposed in Yu (2012) is based only on the local information around γ, and its asymptotic distribution is much easier to obtain.

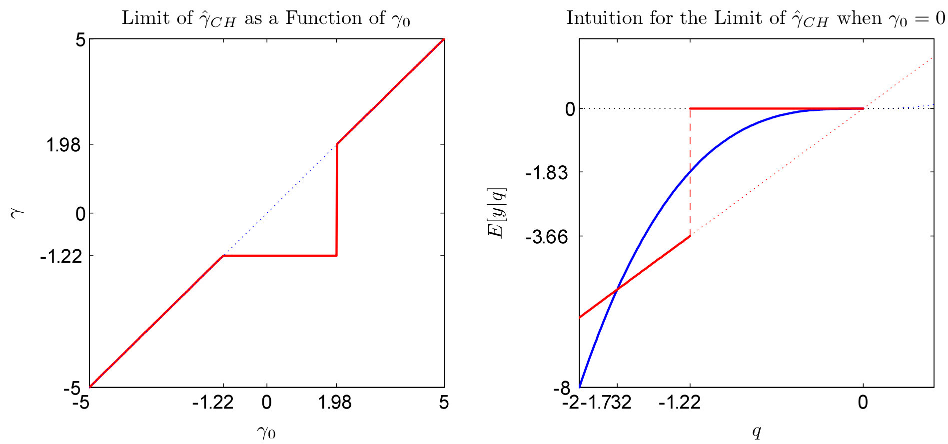

Fig. 4. Limit of $\widehat{\gamma}_{CH}$ as a function of γ_0 and intuition for the limit of $\widehat{\gamma}_{CH}$ when $\gamma_0 = 0$.

In linear regression with endogeneity, $y = x'\beta_0 + \varepsilon$ with $E[x\varepsilon] \neq 0$, the 2SLS procedure utilizes the linear structure of the model. To understand this point, projecting both sides of $y = x'\beta_0 + \varepsilon$ on z, we have $\mathbb{P}(y|z) = \mathbb{P}(x|z)'\beta_0$, so the 2SLS estimator is consistent, and only $\mathbb{P}(\varepsilon|z) = 0$ is required. However, in threshold regression, if we project both sides of (1) on z, we have $\mathbb{P}(y|z) = \mathbb{P}(x1(q \le \gamma)|z)'\beta_1 + \mathbb{P}(x1(q > \gamma)|z)'\beta_2$, which is generally not equal to $\mathbb{P}(x|z)'\beta_1 1(q \le \gamma) + \mathbb{P}(x|z)'\beta_2 1(q > \gamma)$. In other words, the nonlinear structure (in γ) of threshold regression invalidates the usual 2SLS procedure.

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